5 FURTHER APPLICATIONS OF NEWTON'S LAWS: FRICTION, DRAG, AND ELASTICITY

Figure 5.1 Total hip replacement surgery has become a common procedure. The head (or ball) of the patient's femur fits into a cup that has a hard plastic-like inner lining. (credit: National Institutes of Health, via Wikimedia Commons)

Chapter Outline

Introduction: Further Applications of Newton’s Laws

Describe the forces on the hip joint. What means are taken to ensure that this will be a good movable joint? From the photograph (for an adult) in **Figure 5.1**, estimate the dimensions of the artificial device.

It is difficult to categorize forces into various types (aside from the four basic forces discussed in previous chapter). We know that a net force affects the motion, position, and shape of an object. It is useful at this point to look at some particularly interesting and common forces that will provide further applications of Newton’s laws of motion. We have in mind the forces of friction, air or liquid drag, and deformation.

5.1 Friction

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 5.2 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

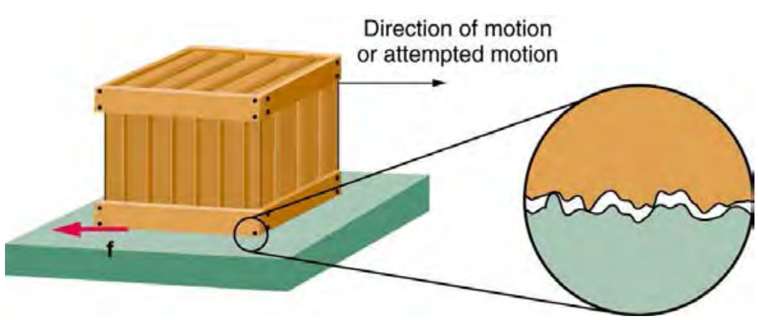

Figure 5.2 Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** $\mathbf{f_s}$ is

$$f_s \leq \mu_s N, \tag{5.1}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction f_s is

$$f_s \leq \mu_s N, \tag{5.2}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_s N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{s(max)}$, the object will move. Thus

$$f_{s(max)} = \mu_s N. \tag{5.3}$$

Once an object is moving, the **magnitude of kinetic friction** $\mathbf{f_k}$ is given by

$$f_k = \mu_k N, \tag{5.4}$$

where μ_k is the coefficient of kinetic friction. A system in which $f_k = \mu_k N$ is described as a system in which *friction behaves simply*.

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k N, \tag{5.5}$$

where μ_k is the coefficient of kinetic friction.

As seen in **Table 5.1**, the coefficients of kinetic friction are less than their static counterparts. That values of μ in **Table 5.1** are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

Table 5.1 Coefficients of Static and Kinetic Friction

System	Static friction μ_s	Kinetic friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\ \text{kg})(9.80\ \text{m/s}^2) = 980\ \text{N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\text{s(max)}} = \mu_\text{s}N = (0.45)(980\ \text{N}) = 440\ \text{N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_\text{k} = \mu_\text{k}N = (0.30)(980\ \text{N}) = 290\ \text{N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (**Figure 5.3**). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

Figure 5.3 Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor’s clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

Example 5.1 Skiing Exercise

A skier with a mass of 62 kg is sliding down a snowy slope. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction was given in to be 45.0 N. Kinetic friction is related to the normal force N as $f_\mathrm{k} = \mu_\mathrm{k} N$; thus, the coefficient of kinetic friction can be found if we can find the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See the skier and free-body diagram in **Figure 5.4**.)

