A Proof of the Roller Coaster Conjecture

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An independent set in a graph is a set of vertices containing no edges. For a graph G, we let $i_t(G)$ be the number of independent sets in G of size t and we define the independence number of G, denoted $\alpha(G)$, to be the size of the largest independent set in G. We call $(i_t(G))_{t=0}^{\alpha(G)}$ the independence sequence of G. For a collection of graphs with independence number is α, we say that the independence sequence for the collection is any-ordered on the index set $S = \{s_1, s_2, \ldots, s_q\} \subseteq [\alpha]$ if, for any permutation π of S, there is a graph G in the collection such that

$$i_{\pi(s_1)}(G) < i_{\pi(s_2)}(G) < \cdots < i_{\pi(s_q)}(G).$$

Alavi, Erdős, Malde, and Schwenk proved that the collection of all graphs with independence number α is any-ordered on $[\alpha]$. A graph is well-covered if every maximal independent set has the same size. Michael and Traves proved that the independence sequence of any well-covered graph is increasing on its first half. They also conjectured that for the collection of well-covered graphs with independence number α, the independence sequence is any-ordered on $\{\lceil \alpha/2 \rceil, \lceil \alpha/2 \rceil + 1, \ldots, \alpha\}$. This conjecture become known as the Roller Coaster Conjecture. In this talk, we will outline a proof of this conjecture, including a graph construction that is related to well-known designs.

This is joint work with Luke Pebody.