Conditional Variation and Option Prices

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Introduction

- Market participants widely agree that an option's market price conveys information about the expected variation of its underlying asset's price over time.
- However, once we move past simple models such as Black Scholes or Bachelier, it is unclear how co-terminal options of different strikes can all determine the variation of the same underlying asset.
- To answer this question, we generalize these models to the case where the underlying's spot price at t is an increasing function of just time t and the time t level of its Brownian driver W_t.
- In this case, we show that a particular combination of the level, slope, and curvature of an option's price in strike price reveals the risk-neutral mean of its underlying's price variation, conditional on its underlying's price being at the strike price at expiry.

Financial Setting

- Consider an options market with no arbitrage, no frictions, and no interest rates. Suppose that the asset underlying a European call option pays no dividends between the call's valuation date t_0 and the call's maturity date $T \geq t_0$.
- For $t \in [t_0, T]$, let S_t denote the spot price of this underlying asset, where only S_0 is known.
- The absence of arbitrage implies the existence of an equivalent martingale measure $\mathbb{Q}$ such that the call price C_t and its underlying asset price S_t are both martingales.
- The choice of the $\mathbb{Q}$ martingale for S over the time interval $[t_0, T]$ determines the conditional call value C_t via the following martingale condition:

$$C_t = E^{\mathbb{Q}}[(S_T - K)^+ | \mathcal{F}_t], \qquad t \in [t_0, T].$$

Path Independent Spot Price Dynamics

- Suppose that the underlying spot price process S is path-independent, i.e there exists a $C^{2,1}$ function $s(x,t) : \mathbb{R} \times [t_0, T] \mapsto \mathbb{R}$ such that:

$$S_t = s(W_t, t), \qquad t \in [t_0, T].$$

- Applying Itô's formula:

$$dS_t = \left[\frac{1}{2}s_{11}(W_t, t) + s_2(W_t, t)\right] dt + s_1(W_t, t) dW_t, \qquad t \in [t_0, T].$$

- Since the process S is a $\mathbb{Q}$ martingale, the function $s(x,t)$ solves the following linear partial differential equation (PDE):

$$\frac{1}{2}s_{11}(x, t) + s_2(x, t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T].$$

Path Independent Spot Price Dynamics (Con'd)

- Recall that when spot price dynamics are path-independent, the spot pricing function $s(x,t)$ solves the following PDE:

$$\frac{1}{2}s_{11}(x,t) + s_2(x,t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T].$$

- To obtain a unique solution, we impose the following terminal condition:

$$s(x,T) = p(x), \qquad x \in \mathbb{R},$$

and require that $p(x)$ be strictly increasing and once differentiable.

- Differentiating the PDE at the top w.r.t. x implies that:

$$\frac{1}{2}s_{111}(x,t) + s_{12}(x,t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T],$$

while differentiating the terminal condition w.r.t. x implies that:

$$s_1(x,T) = p'(x), \qquad x \in \mathbb{R}.$$

- Since $p'(x) > 0$, the Feynman Kac theorem guarantees that $s_1(x,t)$ is positive, so the spot pricing function $s(x,t)$ is increasing in x.

Path-Independence Implies Local Vol

- Recall that the spot pricing function $s(x, t)$ is increasing in x. For each $t \in [t_0, T]$, let $x(S, t)$ be the inverse of $s(x, t)$, i.e. $s(x(S, t), t) = S$.
- From Itô's formula and the PDE, the spot price process S solves:

$$dS_t = s_1(W_t, t)dW_t, \qquad t \in [t_0, T].$$

- Let $n(S, t) = s_1(x(S, t), t), x \in \mathbb{R}, t \in [t_0, T]$ be the function relating the normal volatility of S to the spot price level S and time t.
- Substitution implies: $dS_t = n(S_t, t)dW_t, \qquad t \in [t_0, T]$.
- Thus, our assumption on the existence of a path-independent spot pricing function $s(x, t)$ leads to a local volatility model for the spot price S.
- In contrast to the general family of local volatility models, the local volatility function $n(S, t)$ is not free, but rather is determined by the spot price payoff function $p(x)$.

