CHAPTER 6: UNIFORM CIRCULAR MOTION AND GRAVITATION

6.1 ROTATION ANGLE AND ANGULAR VELOCITY

1. *Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?*

Solution Given:

$$d = 1.15\text{ m} \Rightarrow r = \frac{1.15\text{ m}}{2} = 0.575\text{ m},\ \Delta\theta = 200{,}000\ \text{rot} \times \frac{2\pi\text{ rad}}{1\text{ rot}} = 1.257\times10^{6}\ \text{rad}$$

Find Δs using $\Delta\theta = \frac{\Delta s}{r}$, so that

$$\begin{aligned}\Delta s = \Delta\theta \times r &= (1.257\times10^{6}\ \text{rad})(0.575\text{ m})\\ &= 7.226\times10^{5}\ \text{m} = \underline{723\text{ km}}\end{aligned}$$

7. *A truck with 0.420 m radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?*

Solution Given: $r = 0.420\text{ m}, v = 32.0\text{ m/s}$.

Use $\omega = \frac{v}{r} = \frac{32.0\ \text{m/s}}{0.420\text{ m}} = \underline{76.2\text{ rad/s}}.$

Convert to rpm by using the conversion factor:

$$1\,\text{rev} = 2\pi\,\text{rad}\,,$$
$$\omega = 76.2\,\text{rad/s} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} \times \frac{60\,\text{s}}{1\,\text{min}}$$
$$= 728\,\text{rev/s} = \underline{728\,\text{rpm}}$$

6.2 CENTRIPETAL ACCELERATION

18. *Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating: (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min. (b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).*

Solution (a) Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = (0.100\,\text{m})\left(50{,}000\ \text{rev/min} \times \frac{2\,\pi\,\text{rad}}{1\,\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}\right) = 524\,\text{m/s} = \underline{0.524\,\text{km/s}}$$

(b) Given: $\omega = 2\pi\dfrac{\text{rad}}{\text{y}} \times \dfrac{1\,\text{y}}{3.16\times10^{7}\ \text{s}} = 1.988\times10^{-7}\ \text{rad/s}\,;\, r = 1.496\times10^{11}\ \text{m}$

Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = \left(1.496\times10^{11}\ \text{m}\right)\left(1.988\times10^{-7}\ \text{rad/s}\right) = 2.975\times10^{4}\ \text{m/s} = \underline{29.7\ \text{km/s}}$$

6.3 CENTRIPETAL FORCE

26. *What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?*

Solution Using $\tan\theta = \dfrac{v^2}{rg}$ gives:

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)\tan 20.0^\circ} = \underline{18.9\text{ m/s}}$$

6.5 NEWTON'S UNIVERSAL LAW OF GRAVITATION

33. *(a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is* 9.830 m/s^2 *and the radius of the Earth is 6371 km from pole to pole. (b) Compare this with the accepted value of* $5.979\times10^{24}\text{ kg}$.

Solution (a) Using the equation $g = \dfrac{GM}{r^2}$ gives:

$$g = \frac{GM}{r^2} \Rightarrow M = \frac{r^2 g}{G} = \frac{(6371\times10^3\text{ m})^2(9.830\text{ m/s}^2)}{6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = \underline{5.979\times10^{24}\text{ kg}}$$

(b) This is identical to the best value to three significant figures.

39. *Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational. (a) Calculate the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the force on the baby due to Jupiter if it is at its closest distance to Earth, some* $6.29\times10^{11}\text{ m}$ *away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)*

Solution (a) Use $F = \frac{GMm}{r^2}$ to calculate the force:

$$F_\mathrm{f} = \frac{GMm}{r^2} = \frac{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)(100\ \mathrm{kg})(4.20\ \mathrm{kg})}{(0.200\ \mathrm{m})^2} = \underline{7.01\times10^{-7}\ \mathrm{N}}$$

(b) The mass of Jupiter is:

$$m_\mathrm{J} = 1.90\times10^{27}\ \mathrm{kg}$$

$$F_\mathrm{J} = \frac{\left(6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(1.90\times10^{27}\ \mathrm{kg}\right)(4.20\ \mathrm{kg})}{\left(6.29\times10^{11}\ \mathrm{m}\right)^2} = \underline{1.35\times10^{-6}\ \mathrm{N}}$$

$$\frac{F_\mathrm{f}}{F_\mathrm{J}} = \frac{7.01\times10^{-7}\ \mathrm{N}}{1.35\times10^{-6}\ \mathrm{N}} = \underline{0.521}$$

6.6 SATELLITES AND KEPLER'S LAWS: AN ARGUMENT FOR SIMPLICITY

45. *Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.*

Solution Using $\frac{r^3}{T^2} = \frac{G}{4\pi^2}M$, we can solve the mass of Jupiter:

$$M_\mathrm{J} = \frac{4\pi^2}{G}\times\frac{r^3}{T^2}$$

$$= \frac{4\pi^2}{6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}}\times\frac{\left(4.22\times10^{8}\ \mathrm{m}\right)^3}{\left[(0.00485\ \mathrm{y})\left(3.16\times10^{7}\ \mathrm{s/y}\right)\right]^2} = \underline{1.89\times10^{27}\ \mathrm{kg}}$$

This result matches the value for Jupiter's mass given by NASA.

