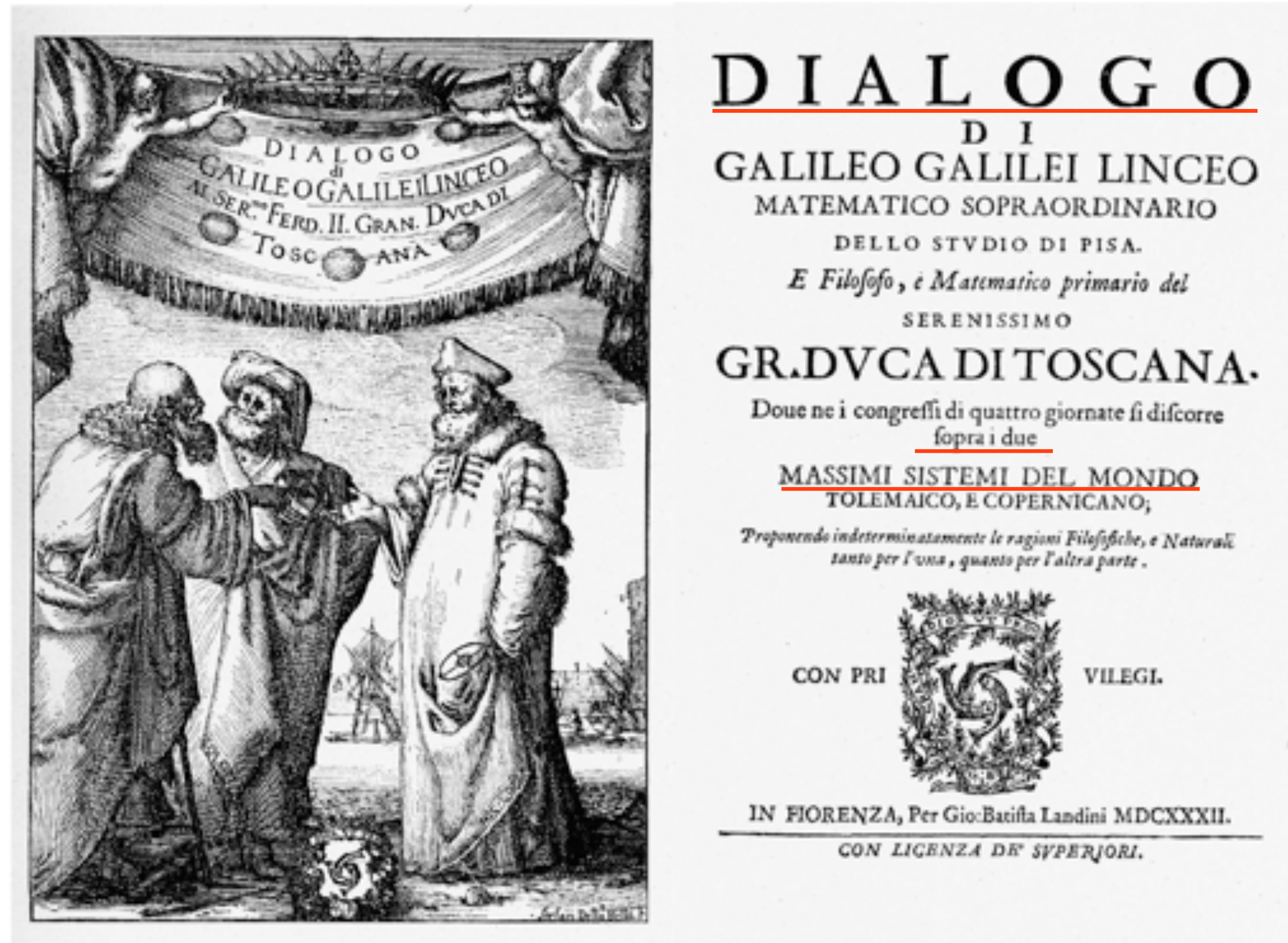
1.1. Galilean Relativity

Galileo Galilei
1564 - 1642



DIALOGO
DI
GALILEO GALILEI LINCEO
MATEMATICO SOPRAORDINARIO
DELLO STVDIO DI PISA.
E Filosofo, e Matematico primario del
SERENISSIMO
GR.DVCA DI TOSCANA.
Doue ne i congressi di quattro giornate si discorre
sopra i due
MASSIMI SISTEMI DEL MONDO
TOLEMAICO, E COPERNICANO;
Proponendo indeterminatamente le ragioni Filosofiche, e Naturali
tanto per l'una, quanto per l'altra parte.
CON PRI VILEGI.
IN FIORENZA, Per Gio:Batista Landini MDCXXXII.
CON LICENZA DE' SVPERIORI.

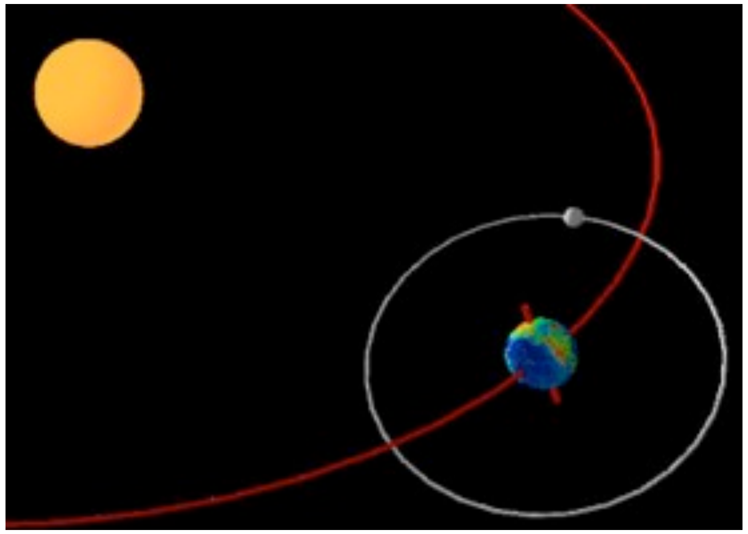
Dialogue Concerning the Two Chief World Systems

The fundamental laws of physics are the same in all frames of reference moving with constant velocity with respect to one another.