Path-Independence Implies Numeraire Vol

- Recall that path-independent spot pricing leads to a local volatility model for the spot price S: $dS_t = n(S_t, t)dW_t, \qquad t \in [t_0, T]$.
- We now show that $N_t = n(S_t, t), t \in [t_0, T]$ is a positive $\mathbb{Q}$ martingale.
- Recall the following PDE from two slides back:

$$\frac{1}{2}s_{111}(x,t) + s_{12}(x,t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T].$$

- From Itô's formula, $s_1(x,t) = n(s(x,t),t), x \in \mathbb{R}, t \in [t_0, T]$.
- Differentiating twice w.r.t. x implies that:

$$s_{111}(x,t) = n_{11}(s(x,t),t)(s_1(x,t))^2 + n_1(s(x,t),t)s_{11}(x,t), x \in \mathbb{R}, t \in [t_0, T].$$

- Differentiating $s_1(x,t) = n(s(x,t),t)$ w.r.t. t implies that:

$$s_{12}(x,t) = n_1(s(x,t),t)s_2(x,t) + n_2(s(x,t),t), \qquad x \in \mathbb{R}, t \in [t_0, T].$$

- Substituting into the top PDE implies that $0 =$:

$$\frac{n^2(s(x,t),t)}{2} n_{11}(s(x,t),t) + n_2(s(x,t),t) + n_1(s(x,t),t)\left[\frac{1}{2}s_{11}(x,t) + s_2(x,t)\right]$$

Path-Independence Implies Numeraire Vol (Con'd)

- Recall from the last slide that for $x \in \mathbb{R}, t \in [t_0, T]$, $0 =$

$$\frac{n^2(s(x,t),t)}{2} n_{11}(s(x,t),t) + n_2(s(x,t),t) + n_1(s(x,t),t)\left[\frac{1}{2}s_{11}(x,t) + s_2(x,t)\right].$$

- However, the PDE $\frac{1}{2}s_{11}(x,t) + s_2(x,t) = 0$ implies that $n(S,t)$ with $S = s(x,t)$ solves the following non-linear PDE:

$$\frac{n^2(S,t)}{2} n_{11}(S,t) + n_2(S,t) = 0, \qquad S \in \mathbb{R}, t \in [t_0, T].$$

- Hence, the process $N_t = n(S_t, t), t \in [t_0, T]$ is a $\mathbb{Q}$ martingale as claimed.
- To show positivity, simply recall that $n(S,t) = s_1(x,t) > 0$.
- Since the volatility process $N_t = n(s(W_t,t),t), t \in [t_0, T]$ is a positive $\mathbb{Q}$ martingale, it is the arbitrage-free value of a numeraire.

Volatility Measure

- We have shown that when the stock price is path-independent i.e. $S_t = s(W_t, t)$ for some increasing function $s(x, t)$, then the normal volatility $N_t = n(S_t, t)$ of the stock is also a numeraire value.
- Suppose now that a probability measure $\mathbb{Q}^*$ is defined by:

$$\frac{d\mathbb{Q}^*}{d\mathbb{Q}} = \frac{n(S_T, T)}{n(S_0, 0)}.$$

- We refer to the probability measure $\mathbb{Q}^*$ as the volatility measure.
- In the following slides, we show that the delta $\triangle_t$ of any contingent claim with a path-independent payoff is a $\mathbb{Q}^*$ martingale.

Delta under Volatility Measure

- Let $V(S,t)$ be the arbitrage-free value of a contingent claim with a path-independent payoff and let $\triangle(S,t) = V_1(S,t)$ be its delta.
- When the underlying spot price is path-independent, then the function $v(x,t) = V(s(x,t),t)$ relates the value of the contingent claim to the SBM W driving the underlying spot price. The function $v(x,t)$ solves the PDE:

$$\frac{1}{2}v_{11}(x,t) + v_2(x,t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T].$$

- However, differentiating w.r.t. x implies that:

$$\frac{1}{2}v_{111}(x,t) + v_{12}(x,t) = 0, \qquad x \in \mathbb{R}, t \in [t_0, T],$$

and hence that $v_1(W_t, t), t \in [t_0, T]$ is a $\mathbb{Q}$ martingale. By the chain rule:

$$v_1(x,t) = V_1(s(x,t),t)s_1(x,t), \qquad x \in \mathbb{R}, t \in [t_0, T].$$

Delta under Volatility Measure (Con'd)

- Recall that $v_1(W_t, t), t \in [t_0, T]$ is a $\mathbb{Q}$ martingale and that:

$$v_1(x,t) = V_1(s(x,t),t)s_1(x,t), \qquad x \in \mathbb{R}, t \in [t_0, T].$$

- The RHS is just the product of the claim's delta and the volatility of its underlying asset:

$$v_1(x,t) = \triangle(S,t)n(S,t), \qquad S \in \mathbb{R}, t \in [t_0, T].$$

- Let $\triangle_t = \triangle(S_t, t), t \in [t_0, T]$ be the stochastic process describing the contingent claim's delta.
- These equations imply that the product $\triangle_t N_t, t \in [t_0, T]$ is a $\mathbb{Q}$ martingale.
- Since N was the positive $\mathbb{Q}$ martingale used to create $\mathbb{Q}^*$ from $\mathbb{Q}$, it follows that the delta process $\triangle_t$ is a $\mathbb{Q}^*$ martingale.

