Atomic Physics

Clicker Questions

Question Q3.01

Description: Distinguishing the Bohr theory’s postulates from related statements.

Question

Which of the following is not one of the Bohr postulates?

1. The laws of classical physics apply to orbital motion of the electrons but not during transitions from one orbit to another.
2. The possible energies that the electron can have are discrete and depend on the orbit radius.
3. Electrons emit electromagnetic radiation only during transitions between orbits and not while in an allowed orbit.
4. Permissible orbits have an angular momentum which is an integer multiple of $\hbar/2\pi$.
5. The frequency of the radiation emitted during a transition is related to the energy difference between the orbits by $f = \Delta E/\hbar$.

Commentary

Purpose: To check your understanding of the fundamental postulates of the Bohr theory (as distinct from their consequences), and stimulate discussion about what actually goes into the theory.

Discussion: Since students usually see the equations for the discrete energies and radii of the Bohr orbits more frequently than they encounter statements of the model’s postulates, they often believe these equations themselves are the foundation of the model. An understanding of the postulates is essential to appreciating how Bohr’s theory relates to both classical and quantum physics.

Statements 1, 3, 4, and 5 are the basis of the theory, though you will often see them with different wording. Statement 2, on the other hand, is not fundamental; quantization of the orbit radii and electron energies is a consequence of the quantization of angular momentum (as described in Statement 4). Bohr’s assertion that angular momentum comes in discrete units of one “h-bar” ($\hbar/2\pi$) was sufficient to predict the discrete energy levels observed in hydrogen.

The quantization condition is sometimes represented as requiring an integer number of electron wavelengths to occur around the circumference of each orbit, where wavelength is related to energy via $E = hf$. This is a heuristic (“hand-waving”) argument designed to make the condition seem “reasonable” or “intuitive,” however, and is not one of the formal postulates of the Bohr theory.
Key Points:

- Distinguishing a theory’s postulates from its derived statements and consequences helps you to understand the theory, know its limits, and appreciate its relationship to other physical theories.
- The Bohr model requires four fundamental postulates, “leaps” which cannot be proven or justified. The value of the model comes entirely from its success in explaining physical phenomena.
- Bohr’s identification of the quantization of angular momentum is a postulate; quantization of orbit radii and electron energies are consequences of that and of classical physics.

For Instructors Only

This is a simple question, but it gets at something students rarely pay attention to: the logical structure and foundation of a physical theory. They are usually far more concerned with the theory’s consequences, since these are used to solve the problems we give them.

If time permits, this question makes an excellent entrée into a general philosophical discussion about physics, theories, model-building, and the role of observables.

QUICK QUIZZES

1. (b). The allowed energy levels in a one-electron atom may be expressed as $E_n = -\frac{Z^2(13.6\ \text{eV})}{n^2}$, where $Z$ is the atomic number. Thus, the ground state ($n = 1$ level) in helium, with $Z = 2$, is lower than the ground state in hydrogen, with $Z = 1$.

2. (a) For $n = 5$, there are 5 allowed values of $\ell$, namely $\ell = 0, 1, 2, 3, \text{ and } 4$.

(b) Since $m_\ell$ ranges from $-\ell$ to $+\ell$ in integer steps, the largest allowed value of $\ell$ ($\ell = 4$ in this case) permits the greatest range of values for $m_\ell$. For $n = 5$, there are 9 possible values for $m_\ell$: -4, -3, -2, -1, 0, +1, +2, +3, and +4.

(c) For each value of $\ell$, there are $2\ell + 1$ possible values of $m_\ell$. Thus, there is 1 distinct pair with $\ell = 0$, 3 distinct pairs with $\ell = 1$, 5 distinct pairs with $\ell = 2$, 7 distinct pairs with $\ell = 3$, and 9 distinct pairs with $\ell = 4$. This yields a total of 25 distinct pairs of $\ell$ and $m_\ell$ that are possible when $n = 5$.

3. (d). Krypton has a closed configuration consisting of filled $n = 1, n = 2$, and $n = 3$ shells as well as filled $4s$ and $4p$ subshells. The filled $n = 3$ shell (the next to outer shell in krypton) has a total of 18 electrons, 2 in the $3s$ subshell, 6 in the $3p$ subshell and 10 in the $3d$ subshell.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Wavelengths of the hydrogen spectrum are given by $\frac{1}{\lambda} = R_n \left(\frac{1/n_f^2 - 1/n_i^2}{n_f^2} \right)$, where the Rydberg constant is $R_n = 1.0973732 \times 10^7$ m⁻¹. Thus, with $n_f = 3$ and $n_i = 5$,

$$\frac{1}{\lambda} = 1.0973732 \times 10^7 \ m^{-1} \left(\frac{1}{3^2} - \frac{1}{5^2} \right) = 7.80 \times 10^4 \ m^{-1},$$

and $\lambda = \frac{1}{(7.80 \times 10^4 \ m^{-1})} = 1.28 \times 10^{-4}$ m, so (a) is seen to be the correct choice.
2. The energy levels in a single electron atom having atomic number $Z$ are

$$E_n = -\frac{Z^2}{n^2}(13.6 \text{ eV})$$

For beryllium, $Z = 4$, and for the ground state, $n = 1$. Thus,

$$E_1 = -\frac{4^2}{1^2}(13.6 \text{ eV}) = -218 \text{ eV}$$

and the correct answer is choice (b).

3. With a principal quantum number of $n = 3$, there are 3 possible values of the orbital quantum number, $\ell = 0, 1, 2$. There are a total of $2(2\ell + 1)$ possible quantum states for each value of $\ell$; $2\ell + 1$ possible values of the orbital magnetic quantum number $m$, and 2 possible spin orientations ($m_s = \pm \frac{1}{2}$) for each value of $m$. Thus, there are 10 $3d$ states (having $n = 3, \ell = 2$), 6 $3p$ states (with $n = 3, \ell = 1$), and 2 $3s$ states (with $n = 3, \ell = 0$), giving a grand total of $10 + 6 + 2 = 18$ $n = 3$ states and the correct choice is (e).

4. There are 6 distinct possible downward transitions with 4 energy levels. These transitions are: $4 \to 1, 4 \to 2, 4 \to 3, 3 \to 1, 3 \to 2,$ and $2 \to 1$. Thus, assuming that each transition has a unique photon energy, $E_{\text{photon}} = |\Delta E| = E_i - E_f$, associated with it, there are 6 different wavelengths $\lambda = h\nu = E_{\text{photon}}$ the atom could emit and (e) is the correct choice.

5. The structure of the periodic table is the result of the Pauli exclusion principle, which states that no two electrons in an atom can ever have the same set of values for the set of quantum numbers $n, \ell, m$, and $m_s$. This principle is best summarized by choice (c).

