22

Reflection and Refraction of Light

Clicker Questions

Question P1.01

Question

The sun appears larger at sunset than it does when overhead because of:

1. Scattering of light.
2. Dispersion.
3. Diffraction.
4. Refraction.
5. An optical illusion.
6. None of the above.

Commentary

Purpose: To connect physics processes to your everyday experience.

Discussion: A careful measurement of the sun's angular size when overhead and setting would reveal that it is not in fact larger at sunset. It just looks larger: an optical illusion, due to the fact that your eye has objects on the horizon to compare it to when setting, but nothing to use as a size reference when it is overhead. The moon, also, looks larger on the horizon than overhead.

Key Points:

- How something appears can be strongly influenced by its surroundings. This is the basis for many optical illusions.
- Not all of your everyday experiences can be explained by physics! Some must be explained by psychology or cognitive science.

For Instructors Only

This question follows a set on scattering and similar optical processes, such as Questions 60 and 61, well. Students may assume from context that the answer to this must be "scattering" or the like; this question helps keeps them thinking.

Question P1.02

Description: Introducing refraction.

Question

Refraction occurs at the interface between two transparent media because:

1. The mass density of the material changes.
2. The frequency of the light changes.
3. The speed of light is different in the two media.
4. The direction of the light changes.
5. Some of the light is reflected.
6. None of the above.

Commentary

Purpose: To probe your understanding of the *cause* of refraction.

Discussion: Answer (4), the changing of the light's direction, is what refraction *means;* it is not an explanation or cause. The light changes direction because the propagation speed of light in the two media is different (answer 3); as the wave front crosses the interface, the part already across the interface is moving at a different rate, "skewing" the front.

The mass density of the materials may differ (answer 1), but this is not necessarily the case and does not directly cause the refraction. Some of the light may indeed reflect (answer 5), but this is a separate phenomenon. Although the speed and wavelength of the light change, its frequency does not, so statement (2) is simply false.

Key Points:

- Refraction occurs when light crosses a boundary between two materials of different *index of refraction:* that is, materials in which light travels at different speeds.
- The *index of refraction* indicates the speed of light in a medium, as compared to the speed of light in vacuum.
- It is important to distinguish between a *cause* of a phenomenon, a *description* of it, phenomena associated with it, and phenomena similar to it.

For Instructors Only

Explaining how refraction arises from a change in the speed of light depends on the level of your class and the time you are willing to spend on it. A wavefront description based on Huygens's construction works. So also can a "marching ants" picture.

QUICK QUIZZES

1. (a). In part (a), you can see clear reflections of the headlights and the lights on the top of the truck. The reflection is specular. In part (b), although bright areas appear on the roadway in front of the headlights, the reflection is not as clear, and no separate reflection of the lights from the top of the truck is visible. The reflection in part (b) is mostly diffuse.

2. Beams 2 and 4 are reflected; beams 3 and 5 are refracted.

3. (b). When light goes from one material into one having a higher index of refraction, it refracts toward the normal line of the boundary between the two materials. If, as the light travels through the new material, the index of refraction continues to increase, the light ray will refract more and more toward the normal line.

4. (c). Both the wave speed and the wavelength decrease as the index of refraction increases. The frequency is unchanged.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The energy of a photon is $E = hf = hc/\lambda$. Thus, if n 800 nm photons have the same energy as four 200 nm photons, it is necessary that $n(hc/800 \text{ nm}) = 4(hc/200 \text{ nm})$, or $n = 4(800 \text{ nm}/200 \text{ nm}) = 16$. Therefore, the correct answer is (e).

2. In going from carbon disulfide ($n_1 = 1.63$) to crown glass ($n_2 = 1.52$), the critical angle for total internal reflection is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.52}{1.63}\right) = 69°$$

so the correct choice is (b).

3. As light travels from one medium to another, both the wavelength of the light and the index of refraction of the medium will change, but the product λn is constant. In going from air into a second medium, $\lambda_2 n_2 = \lambda_{\text{air}} n_{\text{air}}$, or the index of refraction of the second medium is

$$n_2 = n_{\text{air}}\left(\frac{\lambda_{\text{air}}}{\lambda_2}\right) = (1.00)\left(\frac{495 \text{ nm}}{434 \text{ nm}}\right) = 1.14$$

and (c) is the correct choice.

4. When light travels from air ($n_{\text{air}} = 1.00$) into glass ($n_{\text{glass}} > n_{\text{air}}$), both the speed and the wavelength decrease while the frequency is unchanged. Thus, choice (e) is the only true statement among the listed choices.

5. When light is in water, the relationships between the values of its frequency, speed, and wavelength to the values of the same quantities in air are

$$f_{\text{water}} = f_{\text{air}}, \qquad \lambda_{\text{water}} = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\lambda_{\text{air}} \approx \frac{3}{4}\lambda_{\text{air}}, \qquad \text{and} \qquad v_{\text{water}} = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)v_{\text{air}} = \left(\frac{3}{4}\right)c$$

Therefore, only choice (b) is a completely true statement.