48. ***Integrated Concepts*** *Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same*

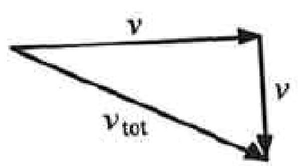
radius that intersects the satellite's orbit at an angle of 90° relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)

Solution (a) Use $F_c = ma_c$, then substitute using $a = \dfrac{v^2}{r}$ and $F = \dfrac{GmM}{r^2}$.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r_S}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.979\times10^{24}\ \text{kg})}{900\times10^3\ \text{m}}} = \underline{2.11\times10^4\ \text{m/s}}$$

(b)



In the satellite's frame of reference, the rivet has two perpendicular velocity components equal to v from part (a):

$$v_{\text{tot}} = \sqrt{v^2 + v^2} = \sqrt{2v^2} = \sqrt{2}\left(2.105\times10^4\ \text{m/s}\right) = \underline{2.98\times10^4\ \text{m/s}}$$

(c) Using kinematics: $d = v_{\text{tot}}t \Rightarrow t = \dfrac{d}{v_{\text{tot}}} = \dfrac{3.00\times10^{-3}\ \text{m}}{2.98\times10^4\ \text{m/s}} = \underline{1.01\times10^{-7}\ \text{s}}$

(d) $\overline{F} = \dfrac{\Delta p}{\Delta t} = \dfrac{mv_{\text{tot}}}{t} = \dfrac{(0.500\times10^{-3}\ \text{kg})(2.98\times10^4\ \text{m/s})}{1.01\times10^{-7}\ \text{s}} = \underline{1.48\times10^8\ \text{N}}$

(e) The energy is generated from the rivet. In the satellite's frame of reference, $v_i = v_{\text{tot}}$, and $v_f = 0$. So, the change in the kinetic energy of the rivet is:

$$\Delta\,\text{KE} = \frac{1}{2}mv_{\text{tot}}^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(0.500\times10^{-3}\ \text{kg})(2.98\times10^4\ \text{m/s})^2 - 0\ \text{J} = \underline{2.22\times10^5\ \text{J}}$$

CHAPTER 7: WORK, ENERGY, AND ENERGY RESOURCES

7.1 WORK: THE SCIENTIFIC DEFINITION

1. *How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.*

Solution Using $W = fd\cos(\theta)$, where $F = 5.00\text{ N}, d = 0.600\text{ m}$ and since the force is applied horizontally, $\theta = 0°$: $W = Fd\cos\theta = (5.00\text{ N})(0.600\text{ m})\cos 0° = \underline{3.00\text{ J}}$

Using the conversion factor $1\text{ kcal} = 4186\text{ J}$ gives:

$$W = 3.00\text{ J} \times \frac{1\text{ kcal}}{4186\text{ J}} = \underline{7.17 \times 10^{-4}\text{ kcal}}$$

7. *A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction $25.0°$ below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?*

Solution (a) The work done by friction is in the opposite direction of the motion, so $\theta = 180°$, and therefore $W_{\text{f}} = Fd\cos\theta = 35.0\text{ N} \times 20.0\text{ m} \times \cos 180° = \underline{-700\text{ J}}$

(b) The work done by gravity is perpendicular to the direction of motion, so $\theta = 90°$, and $W_{\text{g}} = Fd\cos\theta = 35.0\text{ N} \times 20.0\text{ m} \times \cos 90° = \underline{0\text{ J}}$

(c) If the cart moves at a constant speed, no energy is transferred to it, from the work-energy theorem: $\text{net } W = W_{\text{s}} + W_{\text{f}} = 0, \text{ or } W_{\text{s}} = \underline{700\text{ J}}$

(d) Use the equation $W_{\text{s}} = Fd\cos\theta$, where $\theta = 25°$, and solve for the force:

$$F=\frac{W_s}{d\cos\theta}=\frac{700\text{ J}}{20.0\text{ m}\times\cos25°}=38.62\text{ N}=\underline{38.6\text{ N}}$$

(e) Since there is no change in speed, the work energy theorem says that there is no net work done on the cart: $\text{net}\,W=W_f+W_s=-700\text{ J}+700\text{ J}=\underline{0\text{ J}}$

7.2 KINETIC ENERGY AND THE WORK-ENERGY THEOREM

13. *A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.*

Solution Use the work energy theorem, $\text{net}\,W=\frac{1}{2}mv^2-\frac{1}{2}mv_0{}^2=Fd\cos\theta$,

$$F=\frac{mv^2-mv_0{}^2}{2d\cos\theta}=\frac{(900\text{ kg})(0\text{ m/s})^2-(900\text{ kg})(1.12\text{ m/s})^2}{2\,(0.200\text{ m})\cos0°}=\underline{-2.8\times10^3\text{ N}}$$

The force is negative because the car is decelerating.

7.3 GRAVITATIONAL POTENTIAL ENERGY

16. *A hydroelectric power facility (see Figure 7.38) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume 50.0 km^3 ($\text{mass}=5.00\times10^{13}\text{ kg}$), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.*

Solution (a) Using the equation $\Delta\text{PE}_g=mgh$, where
$m=5.00\times10^{13}\text{ kg}$, $g=9.80\text{ m/s}^2$, and $h=40.0\text{ m}$, gives:
$\Delta\text{PE}_g=(5.00\times10^{13}\text{ kg})(9.80\text{ m/s}^2)(40.0\text{ m})=\underline{1.96\times10^{16}\text{ J}}$

(b) From Table 7.1, we know the energy stored in a 9-megaton fusion bomb is $3.8\times10^{16}\text{ J}$, so that $\frac{E_{\text{lake}}}{E_{\text{bomb}}}=\frac{1.96\times10^{16}\text{ J}}{3.8\times10^{16}\text{ J}}=\underline{0.52}$. The energy stored in the lake is approximately half that of a 9-megaton fusion bomb.