Metaphor of Galileo's Ship

Ship traveling at constant speed on a smooth sea. Any observer doing experiments (playing billiard) under deck would not be able to tell if ship was moving or stationary.

Today we can make the same observation on a plane.

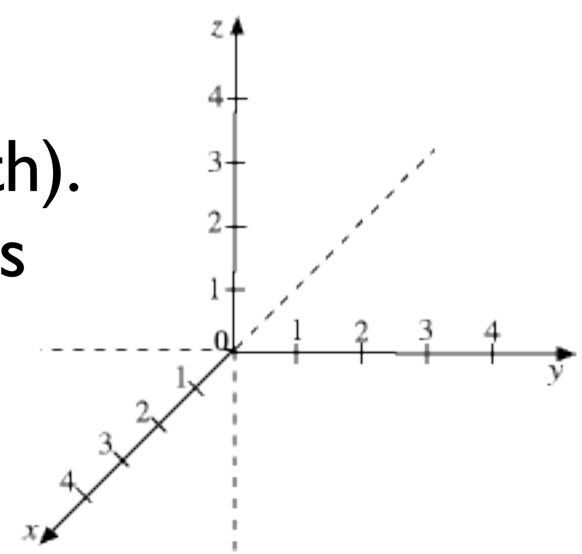


Even better: Earth is orbiting around sun at $v \approx 30 \text{ km/s}$!

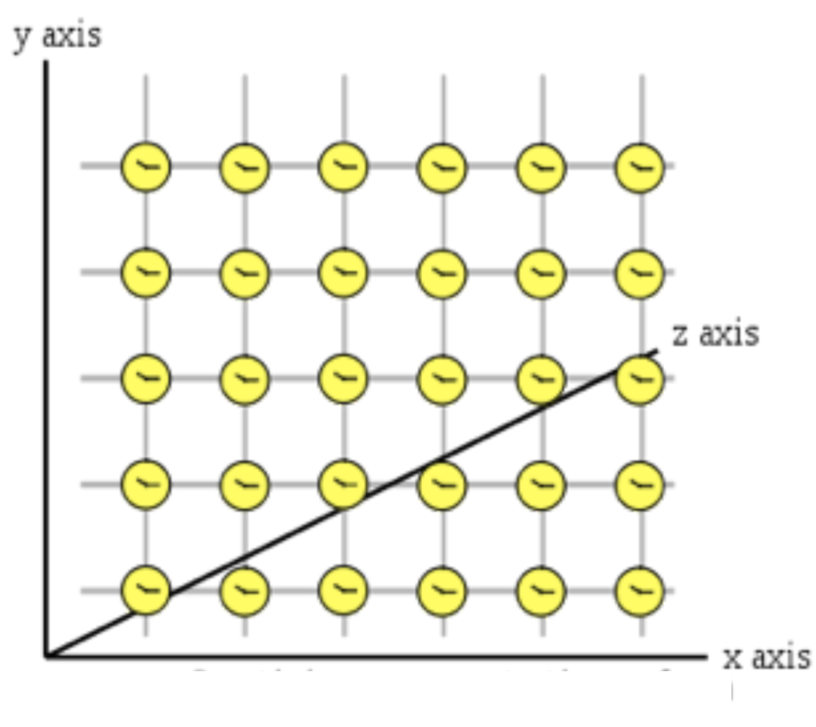
1.2. Frames of Reference

- Special Relativity is concerned with events in space and time
- Events are labeled by a time and a position relative to a particular frame of reference (e.g. the sun, the earth, the cabin under deck of Galileo's ship)

$$E = (t, x, y, z)$$



- Pick spatial coordinate frame (origin, coordinate axes, unit length). In the following, we will always use cartesian coordinate systems
- Introduce clocks to measure time of an event. Imagine a clock at each position in space, all clocks synchronized, define origin of time



<u>Rest frame of an object:</u>
frame of reference in which the object is not moving

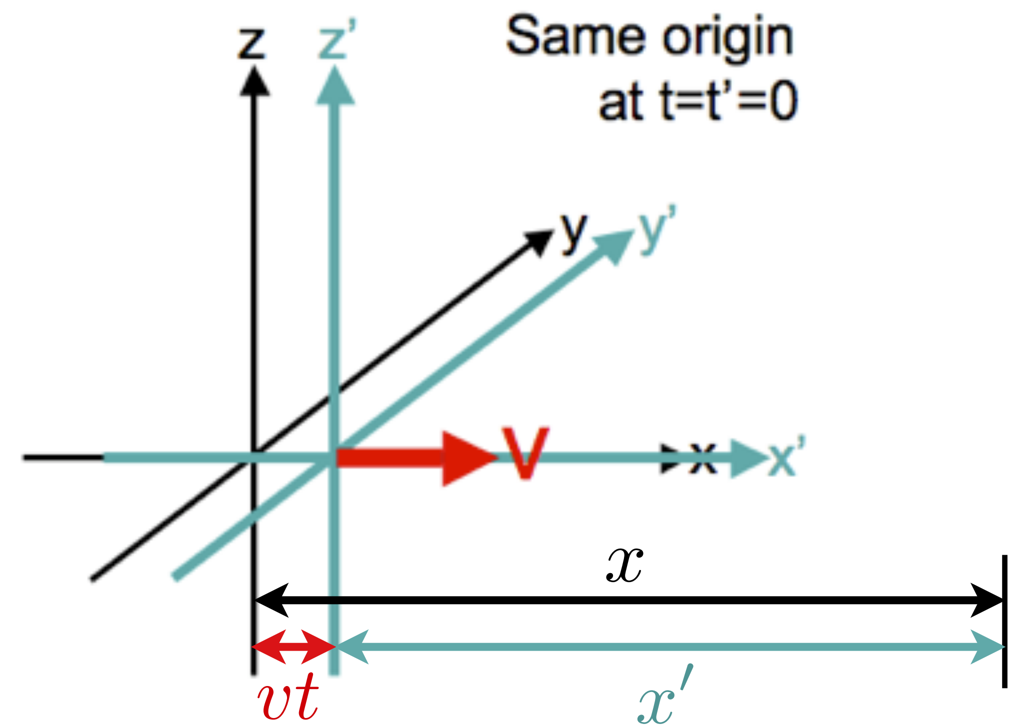
<u>Inertial frame of reference:</u>
frame of reference in which an isolated object experiencing no force moves on a straight line at constant velocity