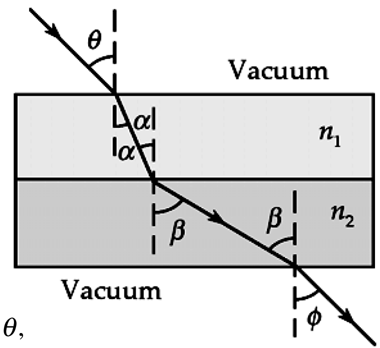
6. Water and air have different indices of refraction, with $n_{\text{water}} \approx 4n_{\text{air}}/3$. In passing from one of these media into the other, light will be refracted (deviated in direction) unless the angle of incidence is zero (in which case, the angle of refraction is also zero). Thus, rays B and D cannot be correct. In refraction, the incident ray and the refracted ray are never on the same side of the line normal to the surface at the point of contact, so ray A cannot be correct. Also in refraction, the ray makes a smaller angle with the normal in the medium having the highest index of refraction. Therefore, ray E cannot be correct, leaving only ray C as a likely path. Choice (c) is the correct answer.

7. Total internal reflection will occur when light, in attempting to go from a medium with one index of refraction n_1 into a second medium where it travels faster than in the first medium (or where $n_2 < n_1$), strikes the surface at an angle of incidence greater than or equal to the critical angle. The geometrical shape of the surface between the two media is irrelevant. The correct choice is (b).

8. For any medium, other than vacuum, the index of refraction for red light is slightly lower than that for blue light. This means that when light goes from vacuum (or air) into glass, the red light deviates from its original direction less than does the blue light. Also, as the light reemerges from the glass into vacuum (or air), the red light again deviates less than the blue light. If the two surfaces of the glass are parallel to each other, the red and blue rays will emerge traveling parallel to each other, but displaced laterally from one another. The sketch that best illustrates this process is C, so (c) is the best answer.

9. In a dispersive medium, the index of refraction is largest for the shortest wavelength. Thus, the violet light will be refracted (or bent) the most as it passes through a surface of the crown glass, making (a) the correct choice.

10. Consider the sketch at the right and apply Snell's law to the refraction at each of the three surfaces. The resulting equations are



$(1.00)\sin\theta = n_1 \sin\alpha$ (1st Surface)

$n_1 \sin\alpha = n_2 \sin\beta$ (2nd Surface)

and $n_2 \sin\beta = (1.00)\sin\phi$ (3rd Surface)

Combining these three equations yields $(1.00)\sin\phi = (1.00)\sin\theta$, and $\phi = \theta$. Hence, choice (c) is the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The spectrum of the light sent back to you from a drop at the top of the rainbow arrives such that the red light (deviated by an angle of 42°) strikes the eye while the violet light (deviated by 40°) passes over your head. Thus, the top of the rainbow looks red. At the bottom of the bow, violet light arrives at your eye and red light is deviated toward the ground. Thus, the bottom part of the bow appears violet.

4. A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction. The different indices of refraction occur because the air has different densities at different temperatures. Two images are seen: one from a direct path from the object to you, and the second arriving by rays originally heading toward Earth but refracted to your eye. On a hot day, the Sun makes the surface of blacktop hot, so the air is hot directly above it, becoming cooler as one moves higher into the sky. The "water" we see far in front of us is an image of the blue sky. Adding to the effect is the fact that the image shimmers as the air changes in temperature, giving the appearance of moving water.

6. The upright image of the hill is formed by light that has followed a direct path from the hill to the eye of the observer. The second image is a result of refraction in the atmosphere. Some light is reflected from the hill toward the water. As this light passes through warmer layers of air directly above the water, it is refracted back up toward the eye of the observer, resulting in the observation of an inverted image of the hill directly below the upright image.

8. The color traveling slowest is bent the most. Thus, *X* travels more slowly in the glass prism.

10. Total internal reflection occurs only when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction. Thus, light moving from air ($n = 1$) to water ($n = 1.33$) cannot undergo total internal reflection.

12. Objects beneath the surface of water appear to be raised toward the surface by refraction. Thus, the bottom of the oar appears to be closer to the surface than it really is, and the oar looks to be bent.