7.7 POWER

30. *The Crab Nebula (see Figure 7.41) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 7.3, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.*

Solution From Table 7.3: $P_{\text{Crab}} = 10^{28}\ \text{W}$, and $P_{\text{Supernova}} = 5\times10^{37}\ \text{W}$ so that $\dfrac{P}{P_0} \approx \dfrac{10^{28} W}{5\times10^{37} W} = \underline{2\times10^{-10}}$. This power today is 10^{10} orders of magnitude smaller than it was at the time of the explosion.

36. *(a) What is the average useful power output of a person who does $6.00\times10^{6}\ \text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)*

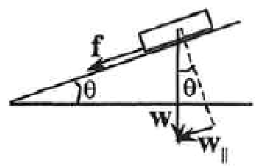
Solution (a) Use $P = \dfrac{W}{t}$ (where *t* is in seconds!):

$$P = \frac{W}{t} = \frac{6.00\times10^{6}\ \text{J}}{(8.00\ \text{h})(3600\ \text{s/1h})} = 208.3\ \text{J/s} = \underline{208\ \text{W}}$$

(b) Use the work energy theorem to express the work needed to lift the bricks: $W = mgh$, where $m = 2000\ \text{kg}$ and $h = 1.50\ \text{m}$. Then use $P = \dfrac{W}{t}$ to solve for the time: $P = \dfrac{W}{t} = \dfrac{mgh}{t} \Rightarrow t = \dfrac{mgh}{P} = \dfrac{(2000\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m})}{(208.3\ \text{W})} = 141.1\ \text{s} = \underline{141\ \text{s}}$

42. *Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.*

Solution The energy supplied by the engine is converted into frictional energy as the car goes up the incline.



$P=\dfrac{W}{t}=\dfrac{Fd}{t}=F\left(\dfrac{d}{t}\right)=Fv,$ where F is parallel to the incline and

$F = f+w=600\text{ N}+mg\sin\theta$. Substituting gives $P=(f+mg\sin\theta)v$, so that:

$$P=\left[600\text{ N}+(950\text{ kg})(9.80\text{ m/s}^2)\sin2°\right](30.0\text{ m/s})=\underline{2.77\times10^4\text{ W}}$$

7.8 WORK, ENERGY, AND POWER IN HUMANS

46. *Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)*

Solution Use the work energy theorem to determine the work done by the shot-putter:

$$\begin{aligned}\text{net }W&=\frac{1}{2}mv^2+mgh-\frac{1}{2}mv_0{}^2-mgh_0\\&=\frac{1}{2}(7.27\text{ kg})(14.0\text{ m/s})^2+(7.27\text{ kg})(9.80\text{ m/s}^2)(0.800\text{ m})=769.5\text{ J}\end{aligned}$$

The power can be found using $P=\dfrac{W}{t}$: $P=\dfrac{W}{t}=\dfrac{769.5\text{ J}}{1.20\text{ s}}=641.2\text{ W}=\underline{641\text{ W}}$.

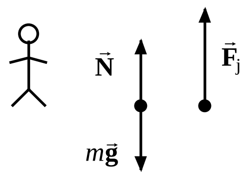
Then, using the conversion 1 hp = 746W, we see that $P=641\text{ W}\times\dfrac{1\text{ hp}}{746\text{ W}}=\underline{0.860\text{ hp}}$

52. *Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.*

Solution Given: $m = 80.0\ \text{kg}, h = 0.600\ \text{m}, \text{and } d = 0.0150\ \text{m}$

Find: net F . Using $W = Fd$ and the work-energy theorem gives: $W = F_\text{j}d = mgh$

$$F = \frac{mgh}{d} = \frac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)(0.600)}{0.0150\ \text{m}} = \underline{3.136 \times 10^4\ \text{N.}}$$



(a) Now, looking at the body diagram: net $F = w + F_\text{j}$

$$\text{net } F = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + 3.136 \times 10^4\ \text{N} = \underline{3.21 \times 10^4\ \text{N}}$$

(b) Now, let $d = 0.300\ \text{m}$ so that $F_\text{j} = \frac{mgh}{d} = \frac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)(0.600)}{0.300\ \text{m}} = 1568\ \text{N.}$

$$\text{net } F = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + 1568\ \text{N} = \underline{2.35 \times 10^3\ \text{N}}$$

(c) In (a), $\frac{\text{net } F}{mg} = \frac{32{,}144\ \text{N}}{784\ \text{N}} = \underline{41.0}$. This could be damaging to the body.

In (b), $\frac{\text{net } F}{mg} = \frac{2352\ \text{N}}{784\ \text{N}} = \underline{3.00}$. This can be easily sustained.