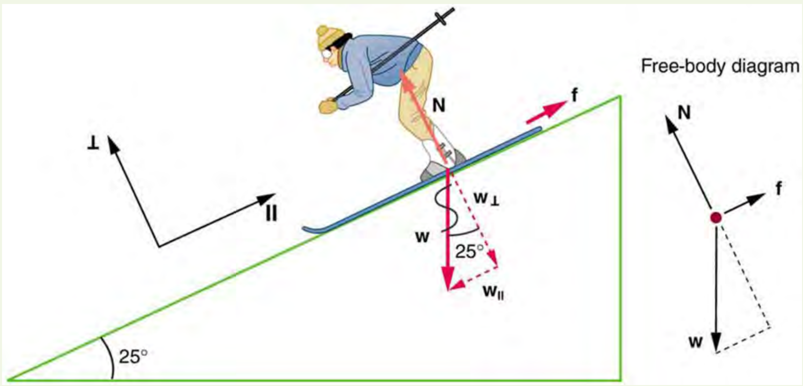

Figure 5.4 The motion of the skier and friction are parallel to the slope and so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). $\mathbf{N}$ (the normal force) is perpendicular to the slope, and $\mathbf{f}$ (the friction) is parallel to the slope, but $\mathbf{w}$ (the skier's weight) has components along both axes, namely $\mathbf{w}_\perp$ and $\mathbf{W}_{//}$. $\mathbf{N}$ is equal in magnitude to $\mathbf{w}_\perp$, so there is no motion perpendicular to the slope. However, $\mathbf{f}$ is less than $\mathbf{W}_{//}$ in magnitude, so there is acceleration down the slope (along the *x*-axis).

That is,

$$N = w_\perp \ = w \cos 25^\mathrm{o} = mg \cos 25^\mathrm{o}. \tag{5.6}$$

Substituting this into our expression for kinetic friction, we get

$$f_\mathrm{k} = \mu_\mathrm{k} mg \cos 25^\mathrm{o}, \tag{5.7}$$

which can now be solved for the coefficient of kinetic friction μ_k .

Solution

Solving for μ_k gives

$$\mu_\mathrm{k} = \frac{f_\mathrm{k}}{N} = \frac{f_\mathrm{k}}{w \cos 25^\mathrm{o}} = \frac{f_\mathrm{k}}{mg \cos 25^\mathrm{o}}. \tag{5.8}$$

Substituting known values on the right-hand side of the equation,

$$\mu_\mathrm{k} = \frac{45.0\ \mathrm{N}}{(62\ \mathrm{kg})(9.80\ \mathrm{m/s}^2)(0.906)} = 0.082. \tag{5.9}$$

Discussion

This result is a little smaller than the coefficient listed in **Table 5.1** for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_\mathrm{k} = \mu_\mathrm{k} mg \cos\theta$. All objects will slide down a slope with constant acceleration under these circumstances. Proof of this is left for this chapter's Problems and Exercises.

Take-Home Experiment

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in **Example 5.1**, the kinetic friction on a slope $f_\mathrm{k} = \mu_\mathrm{k} mg \cos\theta$. The component of the weight down the slope is equal to $mg \sin\theta$ (see the free-body diagram in

Figure 5.4). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out:

$$f_{\mathrm{k}} = Fg_{x} \tag{5.10}$$

$$\mu_{\mathrm{k}} mg \cos\theta = mg \sin\theta. \tag{5.11}$$

Solving for μ_{k} , we find that

$$\mu_{\mathrm{k}} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta. \tag{5.12}$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_{k} . Note that the coin will not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Discuss how this may affect the value for μ_{k} and its uncertainty.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 5.5 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

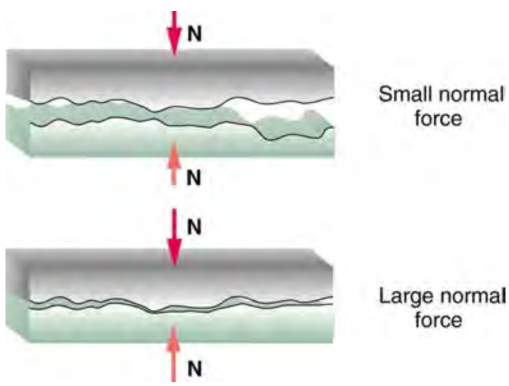

Figure 5.5 Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. Figure 5.6 shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