6. All states associated with $\ell = 2$ are referred to as $d$ states. Thus, all 10 possible quantum states having $n = 3, \ell = 2$ are called $3d$ states (see Question 3 above), and the correct answer is choice (c).

7. Of the electron configurations listed, (b) and (e) are not allowed. Choice (b) is not possible because the Pauli exclusion principle limits the number of electrons in any $p$ subshell to a maximum of 6. Choice (e) is impossible because the selection rules of quantum mechanics limit the maximum value of $\ell$ to $n-1$. Thus, a $2d$ state ($n=2, \ell=2$) cannot exist.

8. Since the electron is in some bound quantum state of the atom, the atom is not ionized and choice (a) is false. The fact that the electron is in a $d$ state means that its orbital quantum number is $\ell = 2$, so choice (b) is false. Also, since the maximum value of $\ell$ is $n-1$, choice (e) is false. Finally, the ground state of hydrogen is a $1s$ state, so choice (d) is false, leaving (c) as the only true statement in the list of choices.

9. If it were possible for the spin quantum number to take on the four values $m_s = \pm \frac{1}{2}$ and $\pm \frac{3}{2}$, the first closed shell would occur for beryllium with 4 electrons in states of $(1,0,0,\frac{1}{2})$, $(1,0,0,\frac{3}{2})$, $(0,0,0,\frac{1}{2})$, and $(0,0,0,\frac{3}{2})$. The correct answer is choice (c).

10. According to de Broglie’s interpretation of Bohr’s quantization postulate, the circumference of the $n = 3$ orbit would be exactly 3 electron wavelengths long. However, the de Broglie wavelength of the electron is given by

$$\lambda = \frac{h}{p_e} = \frac{h}{m_e v_e} = \frac{h}{m_e v_e r_e / r_e} = \frac{h r_e}{n (\hbar/2\pi)} = \frac{2\pi n}{n} = \frac{2\pi n (h^2 \hbar^2)}{m_e c^2} = \frac{n (2\pi n^3)}{m_e^2 c^2}$$

Thus, the wavelength of an electron in the $n = 3$ orbit is 3 times longer than the wavelength of the electron when in the $n = 1$ orbit, and the circumference of the $n = 3$ orbit must be $3(3) = 9$ times greater than that of the $n = 1$ orbit. Choice (c) is the correct answer for this question.
ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Neon signs do not emit a continuous spectrum. They emit many discrete wavelengths, as could be determined by observing the light from the sign through a spectrometer. However, they do not emit all wavelengths. The specific wavelengths and intensities account for the color of the sign.

4. An atom does not have to be ionized to emit light. For example, hydrogen emits light when a transition carries an electron from a higher state to the \( n = 2 \) state.

6. Classically, the electron can occupy any energy state. That is, all energies would be allowed. Therefore, if the electron obeyed classical mechanics, its spectrum, which originates from transitions between states, would be continuous rather than discrete.

8. The de Broglie wavelength of macroscopic objects such as a baseball moving with a typical speed such as 30 m/s is very small and impossible to measure. That is, \( \lambda = \frac{h}{mv} \) is a very small number for macroscopic objects. We are not able to observe diffraction effects because the wavelength is much smaller than any aperture through which the object could pass.

10. In both cases the answer is yes. Recall that the ionization energy of hydrogen is 13.6 eV. The electron can absorb a photon of energy less than 13.6 eV by making a transition to some intermediate state such as one with \( n = 2 \). It can also absorb a photon of energy greater than 13.6 eV, but in doing so, the electron would be separated from the proton and have some residual kinetic energy.

12. It replaced the simple circular orbits in the Bohr theory with electron clouds. More important, quantum mechanics is consistent with Heisenberg’s uncertainty principle, which tells us about the limits of accuracy in making measurements. In quantum mechanics, we talk about the probabilistic nature of the outcome of a measurement of a system, a concept which is incompatible with the Bohr theory. Finally, the Bohr theory of the atom contains only one quantum number \( n \), while quantum mechanics provides the basis for additional quantum numbers to explain the finer details of atomic structure.

14. Each of the given atoms has a single electron in an \( \ell = 0 \) (or \( s \)) state outside a fully closed-shell core, shielded from all but one unit of the nuclear charge. Since they reside in very similar environments, one would expect these outer electrons to have nearly the same electrical potential energies and hence nearly the same ionization energies. This is in agreement with the given data values. Also, since the distance of the outer electron from the nuclear charge should tend to increase with \( Z \) (to allow for greater numbers of electrons in the core), one would expect the ionization energy to decrease somewhat as atomic number increases. This is also in agreement with the given data.

PROBLEM SOLUTIONS

28.1 (a) The wavelengths in the Lyman series of hydrogen are given by \( \frac{1}{\lambda} = R_H (1 - \frac{1}{n^2}) \), where \( n = 2, 3, 4, \ldots \), and the Rydberg constant is \( R_H = 1.097373 \times 10^7 \text{ m}^{-1} \). This can also be written as \( \lambda = \frac{1}{(1/R_H)}(n^2/n^2 - 1) \) so the first three wavelengths in this series are

\[
\begin{align*}
\lambda_1 &= \frac{1}{1.097373 \times 2 \times 10^7 \text{ m}^{-1}} \left( \frac{2^2}{2^2 - 1} \right) = 1.215 \times 10^{-7} \text{ m} = 121.5 \text{ nm} \\
\lambda_2 &= \frac{1}{1.097373 \times 2 \times 10^7 \text{ m}^{-1}} \left( \frac{3^2}{3^2 - 1} \right) = 1.025 \times 10^{-7} \text{ m} = 102.5 \text{ nm} \\
\lambda_3 &= \frac{1}{1.097373 \times 2 \times 10^7 \text{ m}^{-1}} \left( \frac{4^2}{4^2 - 1} \right) = 9.720 \times 10^{-8} \text{ m} = 97.20 \text{ nm}
\end{align*}
\]

(b) These wavelengths are all in the far ultraviolet region of the spectrum.
28.2 (a) The wavelengths in the Paschen series of hydrogen are given by 
\[ \lambda = \frac{1}{n^2 - 9} \frac{R_H}{1} \]
where \( n = 4, 5, 6, \ldots \), and the Rydberg constant is \( R_H = 1.0973732 \times 10^7 \text{ m}^{-1} \). This can also be written as 
\[ \lambda = \frac{1}{n^2 - 9} \frac{9n^2}{R_H} \] so the first three wavelengths in this series are 
\[ \lambda_1 = \frac{1}{4^2 - 9} \frac{9 \times 4^2}{1.0973732 \times 10^7} = 1.875 \times 10^{-6} \text{ m} = 1875 \text{ nm} \]
\[ \lambda_2 = \frac{1}{5^2 - 9} \frac{9 \times 5^2}{1.0973732 \times 10^7} = 1.281 \times 10^{-6} \text{ m} = 1281 \text{ nm} \]
\[ \lambda_3 = \frac{1}{6^2 - 9} \frac{9 \times 6^2}{1.0973732 \times 10^7} = 1.094 \times 10^{-6} \text{ m} = 1094 \text{ nm} \]
(b) These wavelengths are all in the infrared region of the spectrum.