58. *The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about* $7 \times 10^9\ \text{kg}$*. (The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7.45), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet*

was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

Solution (a) To calculate the potential energy use $\mathrm{PE} = mgh$, where $m = 7\times 10^9\ \mathrm{kg}$ and $h = \frac{1}{4}\times 146\ \mathrm{m} = 36.5\ \mathrm{m}$:

$$\mathrm{PE} = mgh = (7.00\times 10^9\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(36.5\ \mathrm{m}) = 2.504\times 10^{12}\ \mathrm{J} = \underline{2.50\times 10^{12}\ \mathrm{J}}$$

(b) First, we need to calculate the energy needed to feed the 1000 workers over the 20 years:

$$E_{\mathrm{in}} = NPt = 1000\times\frac{300\ \mathrm{kcal}}{\mathrm{h}}\times\frac{4186\ \mathrm{J}}{\mathrm{kcal}}\times 20\ \mathrm{y}\times\frac{330\ \mathrm{d}}{\mathrm{y}}\times\frac{12\ \mathrm{h}}{\mathrm{d}} = 9.946\times 10^{13}\ \mathrm{kcal}.$$

Now, since the workers must provide the PE from part (a), use $Eff = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}}$ to calculate their efficiency: $Eff = \frac{W_{\mathrm{out}}}{E_{\mathrm{in}}} = \frac{\mathrm{PE}}{E_{\mathrm{in}}} = \frac{2.504\times 10^{12}\ \mathrm{J}}{9.946\times 10^{13}} = 0.0252 = \underline{2.52\%}$

(c) If each worker requires 3600 kcal/day, and we know the composition of their diet, we can calculate the mass of food required:

$$\begin{aligned} E_{\mathrm{protein}} &= (3600\ \mathrm{kcal})(0.05) = 180\ \mathrm{kcal};\\ E_{\mathrm{carbohydrate}} &= (3600\ \mathrm{kcal})(0.60) = 2160\ \mathrm{kcal};\ \mathrm{and}\\ E_{\mathrm{fat}} &= (3600\ \mathrm{kcal})(0.35) = 1260\ \mathrm{kcal}.\end{aligned}$$

Now, from Table 7.1 we can convert the energy required into the mass required for each component of their diet:

$$\begin{aligned} m_{\mathrm{protein}} &= E_{\mathrm{protein}}\times\frac{1\ \mathrm{g}}{4.1\ \mathrm{kcal}} = 180\ \mathrm{kcal}\times\frac{1\ \mathrm{g}}{4.1\ \mathrm{kcal}} = 43.90\ \mathrm{g};\\ m_{\mathrm{carbohydrate}} &= E_{\mathrm{carbohydrate}}\times\frac{1\ \mathrm{g}}{4.1\ \mathrm{kcal}} = 2160\ \mathrm{kcal}\times\frac{1\ \mathrm{g}}{4.1\ \mathrm{kcal}} = 526.8\ \mathrm{g};\\ m_{\mathrm{fat}} &= E_{\mathrm{fat}}\times\frac{1\ \mathrm{g}}{9.3\ \mathrm{kcal}} = 2160\ \mathrm{kcal}\times\frac{1\ \mathrm{g}}{9.3\ \mathrm{kcal}} = 135.5\ \mathrm{g}.\end{aligned}$$

Therefore, the total mass of food require for the average worker per day is:

$$m_{\mathrm{person}} = m_{\mathrm{protein}} + m_{\mathrm{carbohydrate}} + m_{\mathrm{fat}} = (43.90\ \mathrm{g}) + (526.8\ \mathrm{g}) + (135.5\ \mathrm{g}) = 706.2\ \mathrm{g},$$

and the total amount of food required for the 20,000 workers is:

$$m = Nm_{\text{person}} = 20{,}000 \times 0.7062\ \text{kg} = 1.41 \times 10^4\ \text{kg} = \underline{1.4 \times 10^4\ \text{kg}}$$

CHAPTER 8: LINEAR MOMENTUM AND COLLISIONS

8.1 LINEAR MOMENTUM AND FORCE

1. *(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of* $7.50\ \text{m/s}$*. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of* $600\ \text{m/s}$*. (c) What is the momentum of the 90.0-kg hunter running at* $7.40\ \text{m/s}$ *after missing the elephant?*

Solution (a) $p_\text{e} = m_\text{e}v_\text{e} = 2000\ \text{kg} \times 7.50\ \text{m/s} = \underline{1.50 \times 10^4\ \text{kg} \cdot \text{m/s}}$

(b) $p_\text{b} = m_\text{b}v_\text{b} = 0.0400\ \text{kg} \times 600\ \text{m/s} = 24.0\ \text{kg.m/s, so}$

$$\frac{p_\text{c}}{p_\text{b}} = \frac{1.50 \times 10^4\ \text{kg.m/s}}{24.0\ \text{kg.m/s}} = \underline{625}$$

The momentum of the elephant is much larger because the mass of the elephant is much larger.

(c) $p_\text{b} = m_\text{h}v_\text{h} = 90.0\ \text{kg} \times 7.40\ \text{m/s} = \underline{6.66 \times 10^2\ \text{kg} \cdot \text{m/s}}$

Again, the momentum is smaller than that of the elephant because the mass of the hunter is much smaller.

8.2 IMPULSE

9. *A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.*

Solution (a) Calculate the net force on the hand:

$$\text{net } F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{1.50\text{kg}(0\text{ m/s} - 4.00\text{ m/s})}{2.50\times 10^{-3}\text{ s}} = \underline{-2.40\times 10^{3}\text{ N}}$$

(taking moment toward the leg as positive). Therefore, by Newton's third law, the net force exerted on the leg is $\underline{2.40\times 10^{3}\text{ N}}$, toward the leg.