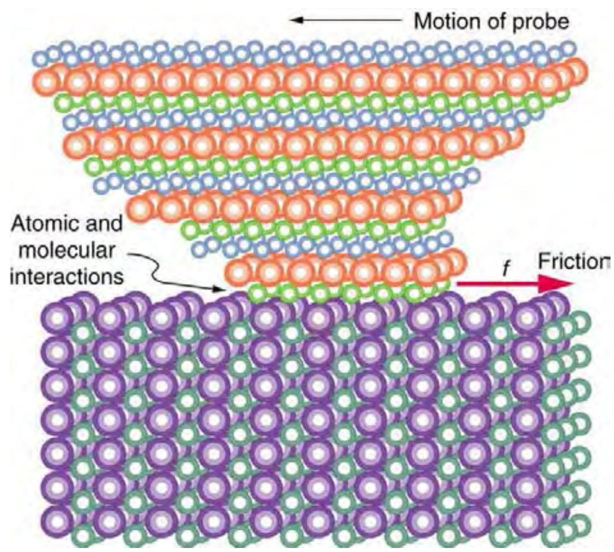

Figure 5.6 The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

PhET Explorations: Forces and Motion

Explore the forces at work when you try to push a filing cabinet. Create an applied force and see the resulting friction force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. Draw a free-body diagram of all the forces (including gravitational and normal forces).

Figure 5.7 Forces and Motion (http://cnx.org/content/m42139/1.7/forces-and-motion_en.jar)

5.2 Drag Forces

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion. Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as bicyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_{D} is found to be proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_{\text{D}} = \frac{1}{2}\text{C}\rho A v^2, \tag{5.13}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_{\text{D}} = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent for these equations as 2 because, when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in a few pages on fluid dynamics, for small particles moving at low speeds in a fluid, the exponent is equal to 1.

Drag Force

Drag force F_{D} is found to be proportional to the square of the speed of the object. Mathematically

$$F_{\text{D}} \propto v^2 \tag{5.14}$$

$$F_{\mathrm{D}} = \frac{1}{2}C\rho Av^2, \tag{5.15}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times. (See **Figure 5.8**). “Aerodynamic” shaping of an automobile can reduce the drag force and so increase a car’s gas mileage.

Figure 5.8 From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed. They are shaped like a bullet with tapered fins. (credit: U.S. Army, via Wikimedia Commons)

The value of the drag coefficient, C , is determined empirically, usually with the use of a wind tunnel. (See **Figure 5.9**).

Figure 5.9 NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we will assume that it is a constant here. **Table 5.2** lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Table 5.2 Drag Coefficient Values **Typical values of drag coefficient C.**

Object	C
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics, and won the gold medal for the 400 m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (See **Figure 5.10**). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Figure 5.10 Body suits, such as this LZR Racer Suit, have been credited with many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as given by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his *terminal velocity* (v_t). Since F_D is proportional to the speed, a heavier skydiver must go faster for F_D to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{net} = mg - F_D = ma = 0. \tag{5.16}$$

Thus,

$$mg = F_D. \tag{5.17}$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}\rho C A v^2. \tag{5.18}$$

Solving for the velocity, we obtain

$$v = \sqrt{\frac{2mg}{\rho CA}}. \tag{5.19}$$

Assume the density of air is $\rho = 1.21\ \text{kg/m}^3$. A 75-kg skydiver descending head first will have an area approximately $A = 0.18\ \text{m}^2$ and a drag coefficient of approximately $C = 0.70$. We find that

$$\begin{aligned} v &= \sqrt{\frac{2(75\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(0.70)(0.18\ \text{m}^2)}} \\ &= 98\ \text{m/s} \\ &= 350\ \text{km/h}. \end{aligned} \tag{5.20}$$

This means a skydiver with a mass of 75 kg achieves a maximum terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

Take-Home Experiment

This interesting activity examines the effect of weight upon terminal velocity. Gather together some nested coffee filters. Leaving them in their original shape, measure the time it takes for one, two, three, four, and five nested filters to fall to the floor from the same height (roughly 2 m). (Note that, due to the way the filters are nested, drag is constant and only mass varies.) They obtain terminal velocity quite quickly, so find this velocity as a function of mass. Plot the terminal velocity v versus mass. Also plot v^2 versus mass. Which of these relationships is more linear? What can you conclude from these graphs?