1.3. Galilean Transformation

Two reference frames (S and S') moving with velocity $\vec{v}$ to each other. If an event has coordinates (t, x, y, z) in S, what are its coordinates (t', x', y', z') in S'?

in the following, we will always assume the "standard configuration":

- Axes of S and S' parallel
- $\vec{v}$ parallel to x-direction
- Origins coincide at $t = t' = 0$



Galileo Galilei
1564 - 1642

Galilean Transf.

$$
\begin{aligned}
t' &= t \\
x' &= x - vt \\
y' &= y \\
z' &= z
\end{aligned}
$$

Time is absolute

This will be modified in Special Relativity!
$\rightarrow$ Lorentz Transformation

1.4. Newton's Laws of Motion

Sir Isaac Newton
1643 - 1727

- Newton adhered to Galileo's relativity principle but believed in a frame of absolute rest
- <u>Time is absolute:</u> no difference of time in different reference frames
- Newton's three laws of motion (1687) dominated scientific view of the physical universe for the next three centuries
- Newton demonstrated the consistency between his theory and Kepler's laws (1609, 1619) of planetary motion

Johannes Kepler
1571 -1630

Newton's laws of motion

(N1) There exists frames of reference relative to which a particle acted on by no forces moves on a straight line at constant speed.

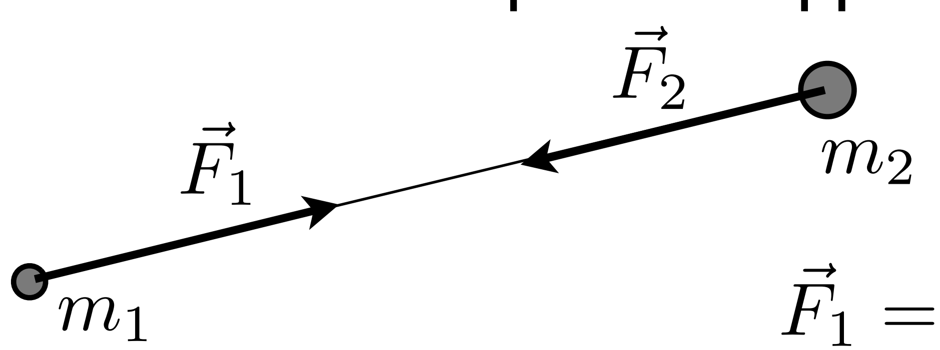
(This law about the existence of inertial frames of reference has not and possible could not been confirmed experimentally but it is nevertheless accepted as a true statement.)

(N2) If a particle is measured in an inertial frame of reference to undergo an *acceleration* $\vec{a}$, then this is a consequence of the action of a *force* $\vec{F}$, where

$$\vec{F} = m\vec{a}$$

with m the *mass* of the particle.

(N3) To every action there is an equal and opposite reaction.



$$\vec{F}_1 = -\vec{F}_2$$

Alternative Formulation of Newton's 3rd law

(N3')

In the absence of any external forces, the *total momentum*

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i$$

of a system is constant.

<u>Proof:</u>

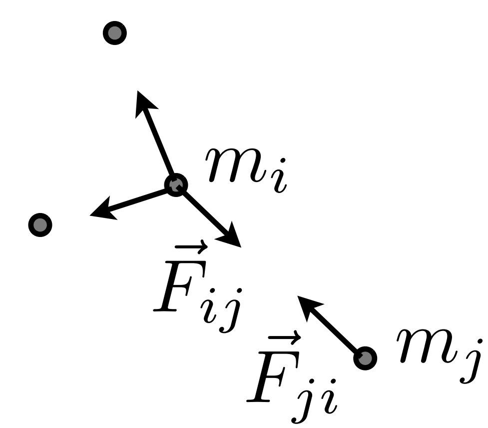
(N2)

$$\frac{\mathrm{d}\vec{P}}{\mathrm{d}t} = \sum_i m_i \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i$$

$$= \sum_i \sum_{j(\neq i)} \vec{F}_{ij} = \sum_{ij}^{i \neq j} \vec{F}_{ij} = 0 \Rightarrow \vec{P} = \mathrm{const}$$

(N3)

$$\vec{F}_{ij} = -\vec{F}_{ji}$$



1.5. Galilean invariance of Newton's laws

S: $(t,x,y,z)=(t,\vec{r})$ $\quad \xrightarrow{\; t' = t, \;\; \vec{r}\,' = \vec{r}-\vec{v}t \;} \quad$ S': $(t',x',y',z')=(t',\vec{r}\,')$

$$t' = t \qquad \vec{r}\,' = \vec{r}-\vec{v}t$$

(N1): S inertial frame of reference $\Longleftrightarrow$ $(\vec{F}=0 \Rightarrow \vec{r}=\vec{r}_0+\vec{v}_0 t)$ (straight line, const. velocity)

in S': $\vec{r}\,' = \vec{r}-\vec{v}t = \vec{r}_0+\vec{v}_0 t-\vec{v}t = \vec{r}_0+(\vec{v}_0-\vec{v})t$

$\Rightarrow$ S' is inertial frame of reference

(N2): in S: $\vec{F} = m\vec{a} = m\dfrac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$

in S': $\vec{F} = m\vec{a}\,' = m\dfrac{\mathrm{d}^2\vec{r}\,'}{\mathrm{d}t^2} = m\dfrac{\mathrm{d}^2}{\mathrm{d}t^2}(\vec{r}-\vec{v}t) = m\dfrac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$

same equation of motion in S and S'

