ALGEBRAICALLY DEFINED DIRECTED GRAPHS

ALEX KODESS

Let q be a prime power, $\mathbb{F}_q$ denote the finite field of q elements. Let $f_i\colon \mathbb{F}_q^2 \to \mathbb{F}_q$ be arbitrary functions, $1 \le i \le l$. The digraph $D = D(q;\mathbf{f})$, where $\mathbf{f} = (f_1, \dots, f_l)\colon \mathbb{F}_q^2 \to \mathbb{F}_q^l$, is defined as follows. The vertex set V of D is $\mathbb{F}_q^{l+1}$. There is an arc from $(x_1, \dots, x_{l+1}) \in V$ to $(y_1, \dots, y_{l+1}) \in V$ if and only if $x_i + y_i = f_{i-1}(x_1, y_1)$ for all $i,\ 2 \le i \le l+1$.

When $l = 1$ and $f = f_1$ can be represented by the polynomial $f(X_1, X_2) = X_1^m X_2^n$, the corresponding digraph $D = D(q; m, n)$ is called a *monomial* digraph.

The digraphs $D(q;\mathbf{f})$ are directed analogues of a well studied class of algebraically defined undirected bipartite graphs $B\Gamma(q; f)$ (proposed by Lazebnik and Woldar) having many applications, most noticeably in extremal graph theory supplying a lower bound of the best magnitude on some $\mathrm{ex}(\nu, C_{2l})$.

We present a number of results on the strong connectivity of the general algebraic digraph $D(q;\mathbf{f})$ and the diameters of monomial digraphs. We also discuss the isomorphism problem for all monomial digraphs for a fixed q.