Example 5.2 A Terminal Velocity

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_{\text{net}} = 0$. Thus the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho C A v^2$.

Thus the terminal velocity v_t can be written as

$$v_t = \sqrt{\frac{2mg}{\rho CA}}. \tag{5.21}$$

Solution

All quantities are known except the person's projected area. This is an adult (82 kg) falling spread eagle. We can estimate the frontal area as

$$A = (2\ \text{m})(0.35\ \text{m}) = 0.70\ \text{m}^2. \tag{5.22}$$

Using our equation for v_t, we find that

$$\begin{aligned} v_t &= \sqrt{\frac{2(85\ \text{kg})(9.80\ \text{m/s}^2)}{(1.21\ \text{kg/m}^3)(1.0)(0.70\ \text{m}^2)}} \\ &= 44\ \text{m/s}. \end{aligned} \tag{5.23}$$

Discussion

This result is consistent with the value for v_t mentioned earlier. The 75-kg skydiver going feet first had a $v = 98\ \text{m/s}$. He weighed less but had a smaller frontal area and so a smaller drag due to the air.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You don't reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J.B.S. Haldane, titled “On Being the Right Size.”

To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal’s length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by **Stokes’ law**, which states that

$$F_s = 6\pi r\eta v, \tag{5.24}$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object’s velocity.

Stokes’ Law

$$F_s = 6\pi r\eta v, \tag{5.25}$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object’s velocity.

Good examples of this law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1\ \mu m$) can be about $2\ \mu m/s$. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell. Sediment in a lake can move at a greater terminal velocity (about $5\ \mu m/s$), so it can take days to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fishes, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spear head as the flock forms a streamlined pattern (see **Figure 5.11**). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Figure 5.11 Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: Julo, Wikimedia Commons)

Galileo’s Experiment

Galileo is said to have dropped two objects of different masses from the Tower of Pisa. He measured how long it took each to reach the ground. Since stopwatches weren’t readily available, how do you think he measured their fall time? If the objects were the same size, but with different masses, what do you think he should have observed? Would this result be different if done on the Moon?

PhET Explorations: Masses & Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

PhET Interactive Simulation

Figure 5.12 Masses & Springs (http://cnx.org/content/m42080/1.9/mass-spring-lab_en.jar)

5.3 Elasticity: Stress and Strain

We now move from consideration of forces that affect the motion of an object (such as friction and drag) to those that affect an object's shape. If a bulldozer pushes a car into a wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important characteristics are observed. First, the object returns to its original shape when the force is removed—that is, the deformation is elastic for small deformations. Second, the size of the deformation is proportional to the force—that is, for small deformations, Hooke's law is obeyed. In equation form, **Hooke's law** is given by

$$F = k\Delta L, \tag{5.26}$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force. Note that this force is a function of the deformation ΔL —it is not constant as a kinetic friction force is. Rearranging this to

$$\Delta L = \frac{F}{k} \tag{5.27}$$

makes it clear that the deformation is proportional to the applied force. Figure 5.13 shows the Hooke's law relationship between the extension ΔL of a spring or of a human bone. For metals or springs, the straight line region in which Hooke's law pertains is much larger. Bones are brittle and the elastic region is small and the fracture abrupt. Eventually a large enough stress to the material will cause it to break or fracture. **Tensile strength** is the breaking stress that will cause permanent deformation or fracture of a material.

Hooke's Law

$$F = k\Delta L, \tag{5.28}$$

where ΔL is the amount of deformation (the change in length, for example) produced by the force F, and k is a proportionality constant that depends on the shape and composition of the object and the direction of the force.