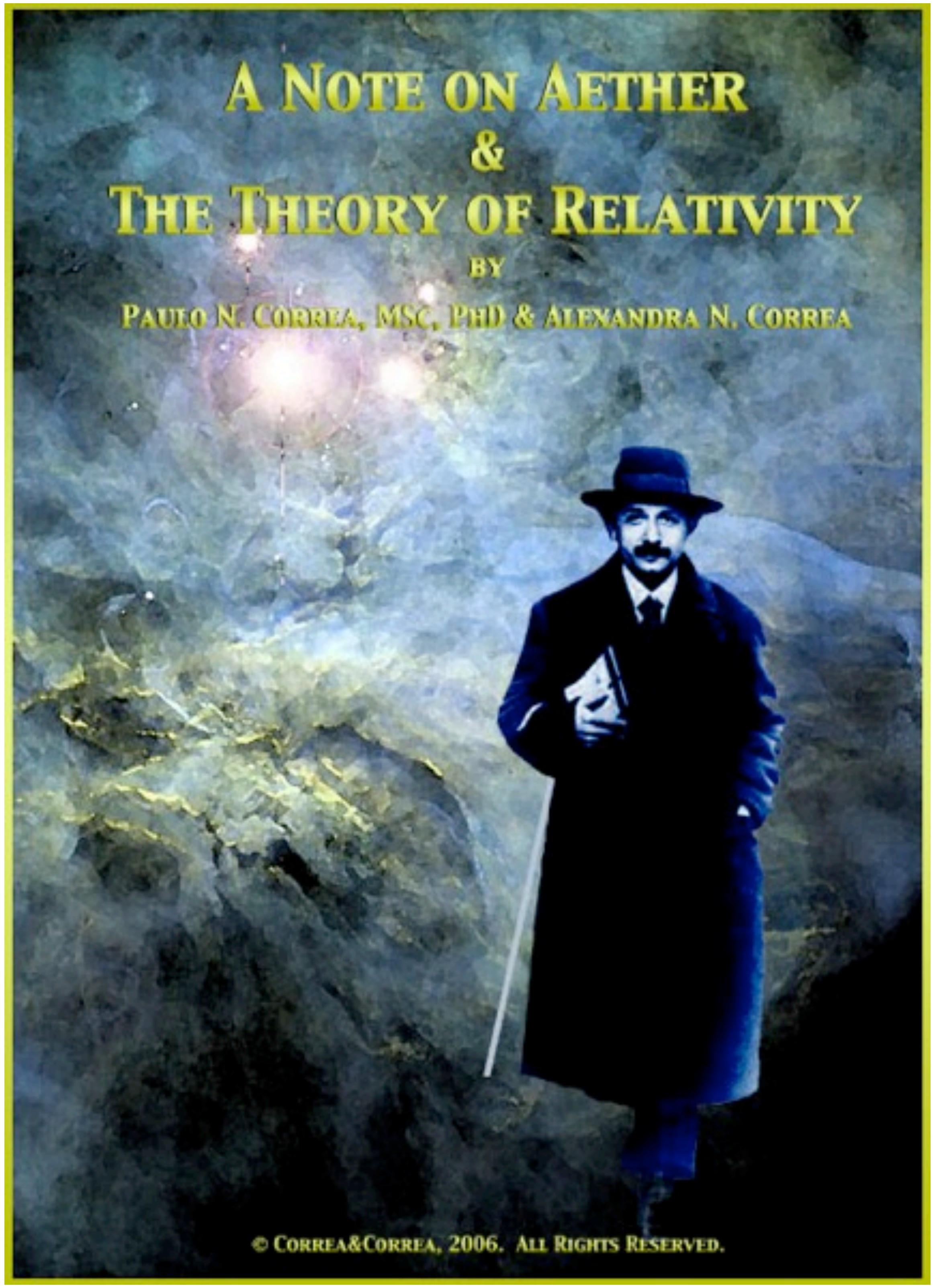
(N3)': total momentum $\vec{P}=\sum_i m_i\vec{v}_i$ conserved in S

in S': $\vec{P}\,' = \sum_i m_i\vec{v}_i\,' = \sum_i m_i\dfrac{\mathrm{d}\vec{r}_i\,'}{\mathrm{d}t} = \sum_i m_i\dfrac{\mathrm{d}}{\mathrm{d}t}(\vec{r}_i-\vec{v}t) = \vec{P}-M\vec{v}$ const.

Newtons's laws of motion are the same in all inertial frames of reference, in agreement with the Galilean relativity principle.

1.6. Light, Maxwell's Equations and the Aether

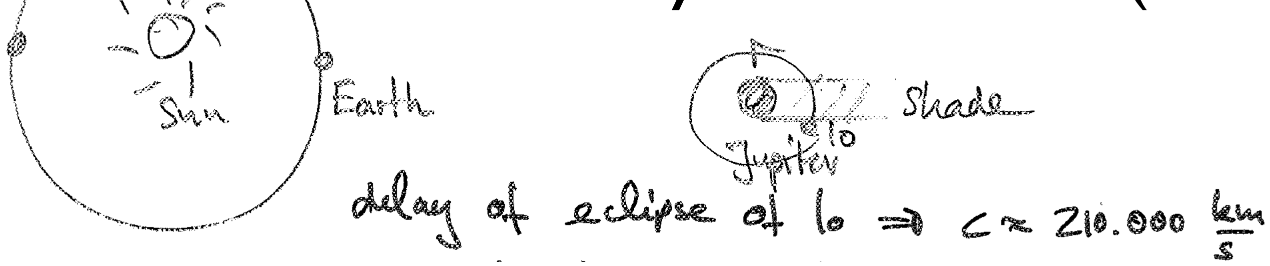
The need for a new relativity principle ...



