

Length Function for Weyl Groups of Extended Affine Root Systems of Type A_1

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Let R be an extended affine root system in a vector space \mathcal{V} . The Weyl group of R is the group generated by reflections based on nonisotropic elements of R . Recently, we found a presentation for the Weyl group of any extended affine root system of type A_1 , which is very similar to Coxeter presentation of finite and affine Weyl groups. We know that Weyl groups of affine Kac-Moody root systems are both Coxeter groups and Weyl groups of extended affine root systems of nullity one. Furthermore, each Coxeter group W has a length function which for any element $w \in W$ gives the minimum number of Coxeter generators needed to build w . In order to show that Coxeter groups and these Weyl groups are “related”, in this talk, we discuss a length function for Weyl groups of extended affine root systems of type A_1 which is the same as the Coxeter group’s length function for Weyl groups of affine Kac-Moody root systems of type A_1 . Our proof of the existence of this length function uses algebraic, combinatorial and geometric properties of these class of Weyl groups and their underlying root systems. To define the length function on these groups, we need several notions on their corresponding root systems such as the notion of positive and negative roots, height function and root bases. These notions on finite and affine Kac-Moody root systems are coming from their corresponding Lie algebras. But for extended affine root systems, they are not obviously defined from their Lie algebras. We generalize these definitions as part of our approach to length function.

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