$$\Delta L = \frac{F}{k} \tag{5.29}$$

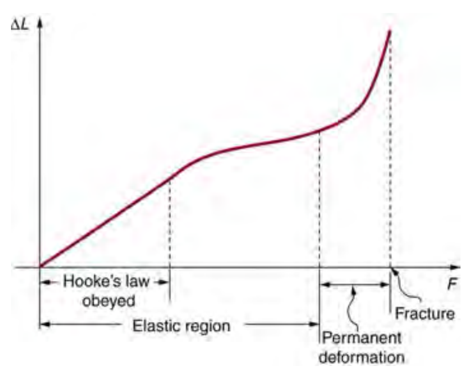

Figure 5.13 A graph of deformation ΔL versus applied force F. The straight segment is the linear region where Hooke's law is obeyed. The slope of the straight region is $\frac{1}{k}$. For larger forces, the graph is curved but the deformation is still elastic— ΔL will return to zero if the force is removed. Still greater forces permanently deform the object until it finally fractures. The shape of the curve near fracture depends on several factors, including how the force F is applied. Note that in this graph the slope increases just before fracture, indicating that a small increase in F is producing a large increase in L near the fracture.

The proportionality constant k depends upon a number of factors for the material. For example, a guitar string made of nylon stretches when it is tightened, and the elongation ΔL is proportional to the force applied (at least for small deformations). Thicker nylon strings and ones made of steel stretch less for the same applied force, implying they have a larger k (see Figure

5.14). Finally, all three strings return to their normal lengths when the force is removed, provided the deformation is small. Most materials will behave in this manner if the deformation is less than about 0.1% or about 1 part in 10^3.

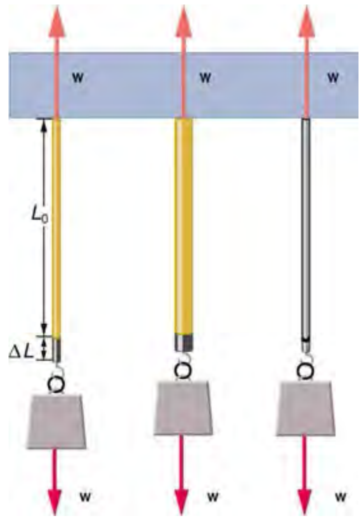

Figure 5.14 The same force, in this case a weight (w), applied to three different guitar strings of identical length produces the three different deformations shown as shaded segments. The string on the left is thin nylon, the one in the middle is thicker nylon, and the one on the right is steel.

Stretch Yourself a Little

How would you go about measuring the proportionality constant k of a rubber band? If a rubber band stretched 3 cm when a 100-g mass was attached to it, then how much would it stretch if two similar rubber bands were attached to the same mass—even if put together in parallel or alternatively if tied together in series?

We now consider three specific types of deformations: changes in length (tension and compression), sideways shear (stress), and changes in volume. All deformations are assumed to be small unless otherwise stated.

Changes in Length—Tension and Compression: Elastic Modulus

A change in length ΔL is produced when a force is applied to a wire or rod parallel to its length L_0, either stretching it (a tension) or compressing it. (See Figure 5.15.)

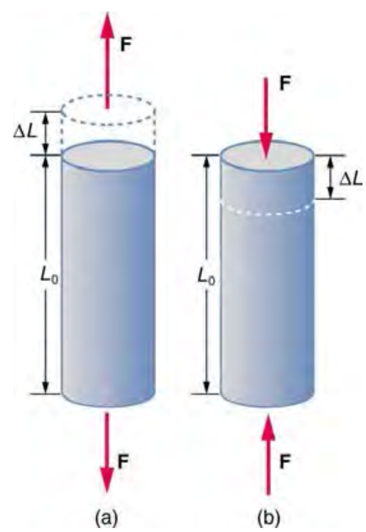

Figure 5.15 (a) Tension. The rod is stretched a length ΔL when a force is applied parallel to its length. (b) Compression. The same rod is compressed by forces with the same magnitude in the opposite direction. For very small deformations and uniform materials, ΔL is approximately the same for the same magnitude of tension or compression. For larger deformations, the cross-sectional area changes as the rod is compressed or stretched.

Experiments have shown that the change in length (ΔL) depends on only a few variables. As already noted, ΔL is proportional to the force F and depends on the substance from which the object is made. Additionally, the change in length is proportional to the original length L_0 and inversely proportional to the cross-sectional area of the wire or rod. For example, a long guitar string will stretch more than a short one, and a thick string will stretch less than a thin one. We can combine all these factors into one equation for ΔL:

$$\Delta L = \frac{1}{Y}\frac{F}{A}L_0, \tag{5.30}$$

where ΔL is the change in length, F the applied force, Y is a factor, called the elastic modulus or Young's modulus, that depends on the substance, A is the cross-sectional area, and L_0 is the original length. **Table 5.3** lists values of Y for several materials—those with a large Y are said to have a large tensile stifness because they deform less for a given tension or compression.