Speed of Light

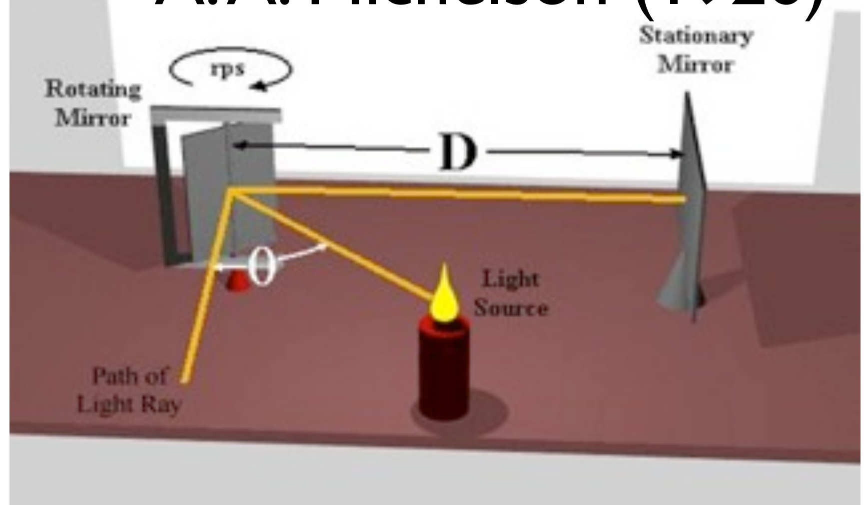
- First attempts to measure speed of light by Galileo Galilei: “Light is either instantaneous or extremely fast”

- First quantitative measurements by Ole Roemer (1675)



Ole Roemer
1644 - 1710

- Rotating mirror: Leon Foucault (1862) - $c = 298000 \pm 500\mathrm{km/s}$
  A.A. Michelson (1926)- $c = 299796 \pm 4\mathrm{km/s}$



Leon Foucault
1819 - 1868

A.A. Michelson
1852 - 1931

- Laser interferometry: Evenson *et al.*, 1972 - $c = 299792.4562 \pm 0.0011\mathrm{km/s}$

- 17th *General Conference on Weights and Measures (1983)*: $c := 299,792,458 \text{ m/s}$

  Redefinition of the meter: "The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

The Nature of Light

- For a long time, two schools of thought:
  1) Light is a wave similar to sound (Christiaan Huygens, 1678)
  2) Light consists of particles (Newton: No bending around obstacles)

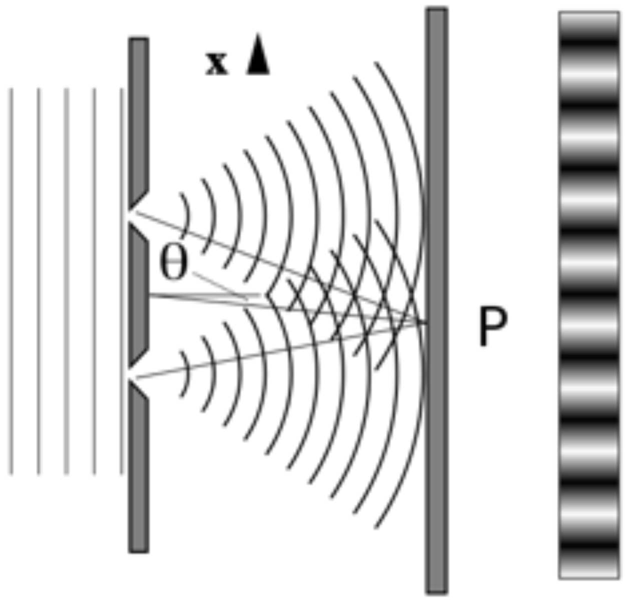
  Both theories were able to explain reflection and refraction of light.

Christiaan Huygens
1629 - 1695

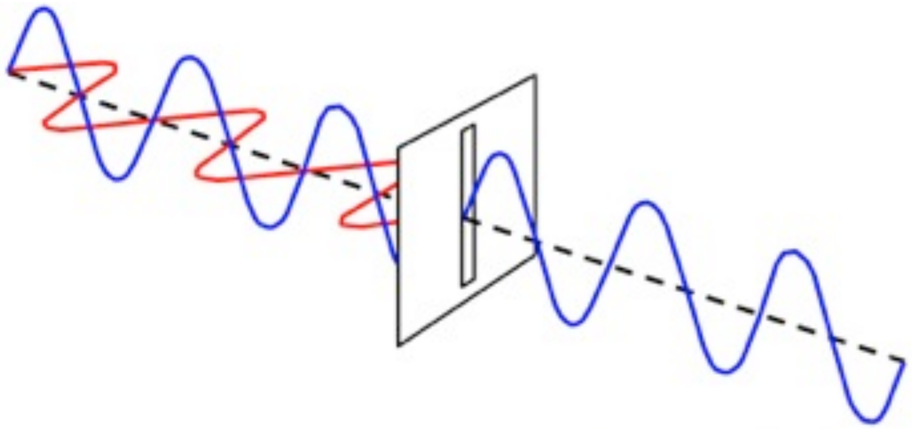
- Huygens suggested that lightwaves propagate in a medium called "luminiferous aether", analogous to soundwaves traveling in air

- Contributions by Thomas Young (1806) and Augustine Fresnel (1816) confirmed wave nature of light

