**Review of Functions**

1. **Functions and their graphs**

1) Find the domain and range of the following functions. Where is the function increasing, decreasing or constant? (Note that the ends go off to infinity even though there are no arrows. This is because graphing software does not put arrows.)

\[ f(x) = x^2 \]

![Graph of \( f(x) = x^2 \)]

\[ f(x) = x^2 + 10 \]

![Graph of \( f(x) = x^2 + 10 \)]

\[ f(x) = x^3 + 3x^2 + 5x + 1 \]

![Graph of \( f(x) = x^3 + 3x^2 + 5x + 1 \)]

\[ f(x) = 3x^4 - 6x^2 \]

![Graph of \( f(x) = 3x^4 - 6x^2 \)]

\[ f(x) = -x^2 + 4 \]

![Graph of \( f(x) = -x^2 + 4 \)]

\[ f(x) = |x - 1| \]

![Graph of \( f(x) = |x - 1| \)]

\[ f(x) = \frac{|x|}{x} \]

![Graph of \( f(x) = \frac{|x|}{x} \)]

\[ f(x) = \sqrt{25 - x^2} \]

![Graph of \( f(x) = \sqrt{25 - x^2} \)]
2) Sketch the graphs of the parabolas using transformations. Mark the $x$ and $y$ intercepts, vertex, axis of symmetry. What is the domain and the range.

a) $f(x) = (x-2)^2 + 4$

b) $f(x) = -(x + 3)^2 - 5$

c) $f(x) = (x + 3)^3 - 1$

d) $f(x) = \sqrt{x} + 8$

e) $f(x) = -\sqrt{x}$

f) $f(x) = \sqrt{-x}$

g) $f(x) = |x - 5| + 2$

3) For each of the following pairs of functions, find $f + g$, $f - g$, $fg$, $\frac{f}{g}$, $f \circ g$, and $g \circ f$.

a) $f(x) = 3x$ and $g(x) = 5x + 10$

b) $f(x) = x^2$ and $g(x) = x - 1$

c) $f(x) = |x|$ and $g(x) = x + 6$
2. Polynomial Functions

1) Solve the polynomial equations. Find real and complex roots.
   a) \( x^3 - 9x = 0 \)
   b) \( x^3 + 5x^2 + 6x + 30 = 0 \)
   c) \( x^3 + 12x^2 - 3x - 36 = 0 \)
   d) \( x^3 + 3x^2 + 3x + 1 = 0 \)
   e) \( x^4 + 5x^2 - 6 = 0 \)
   f) \( x^4 + 11x^2 + 24 = 0 \)
   g) \( x^3 - 9x^2 + 27x - 27 = 0 \)
   h) \( x^5 + 12x^4 - 3x^3 - 36x^2 = 0 \)
   i) \( x^3 + 125 = 0 \)
   j) \( x^4 - 16 = 0 \)

2) Use the rational root test to solve the following equations, if possible.
   a) \( x^3 + x^2 - 5x + 3 = 0 \)
   b) \( -2x^3 + x^2 + 1 = 0 \)
   c) \( x^3 - 6x^2 + 11x - 6 = 0 \)
   d) \( x^3 - 7x^2 - 6 = 0 \)

3) Graph the following functions. Mark the \( x \) and \( y \)-intercepts.
   a) \( y = x^3 - 4x \)
   b) \( y = x^3 - 9x \)
   c) \( y = -x^3 + 9x \)
   d) \( y = x^3 + 5x^2 \)
   e) \( y = -x^3 - 5x^2 \)
   f) \( y = x^4 - 16x^2 \)
   g) \( y = x^5 - 5x^3 + 4x \)
   h) \( y = x^3 + 12x^2 - 3x - 36 \)
3. Rational Functions

1) Graph the following functions using transformations. Mark the $x$ and $y$-intercepts and the horizontal and vertical asymptotes. State the domain and range.

   a) $f(x) = \frac{1}{x}$
   b) $f(x) = \frac{1}{x^2}$
   c) $f(x) = \frac{1}{x} + 2$
   d) $f(x) = \frac{1}{x} - 2$
   e) $f(x) = \frac{1}{x-2}$
   f) $f(x) = \frac{1}{x-4} + 3$
   g) $f(x) = \frac{1}{(x-1)^2}$
   h) $f(x) = -\frac{1}{x}$

2) Determine the vertical asymptotes.