PROBLEM SOLUTIONS

22.1 The total distance the light travels is

$$\Delta d = 2\left(D_{\substack{\text{center to}\\ \text{center}}} - R_{\text{Earth}} - R_{\text{Moon}}\right)$$

$$= 2\left(3.84\times10^{8} - 6.38\times10^{6} - 1.76\times10^{6}\right)\text{ m} = 7.52\times10^{8}\text{ m}$$

Therefore, $v = \dfrac{\Delta d}{\Delta t} = \dfrac{7.52\times10^{8}\text{ m}}{2.51\text{ s}} = \boxed{3.00\times10^{8}\text{ m/s}}$

22.2 (a) The energy of a photon is $E = hf = hc/\lambda$, where Planck's constant is $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$ and the speed of light in vacuum is $c = 3.00\times10^{8}\text{ m/s}$. If $\lambda = 1.00\times10^{-10}\text{ m}$,

$$E = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.00\times10^{-10}\text{ m}} = \boxed{1.99\times10^{-15}\text{ J}}$$

(b) $E = \left(1.99\times10^{-15}\text{ J}\right)\left(\dfrac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right) = \boxed{1.24\times10^{4}\text{ eV}}$

(c) and (d) For the x-rays to be more penetrating, the photons should be more energetic. Since the energy of a photon is directly proportional to the frequency and inversely proportional to the wavelength, $\boxed{\text{the wavelength should decrease}}$, which is the same as saying $\boxed{\text{the frequency should increase}}$.

22.3 (a) $E = hf = \left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(5.00\times10^{17}\text{ Hz}\right)\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{2.07\times10^{3}\text{ eV}}$

(b) $E = hf = \dfrac{hc}{\lambda} = \dfrac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{3.00\times10^{2}\text{ nm}}\left(\dfrac{1\text{ nm}}{10^{-9}\text{ m}}\right) = 6.63\times10^{-19}\text{ J}$

$$E = 6.63\times10^{-19}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{4.14\text{ eV}}$$

22.4 (a) $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{5.45 \times 10^{14}\ \text{Hz}} = \boxed{5.50 \times 10^{-7}\ \text{m}}$

(b) From Table 22.1 the index of refraction for benzene is $n = 1.501$. Thus, the wavelength in benzene is

$$\lambda_n = \frac{\lambda_0}{n} = \frac{5.50 \times 10^{-7}\ \text{m}}{1.501} = \boxed{3.67 \times 10^{-7}\ \text{m}}$$

(c) $E = hf = \left(6.63 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(5.45 \times 10^{14}\ \text{Hz}\right)\left(\dfrac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{2.26\ \text{eV}}$

(d) The energy of the photon is proportional to the frequency, which does not change as the light goes from one medium to another. Thus, when the photon enters benzene, $\boxed{\text{the energy does not change}}$.

22.5 The speed of light in a medium with index of refraction n is $v = c/n$, where c is its speed in vacuum.

(a) For water, $n = 1.333$, and $v = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{1.333} = \boxed{2.25 \times 10^{8}\ \text{m/s}}$

(b) For crown glass, $n = 1.52$, and $v = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{1.52} = \boxed{1.97 \times 10^{8}\ \text{m/s}}$

(c) For diamond, $n = 2.419$, and $v = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{2.419} = \boxed{1.24 \times 10^{8}\ \text{m/s}}$

22.6 (a) From $\lambda f = c$, the wavelength is given by $\lambda = c/f$. The energy of a photon is $E = hf$, so the frequency may be expressed as $f = E/h$, and the wavelength becomes

$$\lambda = \frac{c}{f} = \frac{c}{E/h} = \boxed{\frac{hc}{E}}$$

(b) $\boxed{\text{Higher energy photons have shorter wavelengths.}}$

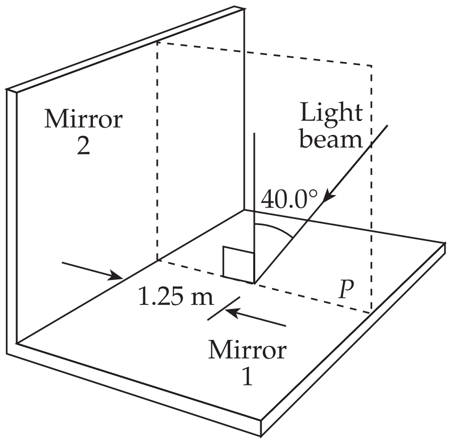
22.7 From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. Thus, when $\theta_1 = 45°$ and the first medium is air $(n_1 = 1.00)$, we have $\sin\theta_2 = (1.00)\sin 45°/n_2$.

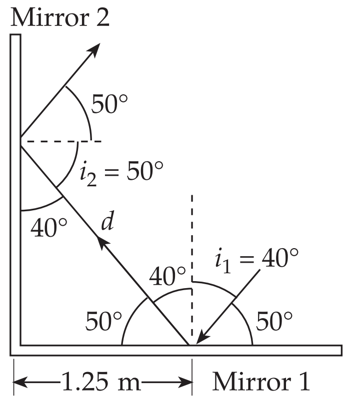
(a) For quartz, $n_2 = 1.458$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45°}{1.458}\right) = \boxed{29°}$

(b) For carbon disulfide, $n_2 = 1.628$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45°}{1.628}\right) = \boxed{26°}$

(c) For water, $n_2 = 1.333$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45°}{1.333}\right) = \boxed{32°}$