Conditional Variation

- We can now address our opening question concerning how co-terminal options of different strikes can all determine the variation of the same underlying asset.
- In particular, we show the relevance of the strike price for the implied variation measure.
- We develop a new measure of the variation of the underlying spot price which is conditional on the spot price being at a particular level K on a fixed date T.
- Assuming that S is path-independent, we show that the risk-neutral mean of this variation can be extracted from knowledge of the normal volatility function $n(K, T)$ and the initial call price level $C_0(K, T)$, the initial strike slope $\frac{\partial}{\partial K} C_0(K)$, and the initial strike curvature $\frac{\partial^2}{\partial K^2} C_0(K)$.

Variation Along a Path

- To develop a measure of the variation of an underlying's spot price path over time, suppose that the time interval $[t_0, T]$ is sub-divided into n time steps of equal length $\triangle t = \frac{T-t_0}{n}$, where n is a positive finite integer.
- Mildly abusing notation, let S_i be the spot price at time $i\triangle t$ for $i = 0, 1, \ldots, n$.
- Consider the following discrete variation measure:

$$DV_n = \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|.$$

- Financially, DV_n is just the cumulative payoff from rolling over single period ATM straddles over the time period $[t_0, T]$.
- As n goes to infinity, DV_n becomes the variation of the sample path of S over $[t_0, T]$. Under the diffusion assumption of the last section, this limiting sum is also infinite.

Normalized Variation

- To deal with the issue of infinite sample path variation, consider the following normalized variation:

$$NFV_n = \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle t}.$$

- As n goes to infinity, the square of this normalized variation approaches the quadratic variation of S at T, which is finite.
- It follows that as n goes to infinity, the normalized variation is also finite and has very little dependence on n for large n.
- Suppose that a market maker issues a variation swap paying the difference between NFV_n and a constant at time T. To make the swap costless to enter at any prior time $t \in [t_0, T]$, the variation swap rate needs to be set at:

$$VS_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle t}.$$

Normalized Variation Swap rate

- Recall that the normalized variation swap rate needs to be set at:

$$VS_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}| \sqrt{\triangle t}.$$

- To determine this mean variation, suppose that S is path-independent.
- Using Monte Carlo, one can simulate the payoff using Euler discretization of the sample path.
- As the number of paths becomes infinite, the average payoff converges to the following approximation of the normalized variation swap rate:

$$\widehat{VS}_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} n(S_i, i\triangle t) |W_{(i+1)\triangle u} - W_{i\triangle u}| \sqrt{\triangle t}.$$

Approximating the Normalized Variation Swap Rate

- Recall the following approximation of the normalized variation swap rate:

$$\widehat{VS}_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} n(S_i, i\triangle t)|W_{(i+1)\triangle u} - W_{i\triangle u}|\sqrt{\triangle t}.$$

- The Bachelier model values an ATM straddle as:

$$E_{i\triangle u}^{\mathbb{Q}}|W_{(i+1)\triangle u} - W_{i\triangle u}| = \sqrt{\frac{2}{\pi}}\sqrt{\triangle u}.$$

- By the law of iterated expectations, substitution implies:

$$\widehat{VS}_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} n(S_i, i\triangle t)\sqrt{\frac{2}{\pi}}\triangle t.$$

- As n goes to infinity in both the variation swap rate $VS_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle t}$ and its approximation $\widehat{VS}_t(n)$, the approximation error goes to zero:

$$\widehat{VS}_t(\infty) = VS_t(\infty) = E_t^{\mathbb{Q}} \int_0^T n(S_t, t)\sqrt{\frac{2}{\pi}}dt.$$

Approximating the Norm'd Variation Swap Rate (Con'd)

- Recall that for large n, the normalized variation swap rate $VS_t(n) = E_t^{\mathbb{Q}} \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle t}$ is approximated by:

$$\widehat{VS}_t(\infty) = VS_t(\infty) = E_t^{\mathbb{Q}} \int_0^T n(S_t, t)\sqrt{\frac{2}{\pi}}dt.$$

- If we pick a normal volatility function $n(S, t)$, we can always determine the normalized variation swap rate by Monte Carlo.
- However, a natural question is to determine the relevance of the initial implied volatility smile of maturity T for the calculation.
- To answer this question, suppose that we condition the variation swap rate on the final stock price:

$$CVS_t(K, \infty) = E_t^{\mathbb{Q}} \left\{ \int_0^T n(S_t, t)\sqrt{\frac{2}{\pi}}dt \middle| S_T = K \right\}.$$

- If we know the smile, we also know the risk-neutral PDF of S_T and so we can easily obtain the continuously monitored variation swap rate if we know the conditional expectation.