(b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the changes in momentum and time interval are the same.

15. *A cruise ship with a mass of* $1.00\times 10^{7}\text{ kg}$ *strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)*

Solution Given: $m = 1.00\times 10^{7}\text{ kg},\ v_0 = 0.75\text{ m/s}, v = 0\text{ m/s}, \Delta x = 6.00\text{ m}.$ Find: net force on the pier. First, we need a way to express the time, Δt, in terms of known quantities. Using the equations $\bar{v} = \frac{\Delta x}{\Delta t}$ and $\bar{v} = \frac{v_0 + v}{2}$ gives:

$$\Delta x = \bar{v}\Delta t = \frac{1}{2}(v + v_0)\Delta t \text{ so that } \Delta t = \frac{2\Delta x}{v + v_0} = \frac{2(6.00\text{ m})}{(0 + 0.750)\text{ m/s}} = \underline{16.0\text{ s.}}$$

$$\text{net } F = \frac{\Delta p}{\Delta t} = \frac{m(v - v_0)}{\Delta t} = \frac{(1.00\times 10^{7}\text{ kg})(0 - 0750)\text{ m/s}}{16.0\text{ s}} = \underline{-4.69\times 10^{5}\text{ N.}}$$

By Newton's third law, the net force on the pier is $\underline{4.69\times 10^{5}\text{ N}}$, in the original direction of the ship.

8.3 CONSERVATION OF MOMENTUM

23. ***Professional Application*** *Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of* -0.120 m/s. *(The minus indicates direction of motion.) What is their final velocity?*

Solution Use conservation of momentum, $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$, since their final velocities are the same.

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(150{,}000\text{ kg})(0.300\text{ m/s}) + (110{,}000\text{ kg})(-0.120\text{ m/s})}{150{,}000\text{ kg} + 110{,}000\text{ kg}} = \underline{0.122\text{ m/s}}$$

The final velocity is in the direction of the first car because it had a larger initial momentum.

8.5 INELASTIC COLLISIONS IN ONE DIMENSION

33. ***Professional Application*** *Using mass and speed data from Example 8.1 and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?*

Solution (a) Use conservation of momentum for the player and the ball:

$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$ so that

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(110\text{ kg})(8.00\text{ m/s}) + (0.410\text{ kg})(25.0\text{ m/s})}{110\text{ kg} + 0.410\text{ kg}} = 8.063\text{ m/s} = \underline{8.06\text{ m/s}}$$

(b) $\Delta\text{KE} = \text{KE}' - (\text{KE}_1 + \text{KE}_2)$

$$= \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v'^2 - \frac{1}{2}(m_1v'^2_1 + m_2v'^2_2)$$

$$= \frac{1}{2}(110.41\text{ kg})(8.063\text{ m/s})^2 - \frac{1}{2}\left[(110\text{ kg})(8.00\text{ m/s})^2 + (0.400\text{ kg})(25.0\text{ m/s})^2\right]$$

$$= \underline{-59.0\text{ J}}$$

(c) (i) $v' = \dfrac{(110\text{ kg})(8.00\text{ m/s}) + (0.410\text{ kg})(-25.0\text{ m/s})}{110.41\text{ kg}} = \underline{7.88\text{ m/s}}$

(ii) $\Delta \mathrm{KE} = \frac{1}{2}(110.41\,\mathrm{kg})(7.877\,\mathrm{m/s})^2 -$

$$\frac{1}{2}\left[(110\,\mathrm{kg})(8.00\,\mathrm{m/s})^2 + \frac{1}{2}(0.410\,\mathrm{kg})(-25.0\,\mathrm{m/s})^2\right] = \underline{-223\,\mathrm{J}}$$

38. *A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) See Example 8.1 and discuss its relationship to this problem.*

Solution (a) Given: $v_1 = v_2 = 0\,\mathrm{m/s},\ m_1 = 3.00\,\mathrm{kg}.$ Use conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$$

$$m_1v'_1 = -m_2v'_2 \Rightarrow v'_1 = \frac{-m_2v'_2}{m_1} = \frac{-(0.0250\,\mathrm{kg})(550\,\mathrm{m/s})}{3.00\,\mathrm{kg}} = -4.583\,\mathrm{m/s} = \underline{4.58\,\mathrm{m/s}}$$

(b) The rifle begins at rest, so $\mathrm{KE_i} = 0\ \mathrm{J}$, and

$$\Delta \mathrm{KE} = \frac{1}{2}m_1v'^2_1 = \frac{1}{2}(3.00\,\mathrm{kg})(-4.58\,\mathrm{m/s})^2 = \underline{31.5\,\mathrm{J}}$$

(c) Now, $m_1 = 28.0\,\mathrm{kg}$, so that $v'_1 = \frac{-m_2v_2}{m_1} = \frac{-(0.0250\,\mathrm{kg})(550\,\mathrm{m/s})}{28.0\,\mathrm{kg}} = \underline{-0.491\,\mathrm{m/s}}$

(d) Again, $\mathrm{KE_i} = 0\ \mathrm{J}$, and

$$\Delta \mathrm{KE} = \frac{1}{2}m_1v'^2_1 = \frac{1}{2}(28.0\,\mathrm{kg})(-0.491\,\mathrm{m/s})^2 = 3.376\,\mathrm{J} = \underline{3.38\,\mathrm{J}}$$

(e) Example 8.1 makes the observation that if two objects have the same momentum the heavier object will have a smaller kinetic energy. Keeping the rifle close to the body increases the effective mass of the rifle, hence reducing the kinetic energy of the recoiling rifle. Since pain is related to the amount of kinetic energy, a rifle hurts less if it held against the body.

