Forbidden intersection patterns in the families of subsets

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Let $[n] = \{1, 2, \ldots, n\}$ be a finite set, families $\mathcal{F}$ of its subsets will be investigated. Sperner's theorem says that if there is no inclusion among the members of $\mathcal{F}$ then the largest family under this condition is the one containing all $\lfloor \frac{n}{2} \rfloor$-element subsets of $[n]$. The present lecture surveys certain generalizations of this theorem. The maximum size of $\mathcal{F}$ is to be found under the condition that a certain configuration is excluded. The configuration here is always described by inclusions. More formally, let P be a poset. The maximum size of a family $\mathcal{F} \subset 2^{[n]}$ which does not contain P as a (non-necessarily induced) subposet is denoted by $\mathrm{La}(n, P)$. We will give account of the newest developments in this theory, among others we will show the surprisingly easy proof for the butterfly $(a < c, a < d, b < c, b < d)$ by Burcsi and D.T. Nagy.