A Practitioner's Guide to Mathematical Finance

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Introduction

- Interest in mathematical finance continues to grow exponentially, both domestically and abroad.

- Mathematical Finance (MF) already has sub-disciplines, financial engineering, computational finance, and Econo-physics (a.k.a. phynance!).

- As these multi-part names suggest, MF is multi-disciplinary, a rich broth of applied math, finance, engineering, physics, and computer science.

- As a representative of both the industry (Bloomberg) and academia (NYU), I welcome BC's ongoing efforts in MF.

A Practitioner's View

- As Casey Stengel once said: "In theory, theory and practice are the same. But in practice, they are very different."

- Having spent 10 years as a practitioner and 8 years as an academic, I have to agree with Casey.

- MF is too important to be left solely to academics; there is also a thriving subculture of quants, happily doing research, trading, and risk management in banking, insurance, software, money management, and even the public sector.

- My talk will address MF from a practitioner's perspective. Accordingly, we will dispense with frivolities such as clarity and logic.

Overview

- A Brief History of MF
- Prominent Mathematicians Contributing to MF
- Prominent Finance Academics Contributing to MF
- Mathematics Used in MF
- Finance Used in MF
- Educational Programs in MF
- MF Books for Mathematicians and Physicists
- Academic and Industry Conferences on MF
- Applications of MF to Industry
- Active Research Areas in MF
- Open Problems in MF

Brief History of Mathematical Finance

- Bachelier (1900)

- Kolmogorov/Lévy/Doeblin/Itô

- Samuelson/Osborne

- Black Scholes/Merton (Nobel 1997)

- Cox Ross/Harrison Kreps/ Harrison Pliska

- HJM/BGM

- Artzner Delbaen Heath Eber

- Li (WSJ)

A-Z of Mathematicians Contributing to MF

- Marco Avellaneda
- Richard Beals
- Freddie Delbaen
- Nicole El Karoui (WSJ)
- Charles Fefferman
- Samuel Howison
- Joseph Keller
- Pierre Lyons
- Terry Lyons
- Paul Malliavin
- Benoit Mandelbrot

A-Z of Mathematicians Contributing to MF (Con'd)

- Henry McKean

- George Papanicolau

- Chris Rogers

- Walter Schachermayer

- Albert Shiryaev

- Jim Simons

- Elias Stein

- Daniel Stroock

- Marc Yor

- Thaleia Zariphopoulou

Prominent Finance Academics Contributing to MF

- Samuelson

- Black and Scholes

- Robert C. Merton

- Cox and Ross

- Leland and Rubinstein

- Duffie and Singleton

- Brennan and Schwartz; Longstaff and Schwartz

- Pliska, Jarrow, Madan

Mathematics Used in Mathematical Finance

- Stochastic Calculus: Markov Processes, Itô's Lemma, Girsanov's Thm
- Linear & Nonlinear PDE's - primarily 2nd order linear, esp. parabolic
- Monte Carlo Simulation
- Finite Differences, Finite Elements, and Spectral Methods
- Functional Analysis - semi-groups
- Integral Transforms Fourier/Gaussian/Hilbert/Laplace/Radon
- Complex Analysis - for inverting transforms
- Pseudo Differential Operators
- Maximum Principle
- Fundamental Theorem of Linear Algebra
- Hahn Banach Theorem

Mathematics Used in MF (Con'd)

- Lie Groups

- Regular and Singular Perturbations

- Optimal Control

- Variational Inequalities

- Differential Geometry

- String Theory

- Game Theory

- Inverse Problems - Calibration

- Statistics, Econometrics, esp. time series.

Finance Used in Mathematical Finance

- MF is often seen as a subdiscipline of finance and finance is often seen as a subdiscipline of economics. Neither need be so.

- Derivatives pricing theory is the most widely used subdiscipline of MF. The basic idea behind this theory is that the price of a derivative security should be given by the cost of replicating its payoff. In the standard theory, no arbitrage and frictionless markets jointly imply the existence of a positive linear pricing operator.

- Derivatives pricing theory has been famously characterized as "ketchup economics": when pricing two bottles of ketchup, first figure out how much one bottle of ketchup costs and then double the price.