28.3 (a) From Coulomb’s law,
\[ F = k \frac{q_1 q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.0 \times 10^{-10} \text{ m})^2} = 2.3 \times 10^{-8} \text{ N} \]
(b) The electrical potential energy is
\[ PE = k \frac{q_1 q_2}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})(-1.60 \times 10^{-19} \text{ C})}{1.0 \times 10^{-10} \text{ m}} \]
\[ = -2.3 \times 10^{-18} \text{ J} \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right) = -14 \text{ eV} \]

28.4 (a) From Coulomb’s law,
\[ F = k \frac{q_1 q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.0 \times 10^{-10} \text{ m})^2} = 2.3 \times 10^{-2} \text{ N} \]
(b) The electrical potential energy is
\[ PE = k \frac{q_1 q_2}{r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{1.0 \times 10^{-10} \text{ m}} \]
\[ = +2.3 \times 10^{-13} \text{ J} \left( \frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} \right) = +1.4 \text{ MeV} \]

28.5 (a) The electrical force supplies the centripetal acceleration of the electron, so
\[ m \frac{v^2}{r} = \frac{k e^2}{r^2} \text{ or } v = \sqrt{\frac{k e^2}{mr}} \]
\[ v = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(1.0 \times 10^{-10} \text{ m})}} = 1.6 \times 10^6 \text{ m/s} \]

continued on next page
(b) \[
\frac{v}{c} = \frac{1.6 \times 10^6 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}} = 5.3 \times 10^{-3} \ll 1, \text{ so the electron is not relativistic.}
\]

(c) The de Broglie wavelength for the electron is \[\lambda = \frac{h}{p} = \frac{h}{mv},\] or
\[\lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.11 \times 10^{-31} \text{ kg} \cdot (1.6 \times 10^{-6} \text{ m/s})} = 4.6 \times 10^{-10} \text{ m} = 0.46 \text{ nm}\]

(d) Yes. The wavelength and the atom are roughly the same size.

28.6
Assuming a head-on collision, the \(\alpha\)-particle comes to rest momentarily at the point of closest approach. From conservation of energy,
\[KE_f + PE_f = KE_i + PE_i\]
or
\[0 + k_e (2e)(79e) = KE_i + k_e (2e)(79e)\]

With \(r_f \to \infty\), this gives the distance of closest approach as
\[r_f = \frac{158k_e^2}{KE_i} = \frac{158(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{5.0 \text{ MeV}(1.60 \times 10^{-13} \text{ J/MeV})} = 4.5 \times 10^{-14} \text{ m} = 45 \text{ fm}\]

28.7
(a) \(r_e = n^2a_0\) yields \(r_e = 4(0.052 \text{ nm}) = 0.212 \text{ nm}\)

(b) With the electrical force supplying the centripetal acceleration, \(m_r^2v_e^2/r_e = k_e e^2/r_e^2\), giving \(v_e = \sqrt{k_e e^2/m_r}\) and \(p_e = m_r v_e = \sqrt{m_e k_e e^2/r_e}\).

Thus,
\[p_e = \sqrt{\frac{m_e k_e e^2}{r_e}} = \sqrt{\frac{9.11 \times 10^{-31} \text{ kg} \cdot (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{0.212 \times 10^{-9} \text{ m}}} = 9.95 \times 10^{-25} \text{ kg} \cdot \text{m/s}\]

(c) \(L_e = n \left(\frac{\hbar}{2\pi}\right) \to L_e = 2 \left(\frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{2\pi}\right) = 2.11 \times 10^{-34} \text{ J} \cdot \text{s}\)

(d) \(KE_2 = \frac{1}{2}mv_e^2 = \frac{9.95 \times 10^{-25} \text{ kg} \cdot \text{m/s}^2}{2(9.11 \times 10^{-31} \text{ kg})} = 5.44 \times 10^{-19} \text{ J}\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 3.40 \text{ eV}\)

(e) \(PE_2 = \frac{k_e (-e)e}{r_1} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{(0.212 \times 10^{-9} \text{ m})} = -1.09 \times 10^{-18} \text{ J} = -6.80 \text{ eV}\)

(f) \(E_2 = KE_2 + PE_2 = 3.40 \text{ eV} - 6.80 \text{ eV} = -3.40 \text{ eV}\)
28.8  (a) With the electrical force supplying the centripetal acceleration, \( m_r v_n^2 / r_n = k_e e^2 / r_n^2 \), giving
\[
 v_n = \sqrt{\frac{k_e e^2}{m_r r_n}} = \sqrt{\frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times (1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg} \times (0.0529 \times 10^{-9} \text{ m})}} = 2.19 \times 10^6 \text{ m/s}
\]

(b) \( KE_n = \frac{1}{2} m_r v_n^2 = \frac{1}{2} (9.11 \times 10^{-31} \text{ kg}) \left(2.19 \times 10^6 \text{ m/s}\right)^2 \)
\[
 = 2.18\times10^{-18} \text{ J} \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 13.6 \text{ eV}
\]

(c) \( PE_n = \frac{k_e (-e)}{r_n} = -\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times (1.60 \times 10^{-19} \text{ C})^2\right)}{(0.0529 \times 10^{-9} \text{ m})} \)
\[
 = -4.35 \times 10^{-18} \text{ J} = -27.2 \text{ eV}
\]

28.9  Since the electrical force supplies the centripetal acceleration, \( \frac{m_r v_n^2}{r_n} = k_e e^2 / r_n^2 \) or \( v_n^2 = \frac{k_e e^2}{m_r r_n} \)

From \( L_n = m_r r_n v_n = n \hbar \), we have \( r_n = n \hbar / m_r v_n \), so
\[
v_n^2 = \frac{k_e e^2}{m_r} \left(\frac{m_r v_n}{n \hbar}\right)
\]
which reduces to \( v_n = k_e e^2 / n \hbar \)

28.10  (a) The Rydberg equation is \( 1/\lambda = R_n \left(1/n_f^2 - 1/n_i^2\right) \), or
\[
\lambda = \frac{1}{R_n} \left(\frac{n_i^2 n_f^2}{n_f^2 - n_i^2}\right)
\]

With \( n_i = 5 \) and \( n_f = 3 \),
\[
\lambda = \frac{1}{1.097 \times 10^{-7} \text{ m}^{-1}} \left[\left(\frac{25}{9}\right)\right] = 1.281 \times 10^{-6} \text{ m} = 1281 \text{ nm}
\]

