ONE-DIMENSIONAL COLLISIONS

Purpose
In this lab we will study conservation of energy and linear momentum in both elastic and perfectly inelastic one-dimensional collisions. To do this, we will consider two frictionless gliders moving on an air track and measure the velocities of the gliders before and after the collision. The velocities measured in these experiments after the collision will be compared with the corresponding theoretical predictions.

Apparatus
1. Air track with blower.
3. Two gliders, a sound reflector, a pair of needle and wax catcher, and an elastic bumper.
4. Timer with photogate.
5. Scale and four 50 g masses.

Discussion and Theoretical Background
- Linear momentum conservation: One of the most important implications of Newton’s Laws is the so called “Theorem of Linear Momentum Conservation” (TLMC). For a single particle of mass \( m \) and velocity \( v \), the linear momentum is defined as \( p = m \cdot v \) (both \( p \) and \( v \) are vectors). For a system of \( n \) masses \( m_1, m_2, \ldots m_n \) with velocities \( v_1, v_2, \ldots v_n \), the linear momentum of the system is defined as \( p_{sys} = p_1 + p_2 + \ldots + p_n = m_1 \cdot v_1 + m_2 \cdot v_2 + \ldots + m_n \cdot v_n \). The TLMC states that “if the sum of all forces acting on a system of particles is zero, then \( p_{sys} \) is constant” (i.e., \( p_{sys} \) is conserved).

In our experiments, we will study the collision of two gliders moving on an air track. Before the collision, glider number 1 will have an initial velocity \( v_{1i} \) while glider number 2 will be at rest (i.e., \( v_{2i} = 0 \)). After the collision, the gliders will have final velocities \( v_{1f} \) and \( v_{2f} \), respectively:

![Before collision](image1)

During the collision, since there is no friction between the gliders and the air track, the sum of all forces acting on the particles is null. Therefore, from the TLMC, \( p_{sys} \) must be conserved, i.e.,

\[
\text{“} p_{sys} \text{ before collision} = p_{sys} \text{ after collision”}.
\]

This equation can be rewritten as (remember that \( v_{2i} = 0 \)):

\[
m_1 \cdot v_{1i} = m_1 \cdot v_{1f} + m_2 \cdot v_{2f}
\]
• Kinetic Energy:
The kinetic energy of a particle of mass \(m\) and velocity \(v\) is defined as \(KE = \frac{1}{2} m v^2\) (note that \(KE\) and \(v^2\) are numbers). The kinetic energy of a system of \(n\) masses \(m_1, m_2, \ldots, m_n\) with velocities \(v_1, v_2, \ldots, v_n\), is defined as \(KE_{sys} = KE_1 + KE_2 + \ldots + KE_n = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \ldots + \frac{1}{2} m_n v_n^2\).

For the case of two gliders of masses \(m_1\) and \(m_2\), this expression can be rewritten as

\[
KE_{sys} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2
\]  

(2)

• Velocity equations for elastic and inelastic collisions between two gliders:

Let us consider two gliders of masses \(m_1\) and \(m_2\) in a perfectly inelastic collision. By definition, in this kind of collision, the gliders stick together after they collide; \(KE_{sys}\) is not conserved. Thus, in this case, both gliders have the same final velocity: \(v_{1f} = v_{2f} = v_f\). Combining this expression with eqn. (1), we get:

\[
v_f = m_1 v_{1i} / (m_1 + m_2)
\]  

(A)  

[perfectly inelastic collision]

By definition, in an elastic collision, \(KE_{sys}\) is conserved, i.e. “\(KE_{sys}\) before collision = \(KE_{sys}\) after collision”. Since in our experiments glider # 2 will be initially at rest (i.e., \(v_{2i} = 0\)), eqn. (2) implies that,

\[
\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]  

(3)

Solving eqns. (1) and (3) simultaneously, we get [elastic collision]:

\[
v_{1f} = (m_1 - m_2) v_{1i} / (m_1 + m_2)
\]  

(B) and

\[
v_{2f} = 2 m_1 v_{1i} / (m_1 + m_2)
\]  

(C)

Procedure
We will study perfectly inelastic collisions in Part I and test the validity of eqn. (A). Elastic collisions will be studied in Part II and eqns. (B) and (C) will be tested. In all experiments, glider #2 (the ‘target’) will be initially at rest at approximately the 100 cm position on the air rack; glider #1 (the ‘projectile’) will be set in motion by hand at about the 20 cm position on the rack.

We will use the computer to measure the velocity of glider #1 before and after the collision. The velocity of glider #2, for Part II only, will be measured using the timer with the photogate.

Part I: Perfectly inelastic collisions

1) Mount the catcher to the bottom hole located at the front of glider #1. Similarly, mount the needle to the bottom hole located at the back of glider #2. This will force the gliders to stick together after the collision. Attach the sound reflector on top of glider #1; the sound reflector must be perpendicular to the air track.
2) Using the balance scale, measure the masses of gliders #1 and #2. Record the results in Table 1.

3) Turn on the blower and test that the air track is horizontal. To do this, place a glider on the air track and be sure that it does not move. Turn the blower off.

4) On the computer screen, double click on the Logger Pro icon. The Logger Pro window will show up with a blank table and a blank graph.