   a) $f(x) = \frac{4}{x + 10}$
   b) $f(x) = \frac{x + 7}{2 - x}$
   c) $f(x) = \frac{x^4 + 2}{x}$
   d) $f(x) = \frac{x^2 + 4}{x(x + 5)(x - 2)}$
   e) $f(x) = \frac{x^2}{2x^2 - x - 3}$
   f) $f(x) = \frac{x + 5}{x^2 + 4x - 32}$

3) Determine the horizontal asymptote, if it exists.

   a) $f(x) = \frac{2x}{3x^2 + 1}$
   b) $f(x) = \frac{2x^2}{3x^2 + 1}$
   c) $f(x) = \frac{3x^2 + 5}{4x^2 - 3}$
   d) $f(x) = \frac{x + 6}{x^3 + 2x^2}$
   e) $f(x) = \frac{x^5}{x^3 + x}$
   f) $f(x) = \frac{x^3 - 2x^2 + x - 1}{x^2 - 16}$

4) Graph the following functions. Mark the $x$ and $y$-intercepts and the horizontal and vertical asymptotes. State the domain and range.

   a) $f(x) = \frac{2x + 1}{x + 1}$
   b) $f(x) = \frac{4x + 8}{2x - 6}$
   c) $f(x) = \frac{x}{x^2 - x - 2}$
   d) $f(x) = \frac{2x + 3}{3x^2 + 7x - 6}$
5) For each of the functions given below, use the graph to identify where \( f(x) \geq 0 \) and \( f(x) \leq 0 \)

a) \( f(x) = x^3 + 6x^2 - x - 30 \)

\[ a = -3, b = 0 \]

b) \( f(x) = x^4 - 27x^2 - 14x + 120 \)

\[ a = -3, b = 0 \]

c) \( f(x) = \frac{8x}{x^2 - 4} \)

\[ a = 0, b = 0 \]

d) \( f(x) = \frac{8}{x^2 - 4} \)

\[ a = 0, b = 0 \]
4. Exponential and Log Functions

1) Sketch the graphs of the following functions. What is the domain and range? Mark x and y intercepts.
   a) \( f(x) = 2^x \)
   b) \( f(x) = 10^x \)
   c) \( f(x) = e^x \)

2) Use transformations to sketch the following functions. Mark x and y intercepts. What is the domain and range?
   a) \( f(x) = -2^x \)
   b) \( f(x) = 2^{-x} \)
   c) \( f(x) = e^{-x} + 1 \)
   d) \( f(x) = 10^x - 3 \)
   e) \( f(x) = 3^{(x-1)} \)
   f) \( f(x) = -2^x \)
   g) \( f(x) = e^{-x} \)
   a) \( f(x) = \log_2 x \)
   b) \( f(x) = \log_{10} x \)
   c) \( f(x) = \log_5 x \)
   d) \( f(x) = \log_3 (-x) \)
   e) \( f(x) = -\log_2 x \)
   f) \( f(x) = \log_3 (x-1) \)
   g) \( f(x) = \log_5 (x-3) \)

3) Solve the exponential equations.
   a) \( 2^x = 32 \)
   b) \( 10^x = 570 \)
   c) \( 2^x = 7 \)
   d) \( 3e^x = 5 \)
   e) \( 4^{x+3} = 7^x \)
   f) \( 2^{x+1} = e^x \)

4) Solve the log equations.
   a) \( \log_2 x = 5 \)
   b) \( 2\log_{10} 5x = 4 \)
   c) \( \log_{10}(2x + 1) - \log_{10}(x + 1) = 1 \)
   d) \( \log_{10}(4x+2) - \log_{10}(x-1) = 1 \)
   e) \( \ln(x-2) + \ln(2x-3) = 2 \ln x \)
5. Trigonometric functions

1) Draw the graphs over the specified intervals. Mark the x-intercepts and asymptotes (if any).
   a) $y = \sin x$
   b) $y = \cos x$
   c) $y = \tan x$
   d) $y = \csc x$
   e) $y = \sec x$
   f) $y = \cot x$

2) Sketch the graphs. State the domain, range, amplitude, period, and phase shift, if relevant.
   a) $y = 2 \sin x$
   b) $y = -\cos x$
   c) $y = -\tan x$
   d) $y = 3 \sin 2x$
   e) $y = \cos \frac{x}{4}$
   f) $y = \tan 2x$
   g) $y = \sin(x - \frac{\pi}{2})$
   h) $y = \cos(x + \frac{\pi}{2})$
   i) $y = \tan(x - \pi)$
   j) $y = \sin(2x - \frac{\pi}{2})$
   k) $y = \cos(2x + \pi)$
   l) $y = \tan(2x - \pi)$