Table 5.3 Elastic Moduli[1]

Material	Young's modulus (tension–compression) *Y* $(10^9\ \text{N/m}^2)$	Shear modulus *S* $(10^9\ \text{N/m}^2)$	Bulk modulus *B* $(10^9\ \text{N/m}^2)$
Aluminum	70	25	75
Bone – tension	16	80	8
Bone – compression	9		
Brass	90	35	75
Brick	15		
Concrete	20		
Glass	70	20	30
Granite	45	20	45
Hair (human)	10		
Hardwood	15	10	
Iron, cast	100	40	90
Lead	16	5	50
Marble	60	20	70
Nylon	5		
Polystyrene	3		
Silk	6		
Spider thread	3		
Steel	210	80	130
Tendon	1		
Acetone			0.7
Ethanol			0.9
Glycerin			4.5
Mercury			25
Water			2.2

Young's moduli are not listed for liquids and gases in **Table 5.3** because they cannot be stretched or compressed in only one direction. Note that there is an assumption that the object does not accelerate, so that there are actually two applied forces of magnitude F acting in opposite directions. For example, the strings in **Figure 5.15** are being pulled down by a force of magnitude w and held up by the ceiling, which also exerts a force of magnitude w.

1. Approximate and average values. Young's moduli Y for tension and compression sometimes differ but are averaged here. Bone has significantly different Young's moduli for tension and compression.

Example 5.3 The Stretch of a Long Cable

Suspension cables are used to carry gondolas at ski resorts. (See **Figure 5.16**) Consider a suspension cable that includes an unsupported span of 3 km. Calculate the amount of stretch in the steel cable. Assume that the cable has a diameter of 5.6 cm and the maximum tension it can withstand is $3.0\times10^6\ \text{N}$.

Figure 5.16 Gondolas travel along suspension cables at the Gala Yuzawa ski resort in Japan. (credit: Rudy Herman, Flickr)

Strategy

The force is equal to the maximum tension, or $F = 3.0\times10^6\ \text{N}$. The cross-sectional area is $\pi r^2 = 2.46\times10^{-3}\ \text{m}^2$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities are known. Thus,

$$\begin{aligned}\Delta L &= \left(\frac{1}{210\times10^9\ \text{N/m}^2}\right)\left(\frac{3.0\times10^6\ \text{N}}{2.46\times10^{-3}\ \text{m}^2}\right)(3020\ \text{m}) \\ &= 18\ \text{m}.\end{aligned} \tag{5.31}$$

Discussion

This is quite a stretch, but only about 0.6% of the unsupported length. Effects of temperature upon length might be important in these environments.

Bones, on the whole, do not fracture due to tension or compression. Rather they generally fracture due to sideways impact or bending, resulting in the bone shearing or snapping. The behavior of bones under tension and compression is important because it determines the load the bones can carry. Bones are classified as weight-bearing structures such as columns in buildings and trees. Weight-bearing structures have special features; columns in building have steel-reinforcing rods while trees and bones are fibrous. The bones in different parts of the body serve different structural functions and are prone to different stresses. Thus the bone in the top of the femur is arranged in thin sheets separated by marrow while in other places the bones can be cylindrical and filled with marrow or just solid. Overweight people have a tendency toward bone damage due to sustained compressions in bone joints and tendons.