(b) \( f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1281 \times 10^{-9} \text{ m}} = 2.34 \times 10^{14} \text{ Hz} \)

(c) \( E_{\text{photon}} = \frac{h c}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \cdot (3.00 \times 10^8 \text{ m/s})}{1281 \times 10^{-9} \text{ m} \cdot \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)} = 0.970 \text{ eV} \)
28.11 The energy of the emitted photon is
\[
E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)}{656 \times 10^{-9} \text{ m}} = 1.89 \text{ eV}
\]
This photon energy is also the difference in the electron's energy in its initial and final orbits. The energies of the electron in the various allowed orbits within the hydrogen atom are
\[
E_n = -\frac{13.6 \text{ eV}}{n^2} \quad \text{where} \quad n = 1, 2, 3, \ldots
\]
giving \(E_1 = -13.6 \text{ eV}, \ E_2 = -3.40 \text{ eV}, \ E_3 = -1.51 \text{ eV}, \ E_4 = -0.850 \text{ eV}, \ldots\)
Observe that \(E_{\text{photon}} = E_3 - E_1\), so the transition was from the \(n = 3\) orbit to the \(n = 2\) orbit.

28.12 The change in the energy of the atom is
\[
\Delta E = E_f - E_i = 13.6 \text{ eV} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)
\]
Transition I: \(\Delta E = 13.6 \text{ eV} \left(\frac{1}{4} - \frac{1}{25}\right) = 2.86 \text{ eV}\) (absorption)
Transition II: \(\Delta E = 13.6 \text{ eV} \left(\frac{1}{25} - \frac{1}{9}\right) = -0.967 \text{ eV}\) (emission)
Transition III: \(\Delta E = 13.6 \text{ eV} \left(\frac{1}{49} - \frac{1}{16}\right) = -0.572 \text{ eV}\) (emission)
Transition IV: \(\Delta E = 13.6 \text{ eV} \left(\frac{1}{16} - \frac{1}{49}\right) = 0.572 \text{ eV}\) (absorption)
(a) Since \(\lambda = \frac{hc}{E_{\text{photon}}} = \frac{hc}{-\Delta E}\), transition II emits the shortest wavelength photon.
(b) The atom gains the most energy in transition I.
(c) The atom loses energy in transitions II and III.

28.13 The energy absorbed by the atom is
\[
E_{\text{photon}} = E_f - E_i = 13.6 \text{ eV} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)
\]
(a) \(E_{\text{photon}} = 13.6 \text{ eV} \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 2.86 \text{ eV}\)
(b) \(E_{\text{photon}} = 13.6 \text{ eV} \left(\frac{1}{4^2} - \frac{1}{6^2}\right) = 0.472 \text{ eV}\)
28.14 (a) The energy absorbed is
\[ \Delta E = E_f - E_i = 13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = 13.6 \text{ eV} \left( \frac{1}{1^2} - \frac{1}{9} \right) = 12.1 \text{ eV} \]

(b) Three transitions are possible as the electron returns to the ground state. These transitions and the emitted photon energies are
\[ n_i = 3 \rightarrow n_f = 1 : \quad \Delta E = 13.6 \text{ eV} \left( \frac{1}{1^2} - \frac{1}{3^2} \right) = 12.1 \text{ eV} \]
\[ n_i = 3 \rightarrow n_f = 2 : \quad \Delta E = 13.6 \text{ eV} \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = 1.89 \text{ eV} \]
\[ n_i = 2 \rightarrow n_f = 1 : \quad \Delta E = 13.6 \text{ eV} \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = 10.2 \text{ eV} \]

28.15 To ionize the atom, it is necessary that \( n_f \rightarrow \infty \). The required energy is then
\[ \Delta E = E_f - E_i = -13.6 \text{ eV} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = -13.6 \text{ eV} \left( \frac{1}{\infty} - \frac{1}{n_i^2} \right) = \frac{13.6 \text{ eV}}{n_i^2} \]

(a) If \( n_i = 1 \), the required energy is \( \Delta E = \frac{13.6 \text{ eV}}{1^2} = 13.6 \text{ eV} \)

(b) If \( n_i = 3 \), \( \Delta E = \frac{13.6 \text{ eV}}{3^2} = 1.51 \text{ eV} \)

28.16 The magnetic force supplies the centripetal acceleration, so
\[ \frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB} \]

If angular momentum is quantized according to
\[ L_n = mv_r r_s = 2n\hbar \], then \( mv_r = \frac{2n\hbar}{r_s} \)

and the allowed radii of the path are given by
\[ r_s = \frac{1}{qB} \left( \frac{2n\hbar}{r_s} \right) \quad \text{or} \quad r_s = \sqrt{\frac{2n\hbar}{qB}} \]

28.17 (a) The energy emitted by the atom is
\[ \Delta E = E_i - E_f = -13.6 \text{ eV} \left( \frac{1}{4^2} - \frac{1}{2^2} \right) = 2.55 \text{ eV} \]

The wavelength of the photon produced is then
\[ \lambda = \frac{hc}{\Delta E} = \frac{hc}{(2.55 \text{ eV})(2.00 \times 10^8 \text{ m/s})} = \frac{6.63 \times 10^{-34} \text{ J \cdot s}}{1.60 \times 10^{-19} \text{ J/eV}} \]
\[ = 4.88 \times 10^{-7} \text{ m} = 488 \text{ nm} \]

*continued on next page*
(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, \( p = m_{\text{atom}} \frac{h}{\lambda} \), or
\[
v = \frac{h}{m_{\text{atom}} \lambda} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.67 \times 10^{-33} \text{ kg(4.88 \times 10^{-3} \text{ m})}} = 0.814 \text{ m/s}
\]

28.18 (a) Starting from the \( n = 4 \) state, there are 6 possible transitions as the electron returns to the ground \( (n = 1) \) state. These transitions are: \( n = 4 \to n = 1 \), \( n = 4 \to n = 2 \), \( n = 4 \to n = 3 \), \( n = 3 \to n = 1 \), \( n = 3 \to n = 2 \), and \( n = 2 \to n = 1 \). Since there is a different change in energy associated with each of these transitions, there will be 6 different wavelengths observed in the emission spectrum of these atoms.

(b) The longest observed wavelength is produced by the transition involving the smallest change in energy. This is the \( n = 4 \to n = 3 \) transition, and the wavelength is
\[
\lambda_{\text{max}} = \frac{hc}{E_4 - E_3} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}(2.998 \times 10^4 \text{ m/s})}{-13.6 \text{ eV}(\frac{1}{4^2} - \frac{1}{3^2})} \left( \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) \left( \frac{1 \text{ nm}}{10^{-9} \text{ m}} \right)
\]
or \( \lambda_{\text{max}} = 1.88 \times 10^3 \text{ nm} \).

