FINITE CONVEX GEOMETRIES OF POINT CONFIGURATIONS

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A closure system $\mathbf{A} = (A, -)$, i.e. a set A with a closure operator $- : 2^A \to 2^A$ defined on A, is called *a convex geometry*, if it satisfies *the anti-exchange axiom*, i.e.,

$$x \in \overline{X \cup \{y\}} \text{ and } x \notin X \text{ imply that } y \notin \overline{X \cup \{x\}}$$
$$\text{for all } x \neq y \text{ in } A \text{ and all closed } X \subseteq A.$$

For convex geometries $\mathbf{A}$ and $\mathbf{B}$, one says that $\mathbf{A}$ is *a sub-geometry of* $\mathbf{B}$, if the lattice of closed subsets of $\mathbf{A}$ is a sublattice of the lattice of closed sets of $\mathbf{B}$.

Given a set of points A in Euclidean n-dimensional space $\mathbb{R}^n$, one defines a closure operator $- : 2^A \to 2^A$ on A as follows: for any $Y \subseteq A$, $\overline{Y} = ch(Y) \cap A$, where ch stands for *the convex hull*. One easily verifies that such an operator satisfies the anti-exchange axiom. Thus, $(A, -)$ is a convex geometry, which we will call *a geometry of relatively convex sets* (assuming that these are convex sets "relative" to A).

It remains to be an open problem *whether every finite convex geometry is a sub-geometry of finite geometry of relatively convex sets* (K.Adaricheva, V.Gorbunov and V.Tumanov, "Join-semidistributive lattices and convex geometries", Adv. Math. 173(2003), 1-49.)

We give a survey of results associated with this problem. We then provide a more detailed account of the most recent paper (http://arxiv.org/abs/1101.1539)

It follows one of the approaches to the solution that studies sub-geometries of geometries of finite points configurations in n-dimensional space, for some fixed n.

We prove that such sub-geometries satisfy the n-Carousel Rule, which is the strengthening of the *n-Carathéodory* property. We also find another property, that is similar to the simplex partition property and does not follow from 2-Carusel Rule. We show that it holds in sub-geometries of geometries of points configurations on a plane.

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