- In a related vein, Nils Hakansson of Berkeley has called this the Catch 22 of derivatives pricing theory. If derivatives are redundant, then why do they exist?

Common Sense Used in Finance

- As Mark Twain once wrote, "common sense ain't so common".

- No practitioners that I know of think that derivatives are redundant. In fact, the closer someone is to the market, the less likely they are to value two ketchup bottles at twice the unit price.

- While the standard pricing models are linear, practitioners keenly appreciate the limited domain in which they operate:

  - Reserves are set aside for model risk.

  - Several models are used simultaneously, and the results are averaged.

  - Fixed model parameters are routinely shocked; risks outside the model are routinely hedged.

- Nobody thinks quants are redundant (fortunately); much time is spent trying to find robust pricing and hedging strategies.

Masters Programs in MF

- To my knowledge, there are no undergrad programs solely devoted to math finance yet.

- The first Master's program in MF was offered by CMU in 1992.

- There are now over 100 Master's programs internationally.

- A typical Master's program requires 1-1.5 years of full-time attendance. Most programs in big cities also offer part-time.

- The programs are usually offered out of math or engineering dept.'s (or both); they are sometimes offered out of the B school as a standalone program (eg. Haas) or as a track (eg. MIT).

- Graduates go to industry: NYU's Courant graduates often go to the big banks or a hedge fund as trader's assistants. Joining a quant group is fairly rare.

Doctoral Programs in Math, Finance, and MF

- Doctoral programs almost never require a Master's degree.

- It is common for doctoral students in math, engineering, computer science, or physics to get interested in MF at the thesis stage. They typically go into industry, primarily as quants in banks, hedge funds, ins. companies, and software companies.

- Some PhD's go into academia, either as a post-doc (eg. at NYU) or less commonly, tenure-track eg. Chicago, Columbia, Cambridge, Cornell, and others. They never go to B school.

- Most Finance PhD's go to finance dept.'s in B school; a few go to industry.

- Some universities now offer PhD's in MF (eg. CMU, Imperial College).

MF Books for Mathematicians and Physicists

- Several books on MF have mathematicians as the target audience:

  - Bjork
  - Duffie
  - Jeanblanc, Yor, and Chesney
  - Musiela and Rutkowski, Bielecki and Rutkowski
  - Karatzas and Shreve, Shreve Vol II.
  - Shiryaev

- There are also books on MF and econo-physics aimed at physicists:

  - Mantegna and Stanley
  - Bouchaud and Potters
  - Ilinksi

Conferences on Mathematical Finance

- Bachelier Finance Congress, every 2 years - in London in 2008.

- The annual CCCP conference: Carnegie, Columbia, Cornell, Princeton

- The annual Derivative Securities Conference: Queens, Cornell, CFTC, Houston.

- The annual FORC conference in Warwick, UK.

- Numerous Practitioner Conferences from Risk, ICBI, and others. In particular, ICBI's Global Derivatives Conference in Paris is the equivalent of the academy awards.

- Bi-annual Journal of Investment Management (JOIM) Conference in San Francisco in Spring 2007.

Academic Journals in MF

- Mathematical Finance

- Finance and Stochastics

- Quantitative Finance

- Journal of Computational Finance

- Review of Derivatives Research

- Applied Mathematical Finance

- International Journal of Theoretical and Applied Finance

Industry Journals in MF

- Journal of Derivatives

- Journal of Fixed Income

- Risk Magazine

- Wilmott Magazine

- Journal of Risk

- Journal of Credit Risk

- Journal of Operational Risk

Applications of MF: The D Word

- Derivatives such as forwards, futures, and options have a long history (Thales).

- Derivatives trade either over-the-counter (eg. currency options) or on a listed exchange (eg. stock options) or both (eg. stock options).

- Derivatives have an underlying. The underlying can be an asset (eg. stock) or not (eg. stock index). Sometimes, the underlying can be stored (eg. wheat) or not, (eg. weather).

