17. Rational Root Theorem and Fundamental Theorem of Algebra

**Rational Root Theorem**

Let \( f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \) be a polynomial function with integer coefficients.

If \( f(x) \) has a rational root, then the rational root has the form \( \frac{p}{q} \) where \( p \) is a factor of the constant \( a_0 \) and \( p \) is a factor of the leading coefficient \( a_n \).

**Note:** The Rational Root Theorem does not guarantee existence of a rational root. All it is saying is that if a rational root exists then it has that particular format.

1) Use the rational root test to list all possible rational zeros of the following functions.

a) \( f(x) = x^3 + x^2 - 5x + 3 \)

b) \( f(x) = x^5 - 4x^3 + 3x^2 - x + 6 \)

c) \( f(x) = x^3 + x^2 - 8 \)

d) \( f(x) = 4x^3 + x^2 - 3 \)

e) \( f(x) = 4x^3 + x^2 - 6 \)

f) \( f(x) = -2x^3 + x^2 + 1 \)

2) Use the rational root test to solve the equations.

a) \( x^3 + x^2 - 5x + 3 = 0 \)

b) \( x^3 - 6x^2 + 11x - 6 = 0 \)

c) \( x^3 - 7x^2 - 6 = 0 \)

d) \( x^3 - 4x^2 - x + 4 = 0 \)

e) \( x^3 - 9x^2 + 20x - 12 = 0 \)

f) \( -2x^3 + x^2 + 1 = 0 \)

**Fundamental Theorem of Algebra**

Let \( f(x) \) be a polynomial function with complex coefficients. Then there exists at least one root for \( f(x) \) in the set of complex numbers.

**Corollary**

Let \( f(x) \) be a polynomial function of degree \( n \) with complex coefficients. Then \( f(x) \) has at most \( n \) roots in the set of complex numbers.

4) Write polynomial functions that have the following zeros of multiplicity 1.

a) \( 0, 1, -5 \)

b) \( 2 + \sqrt{3}, 2 - \sqrt{3} \)

c) \( 2i, -2i \)

d) \( 4 + 3i, 4 - 3i \)

e) \( 2, -2, 3i, -3i \)