Since this transition terminates on the \( n = 3 \) level, this is part of the Paschen series.

28.19 For minimum initial kinetic energy, \( KE_{\text{final}} = 0 \) after collision. Hence, the two atoms must have equal and opposite momenta before impact. The atoms then have the same initial kinetic energy, and that energy is converted into excitation energy of the atom during the collision. Therefore,
\[
KE_{\text{atom}} = \frac{1}{2} m_{\text{atom}} v^2 = E_3 - E_1 = 10.2 \text{ eV}
\]
or \( v = \sqrt{\frac{2(10.2 \text{ eV})}{m_{\text{atom}}}} = \sqrt{\frac{2(10.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 4.42 \times 10^4 \text{ m/s} \)

28.20 (a) \( L = mvr = m \left( \frac{2\pi r}{T} \right) r = \frac{2\pi (7.36 \times 10^{-12} \text{ kg})(3.84 \times 10^8 \text{ m})^2}{2.36 \times 10^6 \text{ s}} = 2.89 \times 10^{35} \text{ kg} \cdot \text{m}^2/\text{s} \)

(b) \( n = \frac{L}{h} = \frac{2\pi L}{h} = \frac{2\pi (2.89 \times 10^{35} \text{ kg} \cdot \text{m}^2/\text{s})}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 2.74 \times 10^{38} \)

continued on next page
(c) The gravitational force supplies the centripetal acceleration so
\[ \frac{mv^2}{r} = \frac{GM_em}{r^2}, \text{ or } rv^2 = GM_e \]

Then, from \( L_m = mv_mr \lambda = nh \) or \( v = \frac{nh}{mr} \),
we have \( r_n \left( \frac{nh}{mr} \right)^2 = GM_e \) which gives \( r_n = n^2 \left( \frac{\hbar^2}{GM_e m^2} \right) = n^2 r_1 \)

Therefore, when \( n \) increases by 1, the fractional change in the radius is
\[ \frac{\Delta r}{r} = \frac{r_{n+1} - r_n}{r_n} = \frac{(n+1)^2 r_n - n^2 r_n}{n^2 r_n} = \frac{2n+1}{n^2} \approx \frac{2}{n} \]

\[ \frac{\Delta r}{r} = \frac{2}{2.74 \times 10^{-10}} = 7.30 \times 10^{-10} \]

28.21 (a) \( r_n = n^2 a_n = n^2 (0.0529 \text{ nm}) \Rightarrow r_1 = 3^2 (0.0529 \text{ nm}) = 0.476 \text{ nm} \)

(b) In the Bohr model, the circumference of an allowed orbit must be an integral multiple of the de Broglie wavelength for the electron in that orbit, or \( 2\pi r_n = n\lambda \). Thus, the wavelength of the electron when in the \( n = 3 \) orbit in hydrogen is
\[ \lambda = \frac{2\pi r_n}{3} = \frac{2\pi (0.476 \text{ nm})}{3} = 0.997 \text{ nm} \]

28.22 (a) The Coulomb force supplies the necessary centripetal force to hold the electron in orbit so \( m \frac{v_e^2}{r_e} = k \frac{e^2}{r_e^2} \), or \( m \frac{v_e^2}{r_e} = k \frac{e^2}{r_e^2} \). But \( m \frac{v_e^2}{r_e} = 2KE_n \) and \( k \frac{e^2}{r_e} = -PE_n \), where \( PE_n \) is the electrical potential energy of the electron-proton system when the electron is in an orbit of radius \( r_e \). We then have \( 2KE_n = -PE_n \), or \( KE_n = -\frac{1}{2} PE_n \).

(b) When the atom absorbs energy, \( E \), and the electron moves to a higher level, both the kinetic and potential energy will change. Conservation of energy requires that \( E = \Delta KE + \Delta PE \). But, from the result of part (a), \( \Delta KE = -\frac{1}{2} \Delta PE \) and we have
\[ E = -\frac{1}{2} \Delta PE + \Delta PE = \frac{1}{2} \Delta PE \quad \text{or} \quad \Delta PE = 2E \]

(c) \( \Delta KE = \frac{1}{2} \Delta PE = -\frac{1}{2} (2E) \quad \text{or} \quad \Delta KE = -E \)

28.23 \( r_e = \frac{n^2 \left( \frac{\hbar^2}{m_e k_e} \right)}{Z} = \frac{n^2 a_0}{Z} \) so \( r_e = \frac{a_0}{Z} = \frac{0.0529 \text{ nm}}{Z} \)

(a) For \( \text{He}^+ \), \( Z = 2 \) and \( r = \frac{0.0529 \text{ nm}}{2} = 0.0265 \text{ nm} \)

(b) For \( \text{Li}^{2+} \), \( Z = 3 \) and \( r = \frac{0.0529 \text{ nm}}{3} = 0.0176 \text{ nm} \)

(c) For \( \text{Be}^{3+} \), \( Z = 4 \) and \( r = \frac{0.0529 \text{ nm}}{4} = 0.0132 \text{ nm} \)
28.24 (a) The energy levels in a single electron atom with nuclear charge \(+Z\) are
\[ E_n = -Z^2(13.6 \text{ eV})/n^2. \]
For doubly-ionized lithium, \(Z = 3\), giving \(E_n = -122 \text{ eV}/n^2\).

(b) \(E_4 = \frac{-122 \text{ eV}}{4^2} = -7.63 \text{ eV}\)

(c) \(E_2 = \frac{-122 \text{ eV}}{2^2} = -30.5 \text{ eV}\)

(d) \(E_{\text{photon}} = E_i - E_f = -7.63 \text{ eV} - (-30.5 \text{ eV}) = 22.9 \text{ eV}\)
\[ E_{\text{photon}} = (22.9 \text{ eV}) \left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 3.66 \times 10^{-15} \text{ J} \]

(e) \(f = \frac{E_{\text{photon}}}{\hbar} = \frac{3.66 \times 10^{-15} \text{ J}}{6.63 \times 10^{-34} \text{ J s}} = 5.52 \times 10^{15} \text{ Hz}\)
\[ \lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.52 \times 10^{15} \text{ Hz}} = 5.43 \times 10^{-8} \text{ m} = 54.3 \text{ nm} \]

(f) This wavelength is in the deep ultraviolet region of the spectrum.

