SIMPLE PENDULUM AND PROPERTY OF SIMPLE HARMONIC MOTION

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Objectives

- Study the properties of a simple pendulum.
- Study what factors determine the period of a simple pendulum.
- Familiarize yourself with oscillation motion and its characteristics.

Apparatus

1. bob with string,
2. support stand,
3. pendulum clamp,
4. digital stopwatch,
5. meter stick,
6. vernier caliper,
7. protractor,
8. air table with blower,
9. puck with string,
10. PC with Logger Pro software,
11. interface,
12. ultrasound motion detector.

Theory

A simple pendulum is an example of an oscillation device. It consists of a point mass suspended to a pivot by an inextensible weightless cord. If the point mass (a bob) is pulled off the equilibrium position and released, the pendulum bob swings around this position. Two forces, the weight \( w = mg \) and the tension of the string, act on the bob. The resulting restoring force \( F = -mg \sin \theta \), which is directed along the arc of the bob moving, where \( \theta \) is an angle of the bob declination from equilibrium. For small angles \( \theta \) (\( \theta < 10^\circ \)) \( \sin \theta \) could be replaced by \( \theta \) and the displacement of the bob off the equilibrium position along the arc is given by \( x = L\theta \), where \( L \) is the length of the pendulum.

![Fig. 1. Simple pendulum](image)
Equation of motion for small angles of the pendulum \( m \ddot{a} = - mg \sin \theta \), can be rewritten as

\[
a + \frac{(g/L)}{x} = 0
\]

or

\[
d^2x/dt^2 + \frac{(g/L)}{x} = 0 \quad (1)
\]

It is an equation of Simple Harmonic Motion (SHM), which has a solution

\[
x = A_x \sin (\omega t + \varphi_x) \quad (2)
\]

The meaning of the values describes a simple harmonic motion: \( A_x \) – amplitude, maximal displacement \( x \); \( \omega \) - angular frequency; \( \varphi_x \) - initial phase. The angular frequency \( \omega \) relates to the period of the oscillation \( T \), the time of whole swing, \( \omega = 2\pi / T \) and is determined by

\[
\omega = \sqrt{g / L} \quad \text{or} \quad T = 2\pi \sqrt{L / g} \quad (3)
\]

The velocity and acceleration of the bob are oscillating functions too and are determined as derivatives of the position from the formula (2)

\[
v = A_v \sin (\omega t + \varphi_v)
\]

and

\[
a = A_a \sin (\omega t + \varphi_a)
\]

where the amplitudes and phases relate to the amplitude and phase of the displacement

\[
A_v = \omega A_x, \quad \varphi_v - \varphi_x = \pi/2 \quad (90^\circ)
\]

and

\[
A_a = \omega^2 A_x, \quad \varphi_a - \varphi_x = \pi \quad (180^\circ)
\]

In the experiment we can easily verify the dependence of the pendulum period on its length. The study of the dependence on the gravity acceleration \( g \) is more complicated, \( g \) is almost constant on the Earth, so we should use a trick. In the lab “Motion on an Air Table” we found the effective acceleration \( g_{\text{eff}} = g \sin \alpha \) on an inclined plane, where \( \alpha \) is the angle of the plane inclination. Therefore if we set a pendulum on the inclined frictionless table, the \( g_{\text{eff}} \) will change. Then \( \sin \alpha = (H - h) / l \), where \( H \) and \( h \) are the heights of the upper and lower edges of the air table, \( l \) is its length. The set up of the experiment is presented in Fig. 2.

**Procedure**

**Part 1. Dependence of the pendulum period on the length**

1. Measure the mass and diameter of the bob and write it down on the data sheet.
2. Attach the pendulum clump to the support stand. Set the string of the pendulum at the pendulum clamp and adjust the length of the string to 0.30 m.
3. Pull the ball at an angle of less than 10° and release it. Simultaneously start the digital stopwatch and measure the time of 10 whole swings. Write down the number and the time of the swings.