a) Interference



b) Polarization ⇒ transversal wave



Thomas Young
1773 - 1829

Augustine Fresnel
1788 - 1827

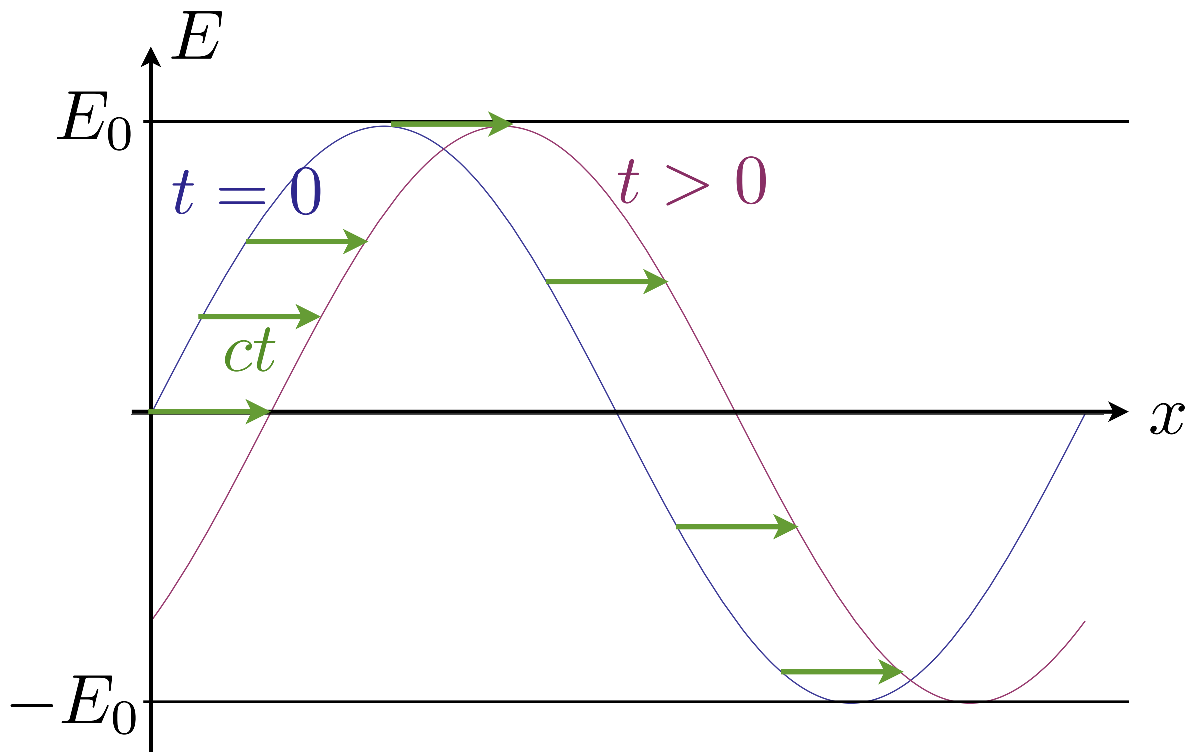
Theory of Electromagnetism

- After considerable work by many scientists, Maxwell (1864) developed an accurate theory of electromagnetism.
- He first proposed that light was e.m. radiation and that there was only one aetheral medium for all e.m. phenomena
- From Maxwell's equations it follows that electric field $E(x,t)$ obeys a wave equation

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$$

James Clerk Maxwell
1831 - 1879

solution: $E(x,t) = E_0 \sin[\omega(x - ct)]$



Electric field induces magnetic field and vise versa, $\vec{E} \perp \vec{B}$

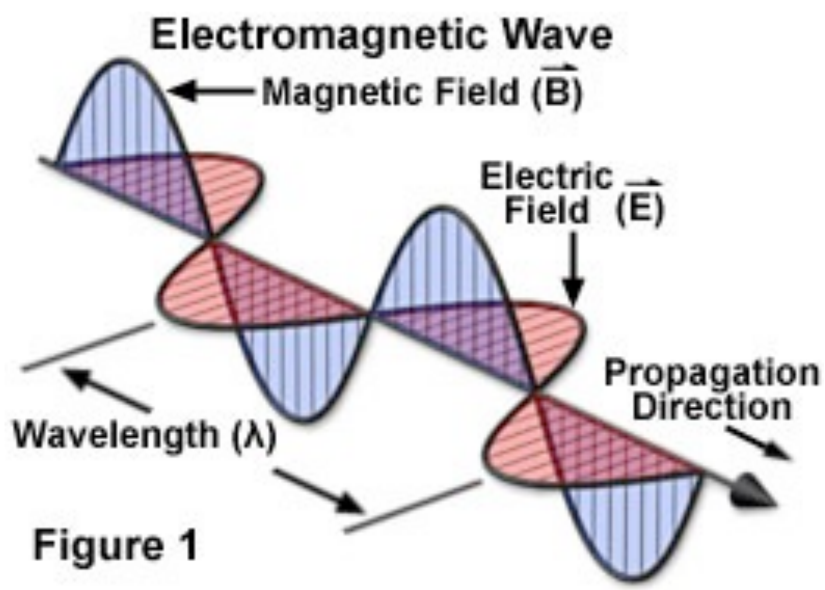
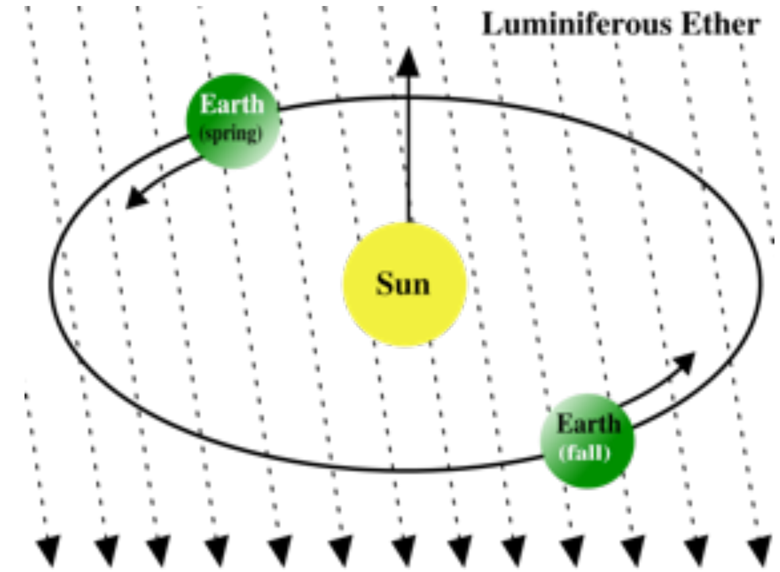


