# Coloring Graphs with Forbidden Induced Subgraphs

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**Abstract**

Efficiently coloring an arbitrary graph is a fundamental and notoriously difficult algorithmic problem. This talk focuses on the restricted problem of determining the complexity of coloring graphs which do not contain a certain induced subgraph. Combining results of Kamiński and Lozin, and Hoyler, it follows that this problem remains NP-complete unless the excluded induced subgraph is a disjoint union of paths. Recently, as a first step towards resolving the more general question, much work has been done to try and determine the complexity of coloring graphs which do not contain induced paths of a certain fixed length. Only one case of this problem remains open if we are allowed to use at least four colors: Determining the complexity of four-coloring graphs with no induced six-vertex path. The remaining, and most interesting, case is the problem of three-coloring graphs without long induced paths. Working with Maria Chudnovsky and Mingxian Zhong, we resolved the first open case for three-coloring. Specifically, we showed that three-coloring graphs with no induced seven-vertex path can be done in polynomial time. Additionally, in recent work with Juraj Stacho, we have also made progress on the open four-coloring question. In this talk, we will discuss some of the ideas and general spirit of these structural coloring algorithms.