22.8

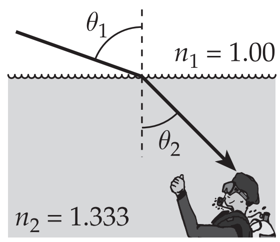




(a) From geometry, $1.25\text{ m} = d\sin 40.0°$, so $d = \boxed{1.94\text{ m}}$

(b) $\boxed{50.0° \text{ above horizontal}}$, or parallel to the incident ray

22.9 $n_1 \sin\theta_1 = n_2 \sin\theta_2$



$\sin\theta_1 = 1.333 \sin 45.0°$

$\sin\theta_1 = (1.333)(0.707) = 0.943$

$\theta_1 = 70.5° \quad \rightarrow \quad \boxed{19.5° \text{ above the horizontal}}$

22.10 (a) $n = \dfrac{c}{v} = \dfrac{3.00\times10^8\text{ m/s}}{2.17\times10^8\text{ m/s}} = \boxed{1.38}$

(b) From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$,

$$\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right) = \sin^{-1}\left[\frac{(1.00)\sin 23.1°}{1.38}\right] = \sin^{-1}(0.284) = \boxed{16.5°}$$

22.11 (a) From Snell's law, $n_2 = \dfrac{n_1\sin\theta_1}{\sin\theta_2} = \dfrac{(1.00)\sin 30.0°}{\sin 19.24°} = \boxed{1.52}$

(b) $\lambda_2 = \dfrac{\lambda_0}{n_2} = \dfrac{632.8\text{ nm}}{1.52} = \boxed{416\text{ nm}}$

(c) $f = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^8\text{ m/s}}{632.8\times10^{-9}\text{ m}} = \boxed{4.74\times10^{14}\text{ Hz}}$ in air and in syrup

(d) $v_2 = \dfrac{c}{n_2} = \dfrac{3.00\times10^8\text{ m/s}}{1.52} = \boxed{1.97\times10^8\text{ m/s}}$

22.12 (a) When light refracts from air $(n_1 = 1.00)$ into the Crown glass, Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\sin 25.0°/n_{\text{Crown glass}}\right)$$

For first quadrant angles, the sine of the angle increases as the angle increases. Thus, from the above equation, note that θ_2 will increase when the index of refraction of the Crown glass decreases. From Figure 22.14, this means that the longer wavelengths have the largest angles of refraction, and $\boxed{\text{deviate the least}}$ from the original path.

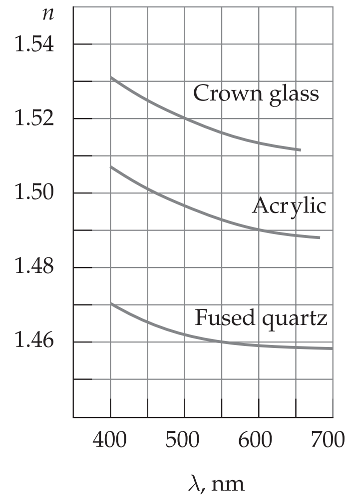


Figure 22.14

(b) From Figure 22.14, observe that the index of refraction of Crown glass for the given wavelengths is:

$$\lambda = 400 \text{ nm:} \quad n_{\text{Crown glass}} = 1.53; \quad \lambda = 500 \text{ nm:} \quad n_{\text{Crown glass}} = 1.52;$$

and $\lambda = 650$ nm: $n_{\text{Crown glass}} = 1.51$

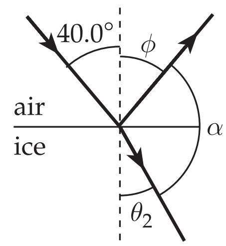
Thus, Snell's law gives: $\lambda = 400$ nm: $\theta_2 = \sin^{-1}\left(\sin 25.0°/1.53\right) = \boxed{16.0°}$

$\lambda = 500$ nm: $\theta_2 = \sin^{-1}\left(\sin 25.0°/1.52\right) = \boxed{16.1°}$

$\lambda = 650$ nm: $\theta_2 = \sin^{-1}\left(\sin 25.0°/1.51\right) = \boxed{16.3°}$

22.13 From Snell's law,

$$\theta_2 = \sin^{-1}\left[\frac{n_1 \sin\theta_1}{n_2}\right] = \sin^{-1}\left[\frac{(1.00)\sin 40.0°}{1.309}\right] = 29.4°$$



and from the law of reflection, $\phi = \theta_1 = 40.0°$

Hence, the angle between the reflected and refracted rays is

$$\alpha = 180° - \theta_2 - \phi = 180° - 29.4° - 40.0° = \boxed{111°}$$

22.14 Using a protractor to measure the angle of incidence and the angle of refraction in Active Figure 22.6b gives $\theta_1 = 55°$ and $\theta_2 = 33°$. Then, from Snell's law, the index of refraction for the Lucite is

