Algorithmic aspects of finite dualities

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Abstract

Let $\mathcal{K}$ be a set partially ordered by a relation '$\leq$'. A finite duality on $\mathcal{K}$ is a pair $(U, \mathcal{F})$, where $U \in \mathcal{K}$, and $\mathcal{F}$ is a finite set, such that for all $H \in \mathcal{K}$, $H \leq U$ if and only if $F \not\leq U$, for all $F \in \mathcal{F}$. We are interested in finite dualities due to the fact that certain problems believed to be intractable (classified as NP-complete) become efficiently solvable, when we restrict our input to be from $\mathcal{K}$, provided that we have a finite duality $(U, \mathcal{F})$ on $\mathcal{K}$. An efficient algorithm is achieved via the finite list of elements in $\mathcal{F}$ that witness whether or not $H \leq U$, for any $H \in \mathcal{K}$. That is, $|\mathcal{F}|$ is a fixed size, and hence the running time is polynomial in $|H|$, rather than being exponential in $|H|$. We shall give some well known examples to make this clear, and present some recent results.