28.25 From \(L = m_n v_n r_n = nh\) and \(r_n = n^2 a_0\),
we find that \(p_n = m v_n = \frac{nh}{r_n} = \frac{n(h/2\pi)}{n^2 a_0} = \frac{h}{2\pi a_0 n}\)
Thus, the de Broglie wavelength of the electron in the \(n\)th orbit is \(\lambda = h/p_n = (2\pi a_0) n\).
For \(n = 4\), this yields \(\lambda = 8\pi a_0 = 8\pi(0.0529 \text{ nm}) = 1.33 \text{ nm}\)

28.26 (a) For standing waves in a string fixed at both ends, \(L = \frac{n\lambda}{2}\)
or \(\lambda = \frac{2L}{n}\). According to the de Broglie hypothesis, \(p = \frac{h}{\lambda}\)
Combining these expressions gives \(p = mv = \frac{nh}{2L}\)

(b) Using \(E = \frac{1}{2}mv^2 = \frac{p^2}{2m}\), with \(p\) as found in (a) above:
\[ E_n = \frac{n^2 h^2}{4 L^2 (2m)} = \frac{n^2 E_0}{8mL} \]
where \(E_0 = \frac{h^2}{8mL}\)
28.27 In the 3d subshell, \( n = 3 \) and \( \ell = 2 \). The 10 possible quantum states are

<table>
<thead>
<tr>
<th>( n = 3 )</th>
<th>( \ell = 2 )</th>
<th>( m_\ell = +2 )</th>
<th>( m_s = +\frac{1}{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = +2 )</td>
<td>( m_s = -\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = +1 )</td>
<td>( m_s = +\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = +1 )</td>
<td>( m_s = -\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = +0 )</td>
<td>( m_s = +\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = +0 )</td>
<td>( m_s = -\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = -1 )</td>
<td>( m_s = +\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = -1 )</td>
<td>( m_s = -\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = -2 )</td>
<td>( m_s = +\frac{1}{2} )</td>
</tr>
<tr>
<td>( n = 3 )</td>
<td>( \ell = 2 )</td>
<td>( m_\ell = -2 )</td>
<td>( m_s = -\frac{1}{2} )</td>
</tr>
</tbody>
</table>

28.28 (a) For a given value of the principal quantum number \( n \), the orbital quantum number \( \ell \) varies from 0 to \( n - 1 \) in integer steps. Thus, if \( n = 4 \), there are 4 possible values of \( \ell \): \( \ell = 0, 1, 2, \) and 3.

(b) For each possible value of the orbital quantum number \( \ell \), the orbital magnetic quantum number \( m_\ell \) ranges from \(-\ell\) to \(+\ell\) in integer steps. When the principal quantum number is \( n = 4 \) and the largest allowed value of the orbital quantum number is \( \ell = 3 \), there are 7 distinct possible values for \( m_\ell \). These values are: \( m_\ell = -3, -2, -1, 0, +1, +2, \) and +3.

28.29 The 3d subshell has \( n = 3 \) and \( \ell = 2 \). For \( \ell \)-mesons, we also have \( s = 1 \). Thus, there are 15 possible quantum states, as summarized in the table below.

\[
\begin{array}{c|ccccccccccc}
 n & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
 \ell & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
 m_\ell & +2 & +2 & +2 & +1 & +1 & 0 & 0 & -1 & -1 & -2 & -2 \\
 m_s & +1 & 0 & -1 & +1 & 0 & -1 & +1 & 0 & -1 & +1 & 0 \\
\end{array}
\]
28.30 (a) The electronic configuration for nitrogen (Z = 7) is $1s^2 2s^2 2p^3$.

(b) The quantum numbers for the 7 electrons can be:

<table>
<thead>
<tr>
<th>States</th>
<th>$n$</th>
<th>$\ell$</th>
<th>$m_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s states</td>
<td>1</td>
<td>0</td>
<td>$m_s = \pm \frac{1}{2}$</td>
</tr>
<tr>
<td>2s states</td>
<td>2</td>
<td>0</td>
<td>$m_s = \pm \frac{1}{2}$</td>
</tr>
<tr>
<td>2p states</td>
<td>2</td>
<td>1</td>
<td>$m_s = \pm \frac{1}{2}$, $m_s = \pm \frac{\sqrt{3}}{2}$</td>
</tr>
</tbody>
</table>

28.31 (a) For Electron #1 and also for Electron #2, $n = 3$ and $\ell = 1$. The other quantum numbers for each of the 30 allowed states are listed in the tables below.

There are 30 allowed states, since Electron #1 can have any of three possible values of $m_s$ for both spin up and spin down, totaling six possible states. For each of these states, Electron #2 can be in either of the remaining five states.

continued on next page
(b) Were it not for the exclusion principal, there would be 36 possible states, six for each electron independently.

28.32

(a) For \( n = 1 \), \( \ell = 0 \) and there are \( 2(2\ell + 1) \) states = \( 2(1) = 2 \) sets of quantum numbers.

(b) For \( n = 2 \), \( \ell = 0 \) for \( 2(2\ell + 1) \) states = \( 2(0 + 1) = 2 \) sets
and \( \ell = 1 \) for \( 2(2\ell + 1) \) states = \( 2(2 + 1) = 6 \) sets

total number of sets = \( 8 \)

(c) For \( n = 3 \), \( \ell = 0 \) for \( 2(2\ell + 1) \) states = \( 2(0 + 1) = 2 \) sets
and \( \ell = 1 \) for \( 2(2\ell + 1) \) states = \( 2(2 + 1) = 6 \) sets
and \( \ell = 2 \) for \( 2(2\ell + 1) \) states = \( 2(4 + 1) = 10 \) sets

total number of sets = \( 18 \)

(d) For \( n = 4 \), \( \ell = 0 \) for \( 2(2\ell + 1) \) states = \( 2(0 + 1) = 2 \) sets
and \( \ell = 1 \) for \( 2(2\ell + 1) \) states = \( 2(2 + 1) = 6 \) sets
and \( \ell = 2 \) for \( 2(2\ell + 1) \) states = \( 2(4 + 1) = 10 \) sets
and \( \ell = 3 \) for \( 2(2\ell + 1) \) states = \( 2(6 + 1) = 14 \) sets

total number of sets = \( 32 \)

(e) For \( n = 5 \), \( \ell = 0 \) for \( 2(2\ell + 1) \) states = \( 2(0 + 1) = 2 \) sets
and \( \ell = 1 \) for \( 2(2\ell + 1) \) states = \( 2(2 + 1) = 6 \) sets
and \( \ell = 2 \) for \( 2(2\ell + 1) \) states = \( 2(4 + 1) = 10 \) sets
and \( \ell = 3 \) for \( 2(2\ell + 1) \) states = \( 2(6 + 1) = 14 \) sets
and \( \ell = 4 \) for \( 2(2\ell + 1) \) states = \( 2(8 + 1) = 18 \) sets

total number of sets = \( 50 \)

For \( n = 1 \): \( 2n^2 = 2 \) 
For \( n = 2 \): \( 2n^2 = 8 \)
For \( n = 3 \): \( 2n^2 = 18 \) 
For \( n = 4 \): \( 2n^2 = 32 \)
For \( n = 5 \): \( 2n^2 = 50 \)

Thus, the total number of sets of quantum states agrees with the \( 2n^2 \) rule.