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1.00)\sin 55°}{\sin 33°} = 1.5$$

(a) $$v_2 = \frac{c}{n_2} = \frac{3.00\times10^8 \text{ m/s}}{1.5} = \boxed{2.0\times10^8 \text{ m/s}}$$

(b) $$f = \frac{c}{\lambda_0} = \frac{3.00\times10^8 \text{ m/s}}{6.328\times10^{-7} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$$

(c) $$\lambda_2 = \frac{\lambda_0}{n_2} = \frac{6.328\times10^{-7} \text{ m}}{1.5} = \boxed{4.2\times10^{-7} \text{ m}}$$

22.15 The index of refraction of zircon is $n = 1.923$.

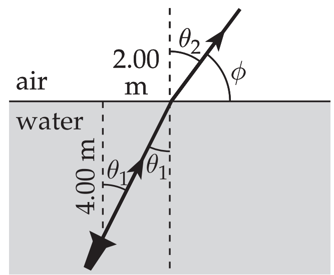
(a) $v = \dfrac{c}{n} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.923} = \boxed{1.56 \times 10^8 \text{ m/s}}$

(b) The wavelength in the zircon is $\lambda_n = \dfrac{\lambda_0}{n} = \dfrac{632.8 \text{ nm}}{1.923} = \boxed{329.1 \text{ nm}}$

(c) The frequency is $f = \dfrac{v}{\lambda_n} = \dfrac{c}{\lambda_0} = \dfrac{3.00 \times 10^8 \text{ m/s}}{632.8 \times 10^{-9} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$

22.16 The angle of incidence is

$$\theta_1 = \tan^{-1}\left[\frac{2.00 \text{ m}}{4.00 \text{ m}}\right] = 26.6°$$



Therefore, Snell's law gives

$$\theta_2 = \sin^{-1}\left[\frac{n_1 \sin\theta_1}{n_2}\right] = \sin^{-1}\left[\frac{(1.333)\sin 26.6°}{1.00}\right] = 36.6°$$

and the angle the refracted ray makes with the surface is

$$\phi = 90.0° - \theta_2 = 90.0° - 36.6° = \boxed{53.4°}$$

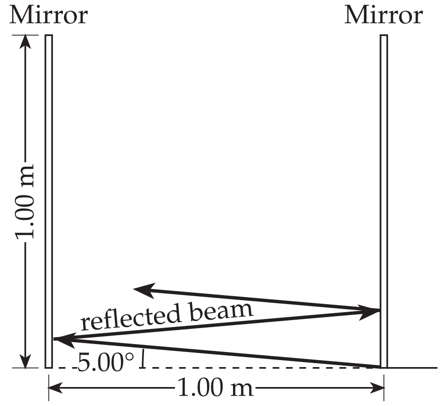
22.17 The incident light reaches the left-hand mirror at distance

$$d/2 = (1.00 \text{ m})\tan 5.00° = 0.087\ 5 \text{ m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

$$d = 2(0.087\ 5 \text{ m}) = 0.175 \text{ m}$$

It bounces between the mirrors with distance d between points of contact with a given mirror.



Since the full 1.00 m length of the right-hand mirror is available for reflections, the number of reflections from this mirror will be

$$N_{\text{right}} = \frac{1.00 \text{ m}}{0.175 \text{ m}} = 5.71 \quad \rightarrow \quad \boxed{5 \text{ full reflections}}$$

Since the first reflection from the left-hand mirror occurs at a height of $d/2 = 0.087\ 5$ m, the total number of reflections that can occur from this mirror is

$$N_{\text{left}} = 1 + \frac{1.00 \text{ m} - 0.087\ 5 \text{ m}}{0.175 \text{ m}} = 6.21 \quad \rightarrow \quad \boxed{6 \text{ full reflections}}$$

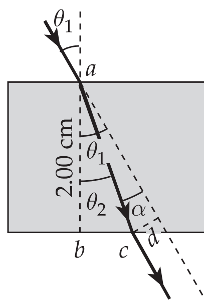
22.18 (a) From Snell's law, the angle of refraction at the first surface is

$$\theta_2 = \sin^{-1}\left[\frac{n_{\text{air}}\sin\theta_1}{n_{\text{glass}}}\right] = \sin^{-1}\left[\frac{(1.00)\sin 30.0^\circ}{1.50}\right] = \boxed{19.5^\circ}$$

(b) Since the upper and lower surfaces are parallel, the normal lines where the ray strikes these surfaces are parallel. Hence, the angle of incidence at the lower surface will be $\theta_2 = \boxed{19.5^\circ}$. The angle of refraction at this surface is then

$$\theta_3 = \sin^{-1}\left[\frac{n_{\text{glass}}\sin\theta_{\text{glass}}}{n_{\text{air}}}\right] = \sin^{-1}\left[\frac{(1.50)\sin 19.5^\circ}{1.00}\right] = \boxed{30.0^\circ}$$

Thus, the light emerges traveling parallel to the incident beam.