5) Click on the blue icon on the upper left side of the toolbar to set up the sensor device. A window will show up. Click on the white square below “Dig/Sonic 1” and select Motion Detector. The motion sensor will start to emit a sound. Close the window.

6) For the first experiment, place a 50 g mass on each side of both gliders. The masses of the two gliders will then be approximately equal.

7) Turn the blower on and place glider #2 at rest, at approximately the 100 cm position on the air track. It should remain at rest if the air track is perfectly leveled but you may want to hold it in position until just before the collision.

8) Click Collect on the computer screen. When you hear that the sound produced by the motion detector changes, set glider #1 in motion towards glider #2. Make sure that nobody is standing in front of the motion detector. The program will collect the data for 5 sec. Data for position and velocity of glider #1 as function of time will appear in the table. The position and velocity of glider #1 will be plotted as function of time in the upper and lower graph, respectively. Make sure that the velocity is approximately constant before and after the collision. Otherwise, repeat the experiment. Turn blower off.

9) Measure the velocities before the collision:
a) The position vs. time graph should show two straight lines of different slopes that meet at the collision time. Mark a region preceding the collision by clicking the left bottom of the mouse and dragging across the region on the graph that you chose; the region is highlighted and square brackets show at the corresponding boundaries. Select Analyze in the toolbar menu and then click Linear Fit. A box will appear showing the slope, \( m \), of the linear fit line. This is the velocity of the glider #1 before the collision.
b) On the velocity vs. time graph, mark a region on the graph before the collision time, as done in part (9a). Select Analyze in the toolbar menu and then click Statistics. A box will appear showing the average value of the velocity in the selected graph region.
Take the average of the values obtained in (a) and (b) as the velocity for glider #1 before the collision and record the value in Table 1.

10) Measure the velocities after the collision: repeat steps (9a) and (9b) using regions in the corresponding graphs after the collision time. The average velocity calculated is the final velocity of the gliders (record the value in Table 1).

11) Move the two masses loaded on glider #1 to glider #2. Glider #2 will have approximately twice the mass of glider #1. Repeat steps 7-10.

12) Move all masses loaded on glider #2 to glider #1. Glider #2 will have approximately half the mass of glider #1. Repeat steps 7-10.

Part II: Elastic collisions

1) Remove the masses, catcher, and needle from the gliders. Mount the elastic bumper to the bottom hole located at the front of glider #1. Measure the masses of gliders #1 and #2. Measure the length of glider #2. Record the results in Table 2.

2) Plug the photogate in. Switch the photogate settings to: mode = “gate”, time = “0.1 ms”, and memory = “on”.

3) Place the photogate at the 145 cm position of the air track. Set the appropriate height of the photogate so glider #2 can pass freely through the gate and it is able to block the light of the photogate (when this happens, the red light of the photogate flashes). In these experiments, the timer is started when the front edge of glider #2 passes through the photogate. The timer is stopped when the back edge of glider #2 passes through the photogate. The velocity of glide #2 is, therefore, the length of glider #2 divided by the time interval measured by the timer.

3) Repeat steps 6-12 of Part I (three experiments in total). The photogate must be “reset” before each experiment is performed. Record your measurements in Table 2.

Computation and Data analysis

a) What is the percentage of linear momentum lost in each experiment? Do your measurements indicate conservation of linear momentum? If not, can you give a possible explanation?

b) What is the percentage of kinetic energy lost in each experiment? In Part II, do your measurements indicate conservation of kinetic energy? If not, can you give a possible explanation?
Student’s name: ____________________________ Date: _______

Table 1: Perfectly inelastic collision

Mass of glider #1: ________ kg  Mass of glide #2: ________ kg

<table>
<thead>
<tr>
<th>BEFORE COLLISION</th>
<th>AFTER COLLISION</th>
<th>Change in $p_{sys}$ [%]</th>
<th>Change in $KE$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$ [kg]</td>
<td>$m_2$ [kg]</td>
<td>$v_{1i}$ [m/s]</td>
<td>$v_{2i}$ [m/s]</td>
</tr>
<tr>
<td>$p_{sys}^i$ [kg m/s]</td>
<td>$KE_{sys}^i$ [J]</td>
<td>$v_f$ [m/s]</td>
<td>$p_{sys}^f$ [kg m/s]</td>
</tr>
<tr>
<td>$KE_{sys}^f$ [J]</td>
<td></td>
<td></td>
<td>$L = $ ________ m</td>
</tr>
</tbody>
</table>

Table 2: Elastic collision

Mass of glider #1: ________ kg  Mass of glide #2: ________ kg  Length of glider #2: $L = $ ________ m

<table>
<thead>
<tr>
<th>BEFORE COLLISION</th>
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<td>$v_{2i}$ [m/s]</td>
</tr>
<tr>
<td>$p_{sys}^i$ [kg m/s]</td>
<td>$KE_{sys}^i$ [J]</td>
<td>$v_{1f}$ [m/s]</td>
<td>$v_{2f} = L/dt$ [m/s]</td>
</tr>
<tr>
<td>$dt$ [sec]</td>
<td>$p_{sys}^f$ [kg m/s]</td>
<td>$KE_{sys}^f$ [J]</td>
<td>$L = $ ________ m</td>
</tr>
<tr>
<td>Change in $p_{sys}$ [%]</td>
<td>Change in $KE$ [%]</td>
<td></td>
<td></td>
</tr>
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