Conditional Variation Swap Rate

- We now show that knowledge of $n(K, T)$ and the level, slope, and curvature of C in K determine the conditional variation swap rate.
- Let $B_t(K)$ be the value at time $t \in [t_0, T]$ of a binary call paying $1(S_T > K)$ at time T.
- Suppose we integrate the product of $S_t - K$ and $B_t(K)$ by parts:
  $(S_T - K)B_T(K) =$

$$(S_t - K)B_t(K) + \int_t^T (S_u - K)dB_u(K) + \int_t^T B_u(K)d(S_u - K) + \langle S, B \rangle_T.$$

- Since $B_T(K) = 1(S_T > K)$, the LHS is
  $(S_T - K)B_T(K) = (S_T - K)1(S_T > K) = (S_T - K)^+$.
- Taking expectations under $\mathbb{Q}$ implies:

$$C_t(K) - (S_t - K)B_t(K) = E_t^{\mathbb{Q}} \langle S, B \rangle_T,$$

since $B(K)$ and $S - K$ are both $\mathbb{Q}$ martingales.

Conditional Variation Swap Rate (Con'd)

- Recall that: $C_t(K) - (S_t - K)B_t(K) = E_t^{\mathbb{Q}}\langle S, B\rangle_T$,
- Let $B(S_u, u; K, T)$ be the binary call's value function. From Itô's formula:

$$dB_u(K) = \frac{\partial}{\partial S}B(S_u, u; K, T)dS_u, \qquad u \in [t, T].$$

- Substituting into the top equation implies:

$$C_t(K) - (S_t - K)B_t(K) = E_t^{\mathbb{Q}} \int_t^T \frac{\partial}{\partial S}B(S_u, u; K, T)d\langle S\rangle_u.$$

- Let $D(S, \sigma, t) = \frac{\partial}{\partial S}B(S, \sigma, t; K, T)$ be the deterministic function describing the delta of a binary call. Correspondingly, let $D_t = D(S_t, \sigma_t, t), t \in [t_0, T]$ be the stochastic process describing the delta of a binary call.
- Since $B_t(K) = -C_t'(K)$ and $d\langle S\rangle_u = N_u^2 du$,

$$C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} \int_t^T N_u D_u N_u du.$$

Conditional Variation Swap Rate (Con'd)

- Recall that: $C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} \int_t^T N_u D_u N_u du.$
- Now recall that the product of the underlying's local volatility N_u and a path-independent claim's delta $\triangle_u$ is a $\mathbb{Q}$ local martingale.
- For $u \in [t, T]$, let $M_u = N_u D_u$ and let $A_u \equiv \int_t^u N_s ds$. The integral on the RHS can be represented as:

$$\int_t^T N_u D_u N_u du = \int_t^T M_u dA_u = M_T A_T - M_t A_t - \int_t^T A_u dM_u,$$

using integration by parts. Taking expectations on both sides implies:

$$E_t^{\mathbb{Q}} \int_t^T N_u D_u N_u du = E_t^{\mathbb{Q}} M_T A_T,$$

since $A_t = 0$ and M is a $\mathbb{Q}$ martingale. Since $M_T = N_T D_T$ and $A_T = \int_t^T N_u du$, substituting implies:

$$C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} N_T D_T \int_t^T N_u du.$$

Conditional Variation Swap Rate (Con'd)

- Recall that: $C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} N_T D_T \int_t^T N_u du$.
- Since the path-independent claim is a binary call, the terminal value of $N_T D_T$, is:

$$N_T D_T = n(S_T, T)\delta(S_T - K) = n(K, T)\delta(S_T - K),$$

by the sifting property.

- Substitution implies:

$$C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} n(K, T)\delta(S_T - K) \int_t^T N_u du.$$

Conditional Variation Swap Rate (Con'd)

- Recall that: $C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} n(K,T)\delta(S_T - K)\int_t^T N_u du.$
- Dividing by $n(K,T)C_t''(K)$ implies that:

$$\frac{C_t(K) + (S_t - K)C_t'(K)}{n(K,T)C_t''(K)} = E_t^{\mathbb{Q}}\left\{\int_t^T N_u du \middle| S_T = K\right\} = \sqrt{\frac{\pi}{2}} CVS_t(K),$$

- Multiplying by $\sqrt{\frac{2}{\pi}}$ gives our desired result:

$$CVS_t(K) = \sqrt{\frac{2}{\pi}} \frac{C_t(K) + (S_t - K)C_t'(K)}{n(K,T)C_t''(K)}.$$

- Thus, the continuously monitored variation, conditional on the spot price finishing at K can be extracted from the level, slope, and curvature of the call price C in its strike price K.

