DESSINS AND CURVES

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Abstract. In 1984, Grothendieck observed that any hand-drawn, "connect-the-dots" picture (he called them "child's drawings" or *dessins d'enfants*) determines a unique algebraic curve (a.k.a. Riemann surface). As a consequence of a theorem of Belyĭ (1979), the curve is defined over a finite extension of the rationals, i.e., an algebraic number field. Conversely, any curve defined over a number field carries a canonical imbedded dessin. It follows that there is a faithful action of the absolute Galois group on dessins, but that is a story for another day.

Dessins have a purely combinatorial description in terms of permutation groups; they also have a purely algebraic description in terms of "triangle groups" – abstract (usually infinite) groups generated by three elements of finite order whose product is the identity. I'll give both descriptions, and an (abridged) dictionary between them. I'll also give an indication of the proof of Belyĭ's theorem, and its relevance to dessins.

The most convenient model of a dessin is a bipartite graph imbedded on a compact surface so that the complement of the graph consists of simply connected regions. I'll quickly specialize to dessins having a single "black" vertex, and all "white" vertices of valence 2. I'll show how this highly restricted class of dessins is sufficient to recover some well-known algebraic curves, such as Wiman's family of curves of genus $g > 1$, admitting automorphisms of maximal order $4g + 2$, and Klein's quartic of genus $g = 3$, admitting $84(g - 1)$ automorphisms.