- The security underlying a derivative can have more liquidity than the derivative (eg. $/Euro) or less (eg. corporate bonds underlying credit default swaps. The underlying can be considered a primary asset (eg. a bond underlying a bond option) or it can be a derivative itself (eg. swaptions, options on VIX).

Applications of MF: Risk Measurement and Management

- Banks are exposed to *market risk*, the possibility that losses will arise due to adverse marking of its assets and liabilities. As a result, banks are required to calculate a risk measure known as Value at Risk on a daily basis.

- Widely criticized for its properties, the wide use of VaR has lead to much academic and industrial research in alternative risk measures.

- In the wake of corporate scandals such as Worldcom and Enron, the Sarbanes Oxley act of 2002 forced CEO's to sign off on the accuracy of their firm's financial statements. As intended, this has lead to a renewed emphasis on the risk management function inside many corporations.

- Continuing banking reforms from Basel have also lead to research in other forms of risk, such as counterparty credit risk & operational risk.

Applications of MF: ALM

- Pension plans have long term "fixed" liabilities whose magnitude is based on actuarial estimates of retirement age, future salaries, mortality rates, etc.

- Similarly, insurance companies promise fixed annuity payments whose length extends from the beneficiary's retirement until death.

- In both cases, it is common practice to invest some fraction of the company's assets in equities to partake of their higher average growth over the long term. This leads to Asset and Liability Management (ALM), a field which has long used quantitative methods, but is just beginning to succumb to market-oriented quantitative techniques.

Applications of MF: Variable Annuities

- To mitigate their market risk, insurance companies offer *variable annuities*, i.e. life insurance policies whose payoff is positively linked to the performance of equity markets.

- Unfortunately, long term path-dependent guarantees (eg. in GMDB's) embedded in the policies lead to large losses during the last bear market.

- As a result, most large insurers now delta hedge their equity exposures.

Applications of MF: Algorithmic Trading

- If market prices are martingales, then one can neither gain nor lose on average from non-anticipating trading strategies.

- However sometimes you can partially anticipate market movements, eg. NAV's of mutual funds with cross country positions.

- Finance academics used to believe that markets are too efficient in order to systematically profit from predictable market movements. The rise in CPU power lead many academics to abandon that view and start hedge funds to exploit "anomalies".

- Simultaneous advances in automated trading and in the theory of market micro-structure have lead to the formation of companies such as Automated Trading Desk (ATD) which place orders hundreds of times per second.

Applications of MF: Portfolio Insurance Again

- A different form of portfolio insurance called Constant Proportion Portfolio Insurance (CPPI) is presently in wide use. While similar to portfolio insurance based on synthetic put replication employed in the 80's, the CPPI trading strategy differs in several important ways:

  1. Insurance is provided contractually rather than on a best efforts basis (insurer is principal, not agent).
  2. Insurance is usually only provided on diversified portfolios, such as funds of hedge funds, or pools of CDO's.
  3. CPPI does not require knowing the underlying asset's volatility.

- Fischer Black invented (68) and popularized (87) CPPI with $e = mc^*$.

- Banks also provide options on CPPI.

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*Note the missing square!

Active Research Areas in Mathematical Finance

- Stochastic Volatility and Lévy Processes (see my website for eg.)

- Market Models (eg. BGM and market models of implied vol)

- Default Risk and Credit Derivatives (see www.defaultrisk.com)

- Liquidity (also hot in academic finance)

- Hedge Funds and Algorithmic Trading

- Inverse Problems/Calibration Issues

Open Problems in Mathematical Finance

- Simple Closed Form Formula for American Put in Black Scholes

- Forward PDE for American Options in Local Volatility model

- Fast Accurate Pricing of Synthetic CDO tranches

- Pricing in Incomplete Markets

Summary and Conclusions

- Continuing demand from industry for quantitatively-oriented students bodes well for Masters and PhD level programs in MF.

- This demand stems from several sources:

  1. ready availability of data (eg. Bloomberg) and computing power
  2. legal reforms (eg. Sarbanes Oxley,Basel II)
  3. the rise (and occasional fall) of hedge funds employing sophisticated and unencumbered trading strategies.
  4. softening of MBA programs?

- MF should benefit considerably from the input provided by BC's quantitative talents.