44. *(a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?*

Solution (a) Use conversation of momentum to find the mass of the barbell:
$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$ where $v_1 = v_2 = 0\ \text{m/s}$, and v_1'=-0.500 m/s (since it recoils backwards), so solving for the mass of the barbell gives:

$$0 = m_1 v_1 + m_2 v_2 \Rightarrow m_2 = \frac{-m_1 v_1}{v_2} = \frac{-(80.0\ \text{kg})(-0.500\ \text{m/s})}{10.0\ \text{m/s}} = \underline{4.00\text{kg}}$$

(b) Find the change in kinetic energy:

$$\Delta \text{KE} = \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 - \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m_2 v_2^2 = \frac{1}{2}\left(m_1 v'^2_1 + m_2 v'^2_2\right)$$
$$= \frac{1}{2}\left[(80.0\ \text{kg})(-0.500\ \text{m/s})^2 + \frac{1}{2}(4.00\ \text{kg})(10.0\ \text{m/s})^2\right] = \underline{210\ \text{J}}$$

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

8.6 COLLISIONS OF POINT MASSES IN TWO DIMENSIONS

49. ***Professional Application*** *Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei* $\left({}^{4}\text{He}\right)$ *from gold-197 nuclei* $\left({}^{197}\text{Au}\right)$. *The energy of the incoming helium nucleus was* $8.00 \times 10^{-13}\ \text{J}$, *and the masses of the helium and gold nuclei were* $6.68 \times 10^{-27}\ \text{kg}$ *and* $3.29 \times 10^{-25}\ \text{kg}$, *respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of* $120°$ *during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?*

Solution (a) $\frac{1}{2}m_1 v_1^2 = \mathrm{KE_i} \Rightarrow v_\mathrm{i} = \left(\frac{2\mathrm{KE_i}}{m_1}\right)^{1/2} = \left[\frac{2\left(8.00\times10^{-13}\ \mathrm{J}\right)}{6.68\times10^{-27}\ \mathrm{kg}}\right]^{1/2} = 1.548\times10^{7}\ \mathrm{m/s}$

Conservation of internal kinetic energy gives:

$$\frac{1}{2}m_1 {v_1}^2 = \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_1 v'^2_2 \qquad \text{(i)}$$

or $$\frac{m_1}{m_2}\left(v'^2_1 - v'^2_{2_1'}\right) = v'^2_2 \qquad \text{(i')}$$

Conservation of momentum along the *x*-axis gives:

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2 \qquad \text{(ii)}$$

Conservation of momentum along the *y*-axis gives:

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2 \qquad \text{(iii)}$$

Rearranging Equations (ii) and (iii) gives:

$$m_1 v_1 - m_1 v'_1 \cos\theta_1 = m_2 v'_2 \cos\theta_2 \qquad \text{(ii')}$$

$$-m_1 v'_1 \sin\theta_1 = m_2 v'_2 \sin\theta_2 \qquad \text{(iii')}$$

Squaring Equation (ii') and (iii') and adding gives:

$$m_2^2 v'^2_2 \cos^2\theta_2 + m_2^2 v'^2_2 \sin^2\theta_2 = (m_1 v_1 - m_1 v'_1 \cos\theta_1)^2 + \left(-m_1 v'_1 \cos\theta_1\right)^2$$
$$\text{or}\quad m_2^2 v'^2_2 = m_1^2 v'^2_1 - 2m_1^2 v_1 v'_1 \cos\theta_1 + m_1^2 v'^2_1$$

Solving for v'^2_2 and substituting into (i'):

$$\frac{m_1}{m_2}\left(v_1^2 - v'^2_1\right) = \frac{m_1^2}{m_2^2}\left(v_1^2 + v'^2_1 - 2v_1 v'_1 \cos\theta_1\right)\text{so that}$$
$$v_1^2 - v'^2_1 = \frac{m_1}{m_2}\left(v_1^2 + v'^2_1 - 2v_1 v'_1 \cos\theta_1\right)$$

Using $v_1 = 1.548\times10^7\ \text{m/s}; \theta_1 = 120°; m_1 = 6.68\times10^{-27}\ \text{kg}; m_2 = 3.29\times10^{-25}\ \text{kg}$

$$\left(1+\frac{m_1}{m_2}\right)v'^2_1 - \left(2\frac{m_1}{m_2}v_1\cos\theta_1\right)v'_1 - \left(1-\frac{m_1}{m_2}\right)v_1^2 = 0$$

$$a = 1+\frac{m_1}{m_2} = 1.0203,\ b = -\frac{2m_1}{m_2}v_1\cos\theta_1 = 3.143\times10^5\ \text{m/s},$$

$$c = -\left(1+\frac{m_1}{m_2}\right)v_1^2 = -2.348\times10^{14}\ \text{m}^2/\text{s}^2 \text{ so that}$$

