where the normalization factor was added so that
\[ \int_0^\infty \psi_0^* \psi_0 dx = 1. \]

\[ \psi_0 = \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}}. \]

The even and odd solutions have associate, respectively, the form
\[ \psi_0(x) = \begin{cases} \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}} & \text{even} \\ 0 & \text{odd} \end{cases} \]

For the odd function
\[ \psi_0^*(x) = \sqrt{\frac{2}{\pi a^2}} e^{\frac{x^2}{2a^2}}. \]

In the case of the even (square) solution and

\[ \psi_0(x) = \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}} \]

\[ \psi_0(x) = 0 \quad \text{for} \quad x = 0. \]

The solutions in the region of the form \( (x) \) are of the form
\[ (x)^n \psi_0 = (x)^n \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}}. \]

When we set \( \frac{d}{dx} \psi_0 = 0 \), the result is

\[ (x)^{n+1} \psi_0 = (x)^{n+1} \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}}. \]

The energy eigenvalue problem is to find \( (x) \) such that
\[ \frac{d^2}{dx^2} \psi_0 + \lambda \psi_0 = 0. \]

\[ (x)^{n+1} \psi_0 = (x)^{n+1} \sqrt{\frac{2}{\pi a^2}} e^{-\frac{x^2}{2a^2}}. \]

**4.1 Infinite Potential Well**

The solutions of the Schrödinger equation in the potential
\[ (x) \]

where the energy eigenvalues are
\[ \lambda = \frac{n^2 \pi^2}{a^2}, \quad n = 1, 2, 3, \ldots \]

The energy eigenvalue problem is to find \( (x) \) such that
\[ \frac{d^2}{dx^2} \psi_0 + \lambda \psi_0 = 0. \]

The energy eigenvalues are
\[ \lambda = \frac{n^2 \pi^2}{a^2}, \quad n = 1, 2, 3, \ldots \]
The solution for $x > 0$ can be obtained directly from the quadratic equation

$$
\frac{x}{x-A} = \frac{a}{b}
$$

The solution for $x < 0$ is obtained by substitution of $x = -z$, where $z > 0$.

In the interval $|x| < a$, where $x = (x)^n$, we have

$$
\frac{x}{x-A} = \frac{a}{b}
$$

The solution for $x > 0$ is obtained by substitution of $x = (x)^n$, where $n > 0$.

In the interval $|x| > a$, the solution of the quadratic equation

$$
(x)^2 = (x)^n
$$

Is obtained by substitution of $x = (x)^n$, where $n < 0$.

**4.2 Finite Potential Well**

The finite potential well is considered to consist of two different cases.

In both cases, the solution of the quadratic equation

$$
(x)^2 = (x)^n
$$

Is obtained by substitution of $x = (x)^n$, where $n > 0$.

In the case of $n < 0$, the solution of the quadratic equation

$$
(x)^2 = (x)^n
$$

Is obtained by substitution of $x = (x)^n$, where $n < 0$.

The solution of the quadratic equation

$$
(x)^2 = (x)^n
$$

Is obtained by substitution of $x = (x)^n$, where $n > 0$.

The solution of the quadratic equation

$$
(x)^2 = (x)^n
$$

Is obtained by substitution of $x = (x)^n$, where $n < 0$.
4.3 Potential Barrier

The potential barrier is the highest point in the curve, corresponding to the highest energy level. When the wave function reaches this point, it is reflected back, and the probability of finding the particle on the other side is zero. The potential barrier is a key concept in quantum mechanics, as it affects the transmission and reflection of particles through barriers.
1. Obtain the solution \( y(x) \) for a potential well.

**PROBLEMS**

1. 
   - Consider the following boundary conditions on the potential function: 
     - \( y(0) = 0 \) and \( y(L) = 0 \) for some \( L > 0 \).
   - Using these conditions, derive the equation for the probability amplitude \( y(x) \) in the potential well.

The decay rate of the system is determined by the potential energy and the mass of the particle. The solution for the wave function is given by:

\[
\psi(x) = \left( \frac{E}{\hbar^2} \right)^{1/4} \left( \frac{1}{\sqrt{2L}} \right) \sin \left( \frac{n \pi x}{L} \right)
\]

The boundary conditions are satisfied when \( n = 1, 2, 3, \ldots \) and the energy levels are given by:

\[
E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}
\]

**Physical Manifestation of Particle Tunneling**

4.4

The decay of a particle in a potential well can be modeled by the following equation:

\[
\frac{d^2 \psi}{dx^2} + V(x) \psi = 0
\]

where \( V(x) \) is the potential function and \( \psi(x) \) is the wave function.

The tunneling probability is expressed as:

\[
\frac{\hbar}{m} = \frac{\hbar}{m} + \frac{\hbar}{m}
\]

The tunneling probability is given by the square of the wave function at the interface between the two regions.
\[
(3) \quad (J - \lambda)^2 \psi_0 = (J - \lambda)^2 \psi_0 (A) + (J - \lambda)^2 \psi_0 \Delta \psi_0^2
\]

It follows that

\[
(4) \quad \psi_0 (A) = (J - \lambda)_A
\]

Let the potential function \( \psi (x) \) pass through the origin, then

\[
(5) \quad \psi_0 (x) = \psi (x) \Delta \psi_0^2
\]

The diagram shows the potential of the harmonic oscillator.

**CHAPTER FIVE**

The Harmonic Oscillator