28.33

(a) Zirconium, with 40 electrons, has 4 electrons outside a closed krypton core. The krypton core, with 36 electrons, has all states up through the \( 4p \) subshell filled. Normally, one would expect the next 4 electrons to go into the \( 4d \) subshell. However, an exception to the rule occurs at this point, and the \( 5s \) subshell fills (with 2 electrons) before the \( 4d \) subshell starts filling. The two remaining electrons in zirconium are in an incomplete \( 4d \) subshell. Thus, \( n = 4 \), and \( \ell = 2 \) for each of these electrons.

(b) For electrons in the \( 4d \) subshell, with \( \ell = 2 \), the possible values of \( m_\ell \) are \( m_\ell = 0, \pm 1, \pm 2 \) and those for \( m_s \) are \( m_s = \pm 1/2 \).

(c) We have 40 electrons, so the electron configuration is:

\[
1s^2 \, 2s^2 \, 2p^6 \, 3s^2 \, 3p^6 \, 3d^{10} \, 4s^2 \, 4p^6 \, 4d^4 \, 5s^2 = [\text{Kr}]4d^4 \, 5s^2
\]
28.34 The photon energy is \( E_{\text{photon}} = E_L - E_K = -951 \text{ eV} - (-8,979 \text{ eV}) = 8,028 \text{ eV} \), and the wavelength is
\[
\lambda = \frac{hc}{E_{\text{photon}}} = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \text{ m/s}\right)}{(8,028 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 1.55 \times 10^{-10} \text{ m} = 0.155 \text{ nm}
\]
To produce the \( K_\alpha \) line, an electron from the K-shell must be excited to the L-shell or higher. Thus, a minimum energy of 8,028 eV must be given to the atom. A minimum accelerating voltage of \( \Delta V = 8,028 \text{ V} = 8.03 \text{ kV} \) is required.

28.35 For nickel, \( Z = 28 \) and
\[
E_K = -(Z - 1)^2 \frac{13.6 \text{ eV}}{(1)^2} = -(27)^2 (13.6 \text{ eV}) = -9.91 \times 10^3 \text{ eV}
\]
\[
E_L = -(Z - 3)^2 \frac{13.6 \text{ eV}}{(2)^2} = -(25)^2 (13.6 \text{ eV}) = -2.13 \times 10^4 \text{ eV}
\]
Thus, \( E_{\text{photon}} = E_L - E_K = -2.13 \text{ keV} - (-9.91 \text{ keV}) = 7.78 \text{ keV} \) and
\[
\lambda = \frac{hc}{E_{\text{photon}}} = \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \text{ m/s}\right)}{7.78 \text{ keV}(1.60 \times 10^{-19} \text{ J/keV})} = 1.60 \times 10^{-10} \text{ m} = 0.160 \text{ nm}
\]

28.36 The energies in the K and M shells are
\[
E_K = -(Z - 1)^2 \frac{13.6 \text{ eV}}{(1)^2} \quad \text{and} \quad E_M = -(Z - 9)^2 \frac{13.6 \text{ eV}}{(3)^2}
\]
Thus, \( E_{\text{photon}} = E_M - E_K = (13.6 \text{ eV}) \left[ \frac{(Z - 9)^2}{9} + (Z - 1)^2 \right] = (13.6 \text{ eV}) \left( \frac{8}{9} Z^2 - 8 \right) \)
and \( E_{\text{photon}} = \frac{hc}{\lambda} \) gives \( Z^2 = \frac{9}{8} \left[ 8 + \frac{hc}{(13.6 \text{ eV}) \lambda} \right] \), or
\[
Z = \sqrt{9 + \frac{9 \left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \text{ m/s}\right)}{8 (13.6 \text{ eV})(0.101 \times 10^{-9} \text{ m})} \left( \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} \right)} = 32.0
\]
The element is germanium.
28.37 The transitions that produce the three longest wavelengths in the K series are shown at the right. The energy of the K shell is $E_K = -69.5 \text{ keV}$.

Thus, the energy of the L shell is

$$E_L = E_K + \frac{hc}{\lambda_L}$$

or

$$E_L = -69.5 \text{ keV} + \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{3.00 \times 10^8 \text{ m/s}} \right) \left( \frac{1 \text{ keV}}{1.60 \times 10^{-16} \text{ J}} \right)$$

$$= -69.5 \text{ keV} + 9.25 \times 10^{-15} \text{ J}$$

$$= -69.5 \text{ keV} + 57.8 \text{ keV} = -11.7 \text{ keV}$$

Similarly, the energies of the M and N shells are

$$E_M = E_K + \frac{hc}{\lambda_M}$$

$$E_M = -69.5 \text{ keV} + \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{3.00 \times 10^8 \text{ m/s}} \right) \left( \frac{1 \text{ keV}}{1.60 \times 10^{-16} \text{ J}} \right) = -10.0 \text{ keV}$$

and

$$E_N = E_K + \frac{hc}{\lambda_N}$$

$$E_N = -69.5 \text{ keV} + \left( \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{3.00 \times 10^8 \text{ m/s}} \right) \left( \frac{1 \text{ keV}}{1.60 \times 10^{-16} \text{ J}} \right) = -2.30 \text{ keV}$$

The ionization energies of the L, M, and N shells are $11.7 \text{ keV}$, $10.0 \text{ keV}$, and $2.30 \text{ keV}$, respectively.

28.38 According to the Bohr model, the radii of the electron orbits in hydrogen are given by

$$r_n = n^2 a_0 \text{ with } a_0 = 0.0529 \text{ nm} = 5.29 \times 10^{-11} \text{ m}$$

Then, if $r_n = 1.00 \mu m = 1.00 \times 10^{-6} \text{ m}$, the quantum number is

$$n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{1.00 \times 10^{-6} \text{ m}}{5.29 \times 10^{-11} \text{ m}}} = 137$$

28.39 (a) $\Delta E = E_2 - E_1 = -13.6 \text{ eV} / (2)^2 - (-13.6 \text{ eV} / (1)^2) = 10.2 \text{ eV}$

(b) The average kinetic energy of the atoms must equal or exceed the needed excitation energy, or $\frac{1}{2} k_B T \geq \Delta E$, which gives

$$T \geq \frac{2(\Delta E)}{3k_B} = \frac{2(10.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{3(1.38 \times 10^{-23} \text{ J/K})} = 7.88 \times 10^4 \text{ K}$$
28.40  
(a) \[ L = c (\Delta t) = (3.00 \times 10^8 \text{ m/s})(1.40 \times 10^{-12} \text{ s}) = 4.20 \times 10^{-5} \text{ m} = 4.20 \text{ mm} \]