4. Adjust to other lengths of the string and repeat step 3. Recommended lengths of the string are 0.4; 0.5; 0.6; 0.8; 1.0; 1.2; 1.4 m.

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**Fig. 2. Set up of the experiment.**

**Part 2. Dependence of the pendulum period on effective g**

1. Attach the puck string to the board string of the air table. The length of the puck string should be about 0.7 m. Measure and write down on the data sheet the length of the string, the length of the air table, and the diameter of the puck.

2. Put a cord loop over the leg centered on side of the air table and a right angle clump of the support stand. Set an approximate angle of inclination of the table, measure the height of the lower and upper edge of the table, and write down the results on the data sheet.

3. Turn on the blower. Pull the puck at the angle less than 10° and release it. Measure the time of 10 whole swings and write down the results on the data sheet. Turn off the blower.

4. Set the next inclination angle by changing the position of the right angle clamp on the support stand and repeat steps 2 and 3. Recommended inclination angles are 10°; 20°; 30°; 40°; 60°.

**Part 3. Study the form of SHM**

1. Attach a pendulum bob with string to pendulum clamp. Place the support stand and adjust the pendulum so that the bob is 35 - 40 cm in front of the motion.
detector. Measure the length of the string and the diameter of the bob and write down the results on the data sheet.

2. Open **Logger Pro** by double clicking the corresponding icon. The Logger Pro window shows up with a blank table and a blank graph. Close **Tip of the Day**.

3. Click on the blue icon on the left of the top toolbar, which says “**Set up sensors**”. A window shows up. Check that a motion detector is connected to **Dig/Sonic 1** input of the interface, click on the box in the window and select **Motion Detector** or click on and drag the icon of **Motion Detector** to the **Dig/Sonic 1** box. Sound clicks indicate that the motion detector works.

4. Pull the bob along the air track and release. The bob should swing along the motion detector beam. Click **Collect** in **Logger Pro** window. Sinusoidal graphs will draw in the windows. If the graphs are not symmetrical, try to adjust the pendulum bob to motion detector and repeat the step. If the graphs look like sinusoid, you can begin to analyze them.

5. Set the appropriate scale of the graph axis. Click on the graph, then on **Options** at toolbar, choose **Graph Options** or right click the mouse on the **Y axis** of the graph. In the appeared window click **Axis Options** of **Y axis**, in **Scaling** choose **Manual**, set for **Position** min = 0, max = 0.5 m and for **Velocity** min = -1, max = 1 m/sec.

6. Highlight the graph of **Position**. Click **Analyze** on toolbar then click **Curve**. In the appeared window look for and click **Sine**. Click **Try Fit**, a fit curve should coincide with the graph. Check that coefficients A (amplitude) and B (frequency) are positive, otherwise change the sign to positive and adjust coefficient C (phase) to coincide graphs by clicking $^\wedge$ or $^\vee$. Then click **Done** and a box with results shows up. Do the same for the **Velocity** graph. Print the graphs and the table. Write down coefficients to the data sheet.

7. Replace the velocity graph to **Acceleration** by clicking on **Velocity** and set appropriate scale. Analyze the graph, print it, and write down coefficients to the data sheet.

8. Change the length of the pendulum string to 0.1 m and adjust the height of the pendulum clamp so as the bob is in front of the motion detector. Write down the new length on data sheet. Repeat steps 4 – 7.

### Computation and Data Analysis

**Part 1. Dependence of the pendulum period on the length**

1. For each trial calculate and write down in the table the length of the pendulum $L$, which equals the sum of the string length and radius of the bob, the period $T$ and the square of the period $T^2$.

2. Plot graph $T^2$ versus $L$ and find the slope.

3. Calculate the experimental value of gravity acceleration

$$g_{exp} = \frac{4\pi^2}{\text{Slope}}$$

Compare it to the standard value.

**Part 2. Dependence of the pendulum period on effective $g$**

1. Calculate the length of the pendulum, which equals the sum of the string length and radius of the puck.
2. For each trial define the difference between higher \( H \) and lower \( h \) edges of the air table. Calculate the \( g_{\text{eff}} \) and its reciprocal
\[
g_{\text{eff}} = g \sin \alpha = g \left( \frac{H - h}{L} \right)
\]
Write down the results in the table.