Another biological example of Hooke's law occurs in tendons. Functionally, the tendon (the tissue connecting muscle to bone) must stretch easily at first when a force is applied, but offer a much greater restoring force for a greater strain. **Figure 5.17** shows a stress-strain relationship for a human tendon. Some tendons have a high collagen content so there is relatively little strain, or length change; others, like support tendons (as in the leg) can change length up to 10%. Note that this stress-strain curve is nonlinear, since the slope of the line changes in different regions. In the first part of the stretch called the toe region, the fibers in the tendon begin to align in the direction of the stress—this is called *uncrimping*. In the linear region, the fibrils will be stretched, and in the failure region individual fibers begin to break. A simple model of this relationship can be illustrated by springs in parallel: different springs are activated at different lengths of stretch. Examples of this are given in the problems at end of this chapter. Ligaments (tissue connecting bone to bone) behave in a similar way.

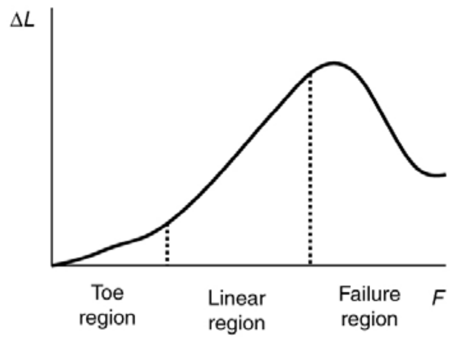

Figure 5.17 Typical stress-strain curve for mammalian tendon. Three regions are shown: (1) toe region (2) linear region, and (3) failure region.

Unlike bones and tendons, which need to be strong as well as elastic, the arteries and lungs need to be very stretchable. The elastic properties of the arteries are essential for blood flow. The pressure in the arteries increases and arterial walls stretch when the blood is pumped out of the heart. When the aortic valve shuts, the pressure in the arteries drops and the arterial walls relax to maintain the blood flow. When you feel your pulse, you are feeling exactly this—the elastic behavior of the arteries as the blood gushes through with each pump of the heart. If the arteries were rigid, you would not feel a pulse. The heart is also an organ with special elastic properties. The lungs expand with muscular effort when we breathe in but relax freely and elastically when we breathe out. Our skins are particularly elastic, especially for the young. A young person can go from 100 kg to 60 kg with no visible sag in their skins. The elasticity of all organs reduces with age. Gradual physiological aging through reduction in elasticity starts in the early 20s.

Example 5.4 Calculating Deformation: How Much Does Your Leg Shorten When You Stand on It?

Calculate the change in length of the upper leg bone (the femur) when a 70.0 kg man supports 62.0 kg of his mass on it, assuming the bone to be equivalent to a uniform rod that is 40.0 cm long and 2.00 cm in radius.

Strategy

The force is equal to the weight supported, or

$$F = mg = (62.0\text{ kg})\left(9.80\text{ m/s}^2\right) = 607.6\text{ N}, \tag{5.32}$$

and the cross-sectional area is $\pi r^2 = 1.257\times10^{-3}\text{ m}^2$. The equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$ can be used to find the change in length.

Solution

All quantities except ΔL are known. Note that the compression value for Young's modulus for bone must be used here. Thus,

$$\begin{aligned}\Delta L &= \left(\frac{1}{9\times10^9\text{ N/m}^2}\right)\left(\frac{607.6\text{ N}}{1.257\times10^{-3}\text{ m}^2}\right)(0.400\text{ m}) \\ &= 2\times10^{-5}\text{ m.}\end{aligned} \tag{5.33}$$

Discussion

This small change in length seems reasonable, consistent with our experience that bones are rigid. In fact, even the rather large forces encountered during strenuous physical activity do not compress or bend bones by large amounts. Although bone is rigid compared with fat or muscle, several of the substances listed in **Table 5.3** have larger values of Young's modulus Y. In other words, they are more rigid.

The equation for change in length is traditionally rearranged and written in the following form:

$$\frac{F}{A} = Y\frac{\Delta L}{L_0}. \tag{5.34}$$

The ratio of force to area, $\frac{F}{A}$, is defined as **stress** (measured in N/m^2), and the ratio of the change in length to length, $\frac{\Delta L}{L_0}$, is defined as **strain** (a unitless quantity). In other words,

$$\text{stress} = Y\times\text{strain}. \tag{5.35}$$

In this form, the equation is analogous to Hooke's law, with stress analogous to force and strain analogous to deformation. If we again rearrange this equation to the form