Reliability Models Associated with the Edge Domination Number

Spy networks can sufficiently operate or carry out missions provided there are sufficiently many spies in communication with each other. Now consider the scenario when a communication link between two spies is intercepted. The identities of the two spies would be compromised, so they can be removed from the network. In graph theoretic terms, if an edge fails, then the endnodes can be removed from the graph, subsequently removing any other edges incident on those nodes. The remaining graph is considered to be in an operating state iff there exists a component of order at least k, where k is a predetermined threshold value. The vulnerability parameter $\lambda_{nc}^{(k)}$ quanitfies this, as it is the minimum number of edges required to fail (such that their endnodes are removed) to yield a surviving subgraph having all components of order at most $k-1$ (a failure state). If $k=1$, then this parameter is equivalent to the edge cover of the nodes, and thus polynomial. However, for $k=2$, the parameter is equivalent to the edge domination number, and thus NP-hard.

If edges can fail independently, all with the same probability ρ, for $0<\rho<1$, the unreliability of the graph is the probability that the surviving subgraph is in a failure state. If n, e, and k are fixed, a graph on n nodes with e edges is uniformly most reliable (UMR) provided its unreliability is minimum among all graphs in its class for all values of ρ. Uniformly least reliable (ULR) graphs are defined analogously. We present results for uniformly optimal trees and unicycles when $k=2$.