$$v'_1 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-3.143\times10^5\ \text{m/s} + \sqrt{\left(3.143\times10^5\ \text{m/s}\right)^2 - 4(1.0203)\left(-2.348\times10^{14}\ \text{m}^2/\text{s}^2\right)}}{2(1.0203)} \text{ or}$$

$$v'_1 = \underline{1.50\times10^7\ \text{m/s}} \text{ and } v'_2 = \sqrt{\frac{m_1}{m_2}\left(v_1^2 - v'_1\right)} = \underline{5.36\times10^5\ \text{m/s}}$$

$$\tan\theta_2 = \frac{-v'_1\sin\theta_1}{v_1 - v'_1\cos\theta_1} = \frac{=\left(1.50\times10^7\ \text{m/s}\right)\sin120°}{1.58\times10^7\ \text{m/s} - \left(1.50\times10^7\ \text{m/s}\right)\cos120°} = -0.56529$$
$$\text{or } \theta_2 = \tan^{-1}(-0.56529) = \underline{-29.5°}$$

(b) The final kinetic energy is then:

$$\text{KE}_\text{f} = (0.5)m_1v'^2_1 = (0.5)\left(6.68\times10^{-27}\ \text{kg}\right)\left(1.50\times10^7\ \text{m/s}\right)^2 = \underline{7.52\times10^{-13}\ \text{J}}$$

8.7 INTRODUCTION TO ROCKET PROPULSION

55. ***Professional Application*** *Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of* $2.00\times10^3\ \text{m/s}$*. You may assume the gravitational force is negligible at the probe's location.*

Solution Use the equation $v = v_0 + v_\text{e}\ln\left(\frac{m_0}{m}\right)$, where

$m_0 = 4000\ \text{kg}, m = 4000\ \text{kg} - 3500\ \text{kg} = 500\ \text{kg}, \text{and } v_\text{e} = 2.00\times10^3\ \text{m/s}$ so that

$$v - v_0 = (2.00\times10^3 \text{ m/s})\ln\left(\frac{4000\text{ kg}}{500\text{ kg}}\right) = 4.159\times10^3 \text{ m/s} = \underline{4.16\times10^3 \text{ m/s}}$$

57. *Derive the equation for the vertical acceleration of a rocket.*

Solution The force needed to give a small mass Δm an acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed v_e requires $v_\text{e} = a_{\Delta m}\Delta t$, so $F = v_\text{e}\dfrac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_\text{thrust} = v_e\dfrac{\Delta m}{\Delta t}$, where all quantities are positive.

Applying Newton's second law to the rocket gives $F_\text{thrust} - mg = ma \Rightarrow a = \dfrac{v_e}{m}\dfrac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

61. ***Professional Application*** *(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?*

Solution (a) First, find v'_1, the velocity after ejecting the fluid:

$$(m_1 + m_2)v = 0 = m_1 v'_1 + m_2 v'_2\text{, so that}$$

$$v'_1 = \frac{-m_2 v'_2}{m_1} = \frac{-(0.250\text{ kg})(10.0\text{ m/s})}{4.75\text{ kg}} = \underline{-0.526\text{ m/s}}$$

Now, the frictional force slows the squid over the 0.100 s

$$\Delta p = f\Delta t = m_1 v'_{1,\text{f}} + m_2 v'_2,\ \text{gives:}$$

$$v'_{1,\text{f}} = \frac{f\Delta t - m_2 v'_2}{m_1} = \frac{(5.00\text{ N})(0.100\text{ s}) - (0.250\text{ kg})(10.0\text{ m/s})}{4.75\text{ kg}} = \underline{-0.421\text{ m/s}}$$

(b) $\Delta\text{KE} = \dfrac{1}{2}m_1 v'^2_{1,\text{f}} - \dfrac{1}{2}m_1 v'^2_1 = \dfrac{1}{2}m_1\left(v'^2_{1,\text{f}} - v'^2_1\right)$

$$= \frac{1}{2}(4.75\text{ kg})\left[(0.421\text{ m/s})^2 - (0.526\text{ m/s})^2\right] = \underline{-0.236\ \text{J}}$$

CHAPTER 9: STATICS AND TORQUE

9.2 THE SECOND CONDITION FOR EQUILIBRIUM

1. *(a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?*

Solution (a) To calculate the torque use $\tau = r_{\perp}F$, where the perpendicular distance is 0.850 m, the force is 55.0 N, and the hinges are the pivot point.

$$\tau = r_{\perp}F \; = 0.850\ \text{m} \times 55.0\ \text{N} = 46.75\ \text{N}\cdot\text{m} = \underline{46.8\ \text{N}\cdot\text{m}}$$

(b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

9.3 STABILITY

6. *Suppose a horse leans against a wall as in Figure 9.32. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal and opposite to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.*

Solution There are four forces acting on the horse and rider: **N** (acting straight up the ground), **w** (acting straight down from the center of mass), **f** (acting horizontally to the left, at the ground to prevent the horse from slipping), and $\mathbf{F}_{\text{wall}}$ (acting to the right). Since nothing is moving, the two conditions for equilibrium apply: $\text{net } F = 0$ and $\text{net } \tau = 0$.