(c) Consider the sketch above and let h represent the distance from point a to c (that is, the hypotenuse of triangle abc). Then,

$$h = \frac{2.00\text{ cm}}{\cos\theta_2} = \frac{2.00\text{ cm}}{\cos 19.5^\circ} = 2.12\text{ cm}$$

Also, $\alpha = \theta_1 - \theta_2 = 30.0^\circ - 19.5^\circ = 10.5^\circ$, so

$$d = h\sin\alpha = (2.12\text{ cm})\sin 10.5^\circ = \boxed{0.386\text{ cm}}$$

(d) The speed of the light in the glass is

$$v = \frac{c}{n_{\text{glass}}} = \frac{3.00\times10^8\text{ m/s}}{1.50} = \boxed{2.00\times10^8\text{ m/s}}$$

(e) The time required for the light to travel through the glass is

$$t = \frac{h}{v} = \frac{2.12\text{ cm}}{2.00\times10^8\text{ m/s}}\left(\frac{1\text{ m}}{10^2\text{ cm}}\right) = \boxed{1.06\times10^{-10}\text{ s}}$$

(f) Changing the angle of incidence will change the angle of refraction, and therefore the distance h the light travels in the glass. Thus, $\boxed{\text{the travel time will also change}}$.

22.19 From Snell's law, the angle of incidence at the air-oil interface is

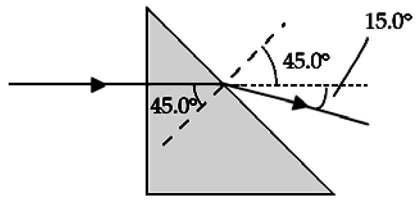
$$\theta = \sin^{-1}\left[\frac{n_{\text{oil}}\sin\theta_{\text{oil}}}{n_{\text{air}}}\right]$$

$$= \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.00}\right] = \boxed{30.4^\circ}$$

and the angle of refraction as the light enters the water is

$$\theta' = \sin^{-1}\left[\frac{n_{\text{oil}}\sin\theta_{\text{oil}}}{n_{\text{water}}}\right] = \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.333}\right] = \boxed{22.3^\circ}$$

22.20 Since the light ray strikes the first surface at normal incidence, it passes into the prism without deviation. Thus, the angle of incidence at the second surface (hypotenuse of the triangular prism) is $\theta_1 = 45.0°$, as shown in the sketch at the right. The angle of refraction is



$$\theta_2 = 45.0° + 15.0° = 60.0°$$

and Snell's law gives the index of refraction of the prism material as

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{(1.00)\sin(60.0°)}{\sin(45.0°)} = \boxed{1.22}$$

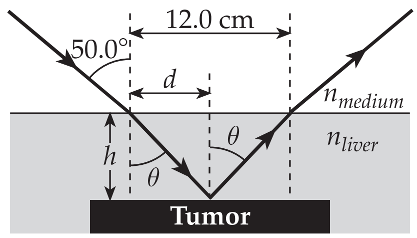
22.21 $\Delta t = (\text{time to travel 6.20 m in ice}) - (\text{time to travel 6.20 m in air})$

$$\Delta t = \frac{6.20 \text{ m}}{v_{\text{ice}}} - \frac{6.20 \text{ m}}{c}$$

Since the speed of light in a medium of refractive index n is $v = \dfrac{c}{n}$

$$\Delta t = (6.20 \text{ m})\left(\frac{1.309}{c} - \frac{1}{c}\right) = \frac{(6.20 \text{ m})(0.309)}{3.00 \times 10^8 \text{ m/s}} = 6.39 \times 10^{-9} \text{ s} = \boxed{6.39 \text{ ns}}$$

22.22 From Snell's law, $\sin\theta = \left(\dfrac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0°$



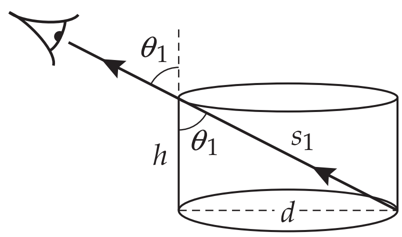
But, $\dfrac{n_{\text{medium}}}{n_{\text{liver}}} = \dfrac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \dfrac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900$

so, $\theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°$

From the law of reflection,

$$d = \frac{12.0 \text{ cm}}{2} = 6.00 \text{ cm, and } h = \frac{d}{\tan\theta} = \frac{6.00 \text{ cm}}{\tan(43.6°)} = \boxed{6.30 \text{ cm}}$$

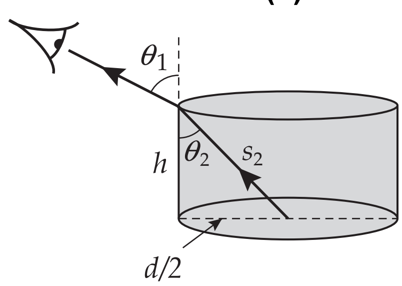
22.23 (a) Before the container is filled, the ray's path is as shown in Figure (a) at the right. From this figure, observe that



