ALGEBRAICALLY DEFINED DIRECTED GRAPHS

ALEX KODESS

Let \( q \) be a prime power, \( \mathbb{F}_q \) denote the finite field of \( q \) elements. Let \( f_i: \mathbb{F}_q^2 \to \mathbb{F}_q \) be arbitrary functions, \( 1 \leq i \leq l \). The digraph \( D = D(q; \mathbf{f}) \), where \( \mathbf{f} = (f_1, \ldots, f_l): \mathbb{F}_q^2 \to \mathbb{F}_q^l \), is defined as follows. The vertex set \( V \) of \( D \) is \( \mathbb{F}_q^{l+1} \). There is an arc from \((x_1, \ldots, x_{l+1}) \in V\) to \((y_1, \ldots, y_{l+1}) \in V\) if and only if
\[
x_i + y_i = f_{i-1}(x_1, y_1) \quad \text{for all} \quad i, \quad 2 \leq i \leq l+1.
\]

When \( l = 1 \) and \( f = f_1 \) can be represented by the polynomial \( f(X_1, X_2) = X_1^m X_2^n \), the corresponding digraph \( D = D(q; m, n) \) is called a monomial digraph.

The digraphs \( D(q; \mathbf{f}) \) are directed analogues of a well studied class of algebraically defined undirected bipartite graphs \( \Gamma(q; \mathbf{f}) \) (proposed by Lazebnik and Woldar) having many applications, most noticeably in extremal graph theory supplying a lower bound of the best magnitude on some \( \text{ex}(\nu, C_{2l}) \).

We present a number of results on the strong connectivity of the general algebraic digraph \( D(q; \mathbf{f}) \) and the diameters of monomial digraphs. We also discuss the isomorphism problem for all monomial digraphs for a fixed \( q \).