Independent sets in hypergraphs.

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Joint work with Robert Morris and Wojciech Samotij.

Many important theorems and conjectures in combinatorics, such as the theorem of Szemerédi on arithmetic progressions and the Erdős-Stone Theorem in extremal graph theory, can be phrased as statements about families of independent sets in certain uniform hypergraphs. In recent years, an important trend in the area has been to extend such classical results to the so-called 'sparse random setting'. This line of research has recently culminated in the breakthroughs of Conlon and Gowers and of Schacht, who developed general tools for solving problems of this type. Although these two papers solved very similar sets of longstanding open problems, the methods used are very different from one another and have different strengths and weaknesses.

In this talk, we explain a third, completely different approach to proving extremal and structural results in sparse random sets that also yields their natural 'counting' counterparts. We give a structural characterization of the independent sets in a large class of uniform hypergraphs by showing that every independent set is almost contained in one of a small number of relatively sparse sets. We then derive many interesting results as fairly straightforward consequences of this abstract theorem. In particular, we prove the well-known conjecture of Kohayakawa, Łuczak, and Rödl, a probabilistic embedding lemma for sparse graphs, for all 2-balanced graphs. We also give alternative proofs of many of the results of Conlon and Gowers and of Schacht, such as sparse random versions of Szemerédi's theorem, the Erdős-Stone Theorem and the Erdős-Simonovits Stability Theorem, and obtain their natural 'counting' versions, which in some cases are considerably stronger. We also obtain new results, such as a sparse version of the Erdős-Frankl-Rödl Theorem on the number of H-free graphs and, as a consequence

of the KŁR conjecture, we extend a result of Rödl and Ruciński on Ramsey properties in sparse random graphs to the general, non-symmetric setting. Similar results have been discovered independently by Saxton and Thomason.