(b) \[ N = \frac{E_{\text{pulse}}}{E_{\text{photon}}} = \frac{E_{\text{pulse}}}{h/c/\lambda} = \frac{(6.94 \times 10^{-4} \text{ m})/(3.00 \text{ J})}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})} = 1.05 \times 10^{19} \text{ photons} \]

(c) \[ n = \frac{N}{V} = \frac{N}{L(\pi d^2/4)} = \frac{4(1.05 \times 10^{19} \text{ photons})}{(4.20 \text{ mm})(\pi)(6.00 \text{ mm})^2} = 8.84 \times 10^{10} \text{ photons/mm}^2 \]

28.41  
(a) With one vacancy in the K shell, an electron in the L shell has one electron shielding it from the nuclear charge, so \[ Z_{\text{eff}} = Z - 1 = 24 - 1 = 23 \]. The estimated energy the atom gives up during a transition from the L shell to the K shell is then

\[ \Delta E = E_i - E_f = -\frac{Z_{\text{eff}}^2 (13.6 \text{ eV})}{n_i^2} - \left[ -\frac{Z_{\text{eff}}^2 (13.6 \text{ eV})}{n_f^2} \right] = Z_{\text{eff}}^2 (13.6 \text{ eV}) \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \]

or

\[ \Delta E = (23)^2 (13.6 \text{ eV}) \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] = 5.40 \times 10^4 \text{ eV} = 5.40 \text{ keV} \]

(b) With a vacancy in the K shell, we assume that \( Z = 24 = 22 \) electrons shield the outermost electron (in a 4s state) from the nuclear charge. Thus, for this outer electron, \( Z_{\text{eff}} = 24 - 22 = 2 \) and the estimated energy required to remove this electron from the atom is

\[ E_{\text{ionization}} = E_f - E_i = -\frac{Z_{\text{eff}}^2 (13.6 \text{ eV})}{n_i^2} = \frac{2^2 (13.6 \text{ eV})}{4^2} = 3.40 \text{ eV} \]

(c) \[ KE = \Delta E - E_{\text{ionization}} = 5.40 \text{ keV} - 3.40 \text{ eV} = 5.40 \text{ keV} \]

28.42  
(a) The energy levels of a hydrogen-like ion whose charge number is \( Z \) are given by

\[ E_n = (-13.6 \text{ eV}) \frac{Z^2}{n^2} \]

For helium, \( Z = 2 \) and the energy levels are

\[ n = \infty \quad 0 \]

\[ n = 5 \quad -2.18 \text{ eV} \]

\[ n = 4 \quad -3.40 \text{ eV} \]

\[ n = 3 \quad -6.04 \text{ eV} \]

\[ n = 2 \quad -13.6 \text{ eV} \]

\[ n = 1 \quad -54.4 \text{ eV} \]

(b) For He\(^+ \), \( Z = 2 \), so we see that the ionization energy (the energy required to take the electron from the \( n = 1 \) to the \( n = \infty \) state) is

\[ E = E_n - E_i = 0 - \frac{(-13.6 \text{ eV})(2)^2}{(1)^2} = 54.4 \text{ eV} \]
28.43  (a)  \[ I = \frac{P}{A} \left( \frac{\Delta E}{\Delta t} \right) = \frac{4 \left( 3.00 \times 10^{-3} \text{J}/1.00 \times 10^{-9} \text{s} \right)}{\pi \left( 30.0 \times 10^{-6} \text{m} \right)^2} = 4.24 \times 10^{12} \text{W/m}^2 \]

(b)  \[ E = I A \left( \Delta t \right) = \left( 4.24 \times 10^{14} \frac{\text{W}}{\text{m}^2} \right) \left( \frac{\pi}{4} \left( 0.600 \times 10^{-9} \text{m} \right)^2 \right) \left( 1.00 \times 10^{-9} \text{s} \right) = 1.20 \times 10^{12} \text{J} \]

28.44  (a)  Given that the de Broglie wavelength is \( \lambda = 2 a_o \), the momentum is \( p = \hbar/\lambda = \hbar/2 a_o \). The kinetic energy of this non-relativistic electron is

\[ KE = \frac{p^2}{2 m_e} = \frac{\hbar^2}{8 m_e a_o^2} \]

\[ = \left( \frac{6.63 \times 10^{-34} \text{J} \cdot \text{s}}{1 \text{ eV}/1.60 \times 10^{-19} \text{J}} \right) \left( 1.60 \times 10^{-19} \text{J} \right) = 135 \text{eV} \]

(b)  The kinetic energy of this electron is \[ \approx 10 \text{ times} \] the magnitude of the ground state energy of the hydrogen atom, which is \(-13.6 \text{eV}\).  

28.45  In the Bohr model,

\[ f = \frac{\Delta E}{\hbar} = \frac{E_n - E_{n-1}}{\hbar} \]

\[ = \frac{1}{\hbar} \left[ -\frac{m k^2 e^4}{2 h^2} \left( \frac{1}{n^2} - \frac{1}{(n-1)^2} \right) \right] = \frac{4 \pi^2 m k e^4}{2 h^2} \left[ \frac{1}{n^2} - \frac{1}{(n-1)^2} \right] \]

which reduces to

\[ f = \frac{2 \pi^2 m k e^4}{h^2} \left( \frac{2 n - 1}{(n-1)^2 n^2} \right) \]

28.46  \[ E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left( 6.626 \times 10^{-34} \text{J} \cdot \text{s} \right) \left( 2.998 \times 10^4 \text{m/s} \right)}{\lambda \left( 1.602 \times 10^{-19} \text{J/eV} \right) \left( 10^{-9} \text{m/nm} \right)} = 1.240 \text{eV} \cdot \text{nm} \]

\[ \frac{\lambda}{\lambda} = \Delta E \]

For:

\( \lambda = 310.0 \text{ nm}, \Delta E = 4.000 \text{ eV} \)

\( \lambda = 400.0 \text{ nm}, \Delta E = 3.100 \text{ eV} \)

and \( \lambda = 1.378 \text{ nm}, \Delta E = 0.900 \text{ eV} \)

The ionization energy is \( 4.100 \text{ eV} \). The energy level diagram having the smallest number of levels and consistent with these energy differences is shown below.

\[ \begin{array}{c}
\text{First Excited State} \\
\text{Second Excited State} \\
\text{Ground State}
\end{array} \]

\[ \begin{array}{c}
\downarrow \\
1.378 \text{ nm} \\
310 \text{ nm} \\
400 \text{ nm}
\end{array} \]

\[ \begin{array}{c}
\text{E} = 0 \\
-0.100 \text{ eV} \\
-1.000 \text{ eV} \\
-4.100 \text{ eV}
\end{array} \]

\[ n = \infty \]