3. Define the period of the pendulum \( T \) and \( T^2 \). Write down the results in the table.

4. Plot the graph \( T^2 \) versus \( 1/ g_{\text{eff}} \). Determine the slope of the graph.

5. Calculate theoretical value of the graph slope, which equals \( 4 \pi^2 L/g \). Compare the experimental value of the slope to the theoretical one and calculate the percent error.

**Part 3. Study the form of SHM**

1. Calculate the lengths of the pendulums.

2. For each trial rewrite coefficients of fit sine for position, velocity, and acceleration.

3. Coefficient \( B \) relates to angular frequency \( B = \omega = 2 \pi / T \). Calculate the value of the period
\[
T = 2 \pi / B
\]

4. From the printed out table (Time, Position, Velocity, Acceleration) find the periods \( T_{\text{table}} \) for both trials.

5. Calculate the theoretical value of the period \( T_{\text{theor}} \) on the basis of the formula for a simple pendulum (3).

6. Compare the results of the values for the period.

7. Compare amplitudes of velocity \( A_v \), acceleration \( A_a \) and position \( A_x \). Theoretical ratio of amplitudes
\[
A_v / A_x = \omega = B \quad \text{and} \quad A_a / A_x = \omega^2 = B^2
\]
Compare the ratio to the coefficient \( B \).

8. Calculate the phase differences between velocity and acceleration and position signals using the formula. Compare the graphs.
\[
\Delta \phi_v = (C_v - C_x) * 180^\circ / \pi \quad \text{and} \quad \Delta \phi_a = (C_a - C_x) * 180^\circ / \pi
\]

9. Make conclusion about relation between amplitudes and phases of position, velocity, and acceleration.

**Questions:**

1. What is your opinion about the period of a pendulum dependence on its mass? Explain.

2. Why is it necessary to use more than one vibration in determining the period?

3. Consider the graphs of position, velocity, and acceleration of the pendulum. At what points of its path does it reach maximum velocity? At what points of its path does it reach maximum acceleration?

4. Show that the value defined by the formula (2) is a solution of the equation (1). (For Physics 1.5 only)
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Part 1. Dependence of the pendulum period on the length

<table>
<thead>
<tr>
<th>Trial</th>
<th>Length of string m</th>
<th>Length L of pendulum m</th>
<th>Number of swings, n</th>
<th>Time, t, of n swings sec</th>
<th>Period T = t/n sec</th>
<th>T^2 sec^2</th>
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Slope of the graph T^2 vs. L = ___________ sec^2/m

g_{exp} = 4\pi^2 / Slope = ___________ m/sec^2

% error = __________ %

Part 2. Dependence of the pendulum period on effective g

<table>
<thead>
<tr>
<th>Trial</th>
<th>Height of lower side cm</th>
<th>Height of upper side cm</th>
<th>Difference d cm</th>
<th>g_{eff.} m/sec^2</th>
<th>1/ g_{eff.} sec^2/m</th>
<th>Swing number n</th>
<th>Time, t sec</th>
<th>Period T = t/n sec</th>
<th>T^2 sec^2</th>
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Slope of the graph T^2 vs. 1/ g_{eff.} = ___________ m

Theoretical value of the slope 4\pi^2 L = ___________ m

% error = _______ %
Part 3. Study the Form of SHM

Diameter of the bob = ________ m

<table>
<thead>
<tr>
<th>Trial</th>
<th>Length of string m</th>
<th>Length L of pendulum m</th>
<th>Position Ax m</th>
<th>Bx 1/sec</th>
<th>Cx rad.</th>
<th>Velocity Av m/sec</th>
<th>Bv 1/sec</th>
<th>Cv rad.</th>
<th>Av/Ax 1/sec</th>
<th>Cv-Cx rad.</th>
<th>Δφv degree</th>
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<th>Trail</th>
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<th>Period</th>
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