Discretely Monitored Variation Swap Rate

- Since continuous monitoring is physically impossible, we try to approximate the discretely monitored conditional variation. Recall from the last slide that:

$$C_t(K) + (S_t - K)C_t'(K) = E_t^{\mathbb{Q}} n(K,T)\delta(S_T - K)\int_t^T N_u du, \text{ so:}$$

$$\sqrt{\frac{2}{\pi}}\left[C_t(K) + (S_t - K)C_t'(K)\right] = E_t^{\mathbb{Q}} n(K,T)\delta(S_T - K)\int_t^T N_u \sqrt{\frac{2}{\pi}} du.$$

- Since a sum approximates an integral, $E_t^{\mathbb{Q}} \int_0^T n(S_t, t)\sqrt{\frac{2}{\pi}} dt \approx$:

$$E_t^{\mathbb{Q}} \sum_{i=1}^{n} n(S_i, i\triangle t)\sqrt{\frac{2}{\pi}}\triangle t \approx E_t^{\mathbb{Q}} \sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle t}, \text{ by Euler approx'n.}$$

- Substitution implies $\sqrt{\frac{2}{\pi}}\left[C_t(K) + (S_t - K)C_t'(K)\right] \approx$

$$E_t^{\mathbb{Q}} n(K,T)\delta(S_T - K)\sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle u}.$$

Discretely Monitored VS Rate (Con'd)

- Recall that $\sqrt{\frac{2}{\pi}}\left[C_t(K) + (S_t - K)C_t'(K)\right] \approx$

$$E_t^{\mathbb{Q}} n(K,T)\delta(S_T - K)\sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|\sqrt{\triangle u}.$$

- Dividing by $n(K,T)\sqrt{\triangle u}$:

$$\sqrt{\frac{2}{\pi}}\frac{C_t(K) + (S_t - K)C_t'(K)}{n(K,T)\sqrt{\triangle u}} \approx E_t^{\mathbb{Q}}\delta(S_T - K)\sum_{i=1}^{n} |S_{(i+1)\triangle u} - S_{i\triangle u}|.$$

- The sum on the right is the aggregate payoff from rolling over short-dated ATM straddles. The multiplication by $\delta(S_T - K)$ make this payoff contingent on the final spot price S_T being at K. The denominator $n(K,T)\sqrt{\triangle u}$ on the LHS is the risk-neutral standard deviation of $\triangle S$ conditional on $S_T = K$. In a local vol model such as ours, $n(K,T)$ can be observed from the prices of options struck around K and maturing near T.

Discretely Monitored VS Rate (Con'd)

- Recall that:

$$\sqrt{\frac{2}{\pi}}\frac{C_t(K)+(S_t-K)C_t'(K)}{n(K,T)\sqrt{\triangle u}} \approx E_t^{\mathbb{Q}}\delta(S_T-K)\sum_{i=1}^{n}|S_{(i+1)\triangle u}-S_{i\triangle u}|.$$

- Dividing by $C_t''(K)$ converts the RHS into a conditional mean:

$$\sqrt{\frac{2}{\pi}}\frac{C_t(K)+(S_t-K)C_t'(K)}{C_t''(K)n(K,T)\sqrt{\triangle u}} \approx E_t^{\mathbb{Q}}\left\{\sum_{i=1}^{n}|S_{(i+1)\triangle u}-S_{i\triangle u}|\middle| S_T=K\right\}.$$

- This final result shows that the level, slope, and curvature of the call price C in its strike price K gives a good approximation of the risk-neutral mean of the variation in the underlying's price conditional on the terminal spot price being at K at maturity.
- Clearly, one can integrate against the risk-neutral PDF $C_t''(K)$ to get the unconditional mean variation implied by co-terminal option prices of all strikes.

Summary

- We considered models where the underlying's spot price at t is an increasing function of just time t and the time t level of its Brownian driver W_t.
- In this case, we show that a particular combination of the level, slope, and curvature of an option's price in strike price reveals the risk-neutral mean of its underlying's price variation, conditional on its underlying's price being at the strike price at expiry.
- We also showed how to approximate the value of a discretely monitored variation swap.
- Thanks for listening!

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