Defining sets in combinatorics with emphasis in graph theory

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In a given graph G, a set of vertices S with an assignment of colors is called a *defining set (of a k–coloring)*, if there exists a unique extension of the colors of S to a proper k–coloring of the vertices of G. A defining set with minimum cardinality is called a *minimum defining set.* The cardinality of minimum defining set is the *defining number* denoted by $d(G, k)$. A *critical set* is a minimal defining set. Defining sets are defined and discussed for many concepts and parameters in graph theory and combinatorics. For example in Latin squares a *critical set* is a partial Latin square that has a unique completion to a Latin square, and is minimal with respect to this property. Smallest possible size of a critical set in any Latin square of order n is of interest and is denoted by $\mathrm{scs}(n)$. But it is clear that any $n \times n$ Latin square may be used as an n-coloring of the Cartesian product of K_n by K_n and vice versa. So $\mathrm{d}(K_n \square K_n, n) = \mathrm{scs}(n)$. The following conjecture which is made in 1995, is still open: For any n, $d(K_n \square K_n, n) = \lfloor n^2/4 \rfloor$. In this talk we discuss these concepts in different areas and introduce some more open problems. Our emphasis will be in graph colorings and perfect matching. For some references and a survey, we refer to the author's webpage and papers referenced in there.

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