(a)

$$\sin\theta_1 = \frac{d}{s_1} = \frac{d}{\sqrt{h^2 + d^2}} = \frac{1}{\sqrt{(h/d)^2 + 1}}$$

After the container is filled, the ray's path is shown in Figure (b). From this figure, we find that



(b)

$$\sin\theta_2 = \frac{d/2}{s_2} = \frac{d/2}{\sqrt{h^2 + (d/2)^2}} = \frac{1}{\sqrt{4(h/d)^2 + 1}}$$

From Snell's law, $n_{\text{air}} \sin\theta_1 = n \sin\theta_2$, or

$$\frac{1.00}{\sqrt{(h/d)^2 + 1}} = \frac{n}{\sqrt{4(h/d)^2 + 1}} \quad \text{and} \quad 4(h/d)^2 + 1 = n^2 (h/d)^2 + n^2$$

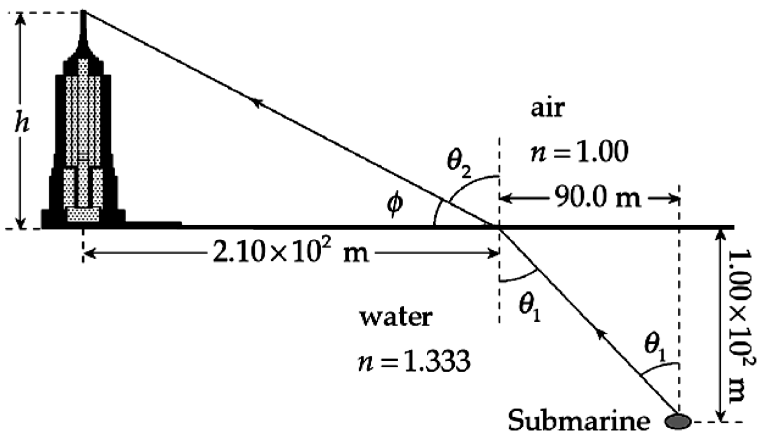
Simplifying, this gives $(4 - n^2)(h/d)^2 = n^2 - 1$ or $\boxed{\dfrac{h}{d} = \sqrt{\dfrac{n^2 - 1}{4 - n^2}}}$

continued on next page

(b) If $d = 8.0$ cm and $n = n_{\text{water}} = 1.333$, then

$$h = (8.0\text{ cm})\sqrt{\frac{(1.333)^2 - 1}{4-(1.333)^2}} = \boxed{4.7\text{ cm}}$$

22.24 (a) A sketch illustrating the situation and the two triangles needed in the solution is given below:



(b) The angle of incidence at the water surface is

$$\theta_1 = \tan^{-1}\left(\frac{90.0\text{ m}}{1.00\times 10^2\text{ m}}\right) = 42.0°$$

(c) Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{water}}\sin\theta_1}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.333)\sin 42.0°}{1.00}\right) = \boxed{63.1°}$$

(d) The refracted beam makes angle $\phi = 90.0° - \theta_2 = \boxed{26.9°}$ with the horizontal.

(e) Since $\tan\phi = h/(2.10\times 10^2\text{ m})$, the height of the target is

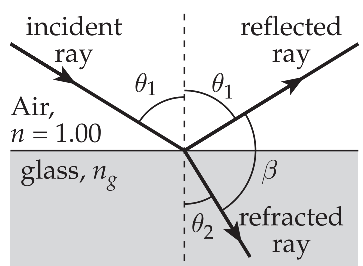
$$h = (2.10\times 10^2\text{ m})\tan(26.9°) = \boxed{107\text{ m}}$$

22.25 As shown at the right, $\theta_1 + \beta + \theta_2 = 180°$.

When $\beta = 90°$, this gives $\theta_2 = 90° - \theta_1$

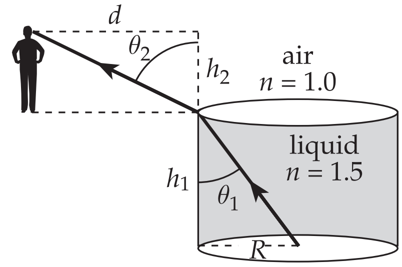
Then, from Snell's law

$$\sin\theta_1 = \frac{n_g \sin\theta_2}{n_{\text{air}}}$$
$$= n_g \sin(90° - \theta_1) = n_g \cos\theta_1$$



Thus, when $\beta = 90°$, $\dfrac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = n_g$ or $\boxed{\theta_1 = \tan^{-1}(n_g)}$