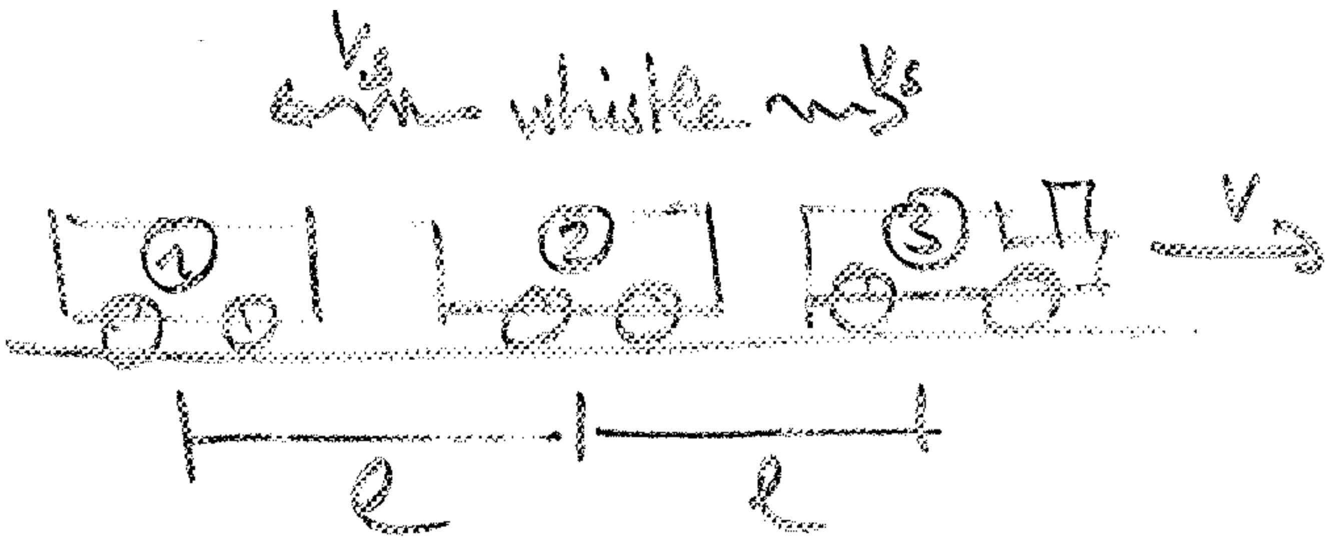
Figure 1

Problem: Maxwell's equations are NOT invariant under the Galilean transformation! Is this theory valid only in the rest frame of the aether?

Search for the Aether



- Earth orbits around the sun, therefore it should move relative to the aether. We should be able to measure this relative motion!
- Analogy with sound propagating in air



- velocity of sound always v_s with respect to air
- before set-off 1,2 and 3 synchronize watches
- 2 notes when he blows whistle, 1 and 3 note when they hear it

$$\Delta t_1 = \frac{l}{v_s + v}$$

$$\Delta t_3 = \frac{l}{v_s - v}$$

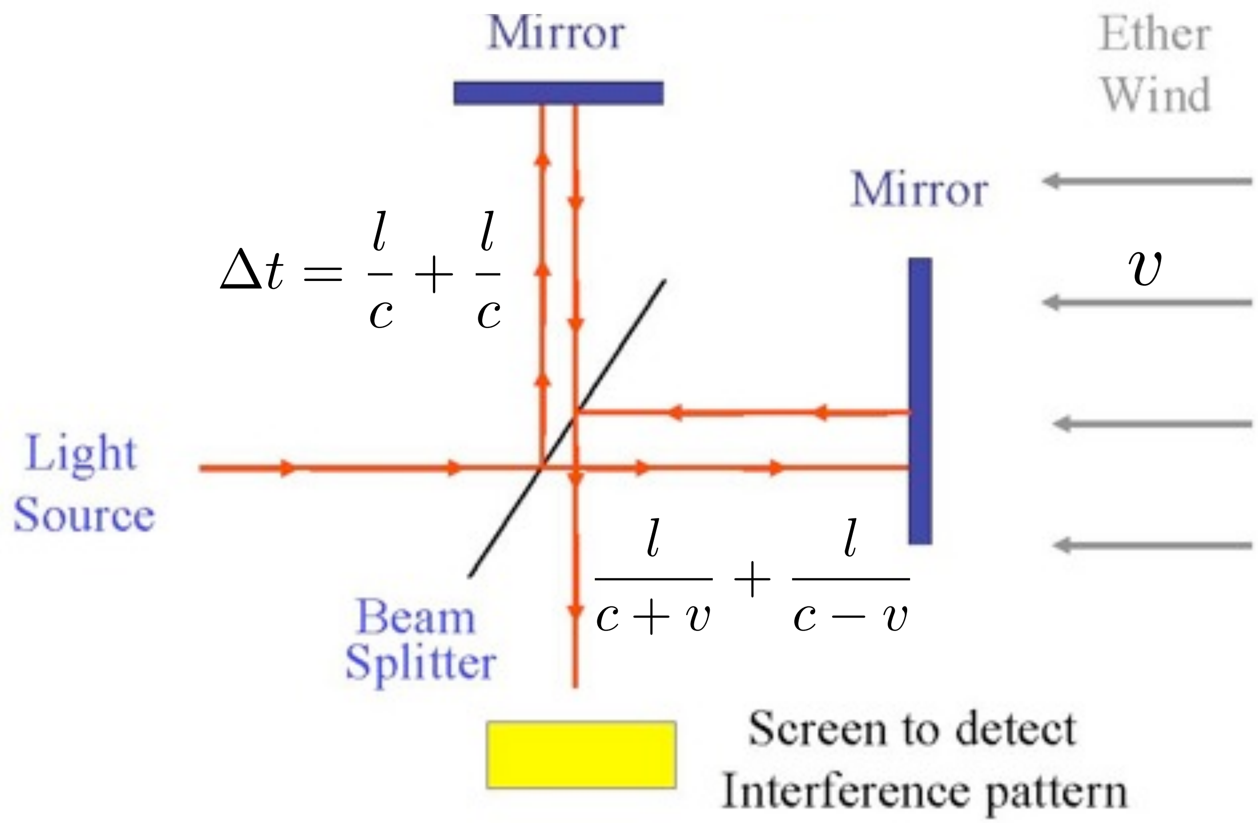
$$\Rightarrow \Delta t_1 < \Delta t_3$$

Because of this result we can tell that train is moving relative to the air.
Alternatively, we can stick our head out of the window and feel the wind!

Can we stick our head out of the cosmic window and feel the aetheral wind?