The first condition leads to two equations (one for each direction):

$$\text{net } F_x = F_{\text{wall}} - f = 0 \text{ and net } F_y = N - w = 0$$

The torque equation (taking torque about the center of gravity, where CCW is positive) gives: $\text{net } \tau = F_{\text{wall}}(1.40 - 1.20) - f(1.40 \text{ m}) + N(0.350 \text{ m}) = 0$

The first two equations give: $F_{\text{wall}} = f$, and $N = w = mg$

Substituting into the third equation gives:

$$F_{\text{wall}}(1.40 \text{ m} - 1.20 \text{ m}) - F_{\text{wall}}(1.40 \text{ m}) = -mg(0.350 \text{ m})$$

So, the force on the wall is:

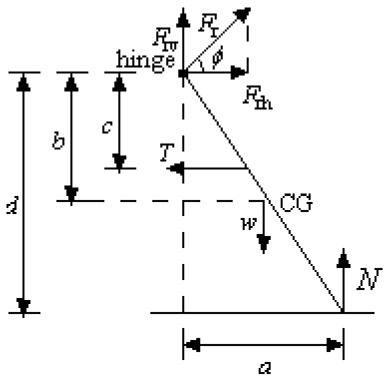
$$F_{\text{wall}} = \frac{mg(0.350 \text{ m})}{1.20 \text{ m}} = \frac{(500 \text{ kg})(9.80 \text{ m/s}^2)(0.350 \text{ m})}{1.20 \text{ m}} = 1429 \text{ N} = \underline{1.43 \times 10^3 \text{ N}}$$

14. *A sandwich board advertising sign is constructed as shown in Figure 9.36. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?*

Solution Looking at Figure 9.36, there are three forces acting on the entire sandwich board system: $\mathbf{w}$, acting down at the center of mass of the system, $\mathbf{N}_\text{L}$ and $\mathbf{N}_\text{R}$, acting up at the ground for EACH of the legs. The tension and the hinge exert internal forces, and therefore cancel when considering the entire sandwich board. Using the first condition for equilibrium gives: $\text{net } F = N_\text{L} + N_\text{R} - w_\text{S}$.

The normal forces are equal, due to symmetry, and the mass is given, so we can determine the normal forces: $2N = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) \Rightarrow N = 39.2N$

Now, we can determine the tension in the chain and the force due to the hinge by using the one side of the sandwich board:



$$a=\frac{1.10}{2}=0.550\text{ m}, b=\frac{1.30\text{ m}}{2}=0.650\text{ m}, c=0.500\text{ m}, d=1.30\text{ m},$$

$$N=39.2\text{ N}, w=mg=\frac{8.00\text{ kg}}{2}\left(9.80\text{ m/s}^2\right)=39.2\text{ N (for one side)}$$

$$F_{\text{rv}}=F_{\text{r}}\sin\phi, F_{\text{rh}}=F_{\text{r}}\cos\phi$$

The system is in equilibrium, so the two conditions for equilibrium hold:

$$\text{net } F=0 \text{ and net } \tau=0$$

This gives three equations:

$$\text{net } F_x=F_{\text{rh}}-T=0$$
$$\text{net } F_y=F_{\text{rv}}-w+N=0$$
$$\text{net } \tau=-Tc-w\frac{a}{2}+Na=0$$

(Pivot at hinge)

$$\text{Giving } F_{\text{rh}}=T=F_{\text{r}}\cos\phi,$$
$$F_{\text{rv}}=F_{\text{r}}\sin\phi, F_{\text{rh}}=F_{\text{r}}\cos\phi$$
$$F_{\text{rv}}=w-N=F_{\text{r}}\sin\phi, \text{ and } Tc+w\frac{a}{2}=Na$$

(a) To solve for the tension, use the third equation:

$$Tc=Na-w\frac{a}{2}\Rightarrow T=\frac{Na}{c}-\frac{wa}{2c}=\frac{wa}{2c}$$

Since $N=w$

Therefore, substituting in the values gives: $T = \dfrac{(39.2\ \text{N})(0.550\ \text{m})}{2(0.500\ \text{m})} = \underline{21.6\ \text{N}}$

(b) To determine the force of the hinge, and the angle at which it acts, start with the second equation, remembering that $N = w$, $F_{\text{rv}} = w - N \Rightarrow F_{\text{rv}} = 0$.

Now, the first equation says: $F_{\text{rh}} = T$, so F_{r} cannot be zero, but rather $\phi = 0$, giving a force of $F_r = \underline{21.6\ \text{N (acting horizontally)}}$

9.6 FORCES AND TORQUES IN MUSCLES AND JOINTS

32. *Even when the head is held erect, as in Figure 9.42, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?*

Solution (a) Use the second condition for equilibrium:

$$\text{net } \tau = F_{\text{M}}(0.050\ \text{m}) - w(0.025\ \text{m}) = 0, \text{ so that}$$
$$F_{\text{M}} = w\frac{0.025\ \text{m}}{0.050\ \text{m}} = (50\ \text{N})\frac{0.025\ \text{m}}{0.050\ \text{m}} = \underline{25\ \text{N downward}}$$

(b) To calculate the force on the joint, use the first condition of equilibrium:

$$\text{net } F_y = F_{\text{J}} - F_{\text{M}} - w = 0, \text{ so that}$$
$$F_{\text{J}} = F_{\text{M}} + w = (25\ \text{N}) + (50\ \text{N}) = \underline{75\ \text{N upward}}$$