Michelson/Morley experiment

- Lots of experiments equivalent to the train-whistle experiment have been performed with light, ALWAYS find $\Delta t_1 = \Delta t_3$
- Most prominently, the experiment by Michelson and Morley (1887)



A.A. Michelson
1852 - 1931

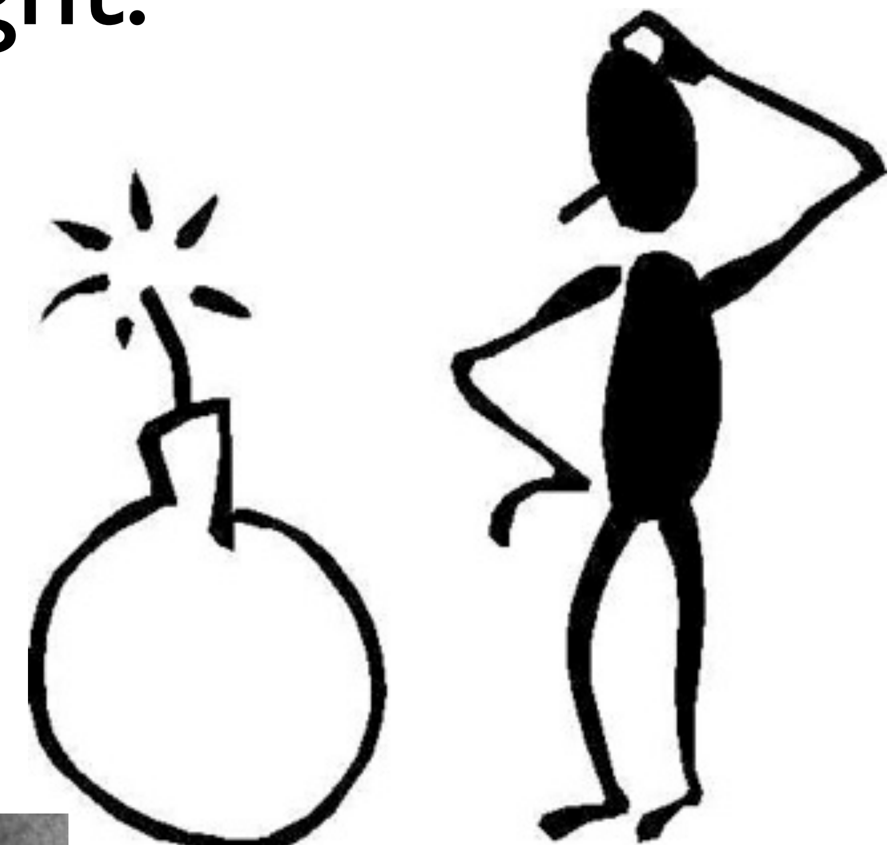
E. Morley
1838 - 1923

<u>Simplest interpretation:</u> Light does not require a medium!

Other ideas have been proposed, e.g. that the earth dragged the aether in its immediate vicinity along with it (Stokes 1845), but no theory was able to explain such mechanical properties.

Without the Aether, there is no preferred reference frame for light.

How to reconcile Maxwell's equations with the notion of relativity?



Galileo Galilei
1564 - 1642

James Clerk Maxwell
1831 - 1879

Three possibilities...

(A) The Galilean transformation was correct and there was something wrong with Maxwell's equations.

NO! Maxwell's equation proved to be extremely successful in application.

(B) The Galilean transformation applied to Newtonian mechanics only.

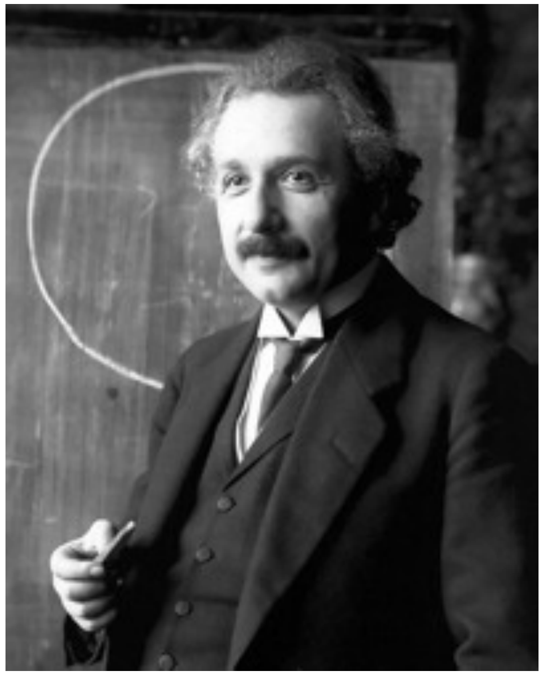
NO! The transformation between frames of reference should be fundamental and not depend on which physical phenomena we are looking at.

(C) The Galilean transformation and the Newtonian relativity principle based on this transformation were wrong. There exists a new relativity principle for both mechanics and electrodynamics that was not based on the Galilean transformation.

1.7. Einstein's Postulates

- Einstein developed axiomatic Theory of Special Relativity (1905) specifying properties of space and time

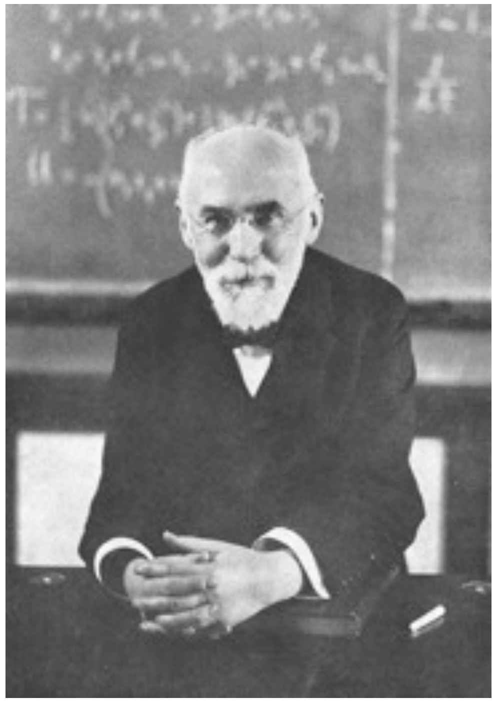
  $\Longrightarrow$ Unifying relativity principle based on the Lorentz Transformation (1899,1904)



Albert Einstein
1879 - 1955

- Lorentz was the first to realize that Maxwell's equations are invariant under this transformation

- In 1905, Poincare was the first to recognize that the transformation has the properties of a mathematical group and named it after Lorentz



Hendrik Lorentz
1853 - 1928

Henri Poincare
1854 - 1912

Einstein's Postulates:

(E1) ALL the laws of physics are the same in every inertial frame of reference.

(E2) The speed of light is independent of the motion of its source.