22.26 From the drawing, observe that



$$\theta_1 = \tan^{-1}\left(\frac{R}{h_1}\right) = \tan^{-1}\left(\frac{1.5\ \text{m}}{2.0\ \text{m}}\right) = 37°$$

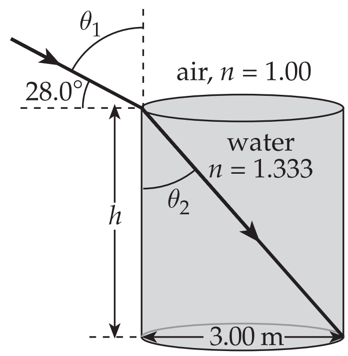
Applying Snell's law to the ray shown gives

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{liquid}}\sin\theta_1}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{1.5\sin 37°}{1.0}\right) = 64°$$

Thus, the distance of the girl from the cistern is

$$d = h_2\tan\theta_2 = (1.2\ \text{m})\tan 64° = \boxed{2.5\ \text{m}}$$

22.27 When the Sun is 28.0° above the horizon, the angle of incidence for sunlight at the air-water boundary is



$$\theta_1 = 90.0° - 28.0° = 62.0°$$

Thus, the angle of refraction is

$$\theta_2 = \sin^{-1}\left[\frac{n_{\text{air}}\sin\theta_1}{n_{\text{water}}}\right]$$

$$= \sin^{-1}\left[\frac{(1.00)\sin 62.0°}{1.333}\right] = 41.5°$$

The depth of the tank is then $h = \dfrac{3.00\ \text{m}}{\tan\theta_2} = \dfrac{3.00\ \text{m}}{\tan(41.5°)} = \boxed{3.39\ \text{m}}$

22.28 The angles of refraction for the two wavelengths are

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_1}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{1.00\,0\sin 30.00°}{1.615}\right) = 18.04°$$

and $$\theta_{\text{blue}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_1}{n_{\text{blue}}}\right) = \sin^{-1}\left(\frac{1.00\,0\sin 30.00°}{1.650}\right) = 17.64°$$

Thus, the angle between the two refracted rays is

$$\Delta\theta = \theta_{\text{red}} - \theta_{\text{blue}} = 18.04° - 17.64° = \boxed{0.40°}$$

22.29 Using Snell's law gives

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.331}\right) = \boxed{48.22°}$$

and $$\theta_{\text{blue}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{blue}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.340}\right) = \boxed{47.79°}$$

22.30 Using Snell's law gives

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.512}\right) = \boxed{34.9°}$$

and $$\theta_{\text{violet}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{violet}}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.530}\right) = \boxed{34.5°}$$

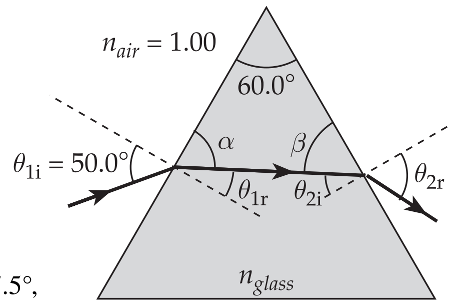
22.31 Using Snell's law gives

$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.455}\right) = \boxed{31.77°}$$

and $$\theta_{violet} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{violet}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.468}\right) = \boxed{31.45°}$$

Thus, the dispersion is $\theta_{red} - \theta_{violet} = 31.77° - 31.45° = \boxed{0.32°}$

22.32 For the violet light, $n_{glass} = 1.66$, and



$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{glass}}\right)$$

$$= \sin^{-1}\left(\frac{1.00\sin 50.0°}{1.66}\right) = 27.5°$$

$\alpha = 90° - \theta_{1r} = 62.5°$, $\beta = 180.0° - 60.0° - \alpha = 57.5°$,

and $\theta_{2i} = 90.0° - \beta = 32.5°$. The final angle of refraction of the violet light is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{2i}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.66\sin 32.5°}{1.00}\right) = 63.2°$$

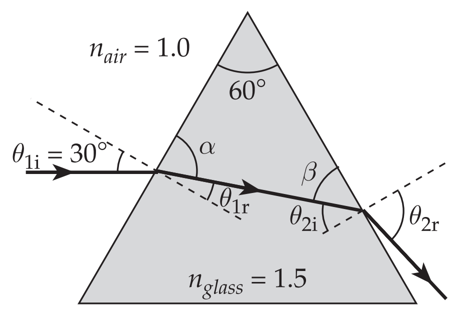
Following the same steps for the red light $(n_{glass} = 1.62)$ gives

$\theta_{1r} = 28.2°$, $\alpha = 61.8°$, $\beta = 58.2°$, $\theta_{2i} = 31.8°$, and $\theta_{2r} = 58.6°$

Thus, the angular dispersion of the emerging light is

$$Dispersion = \theta_{2r}\big|_{violet} - \theta_{2r}\big|_{red} = 63.2° - 58.6° = \boxed{4.6°}$$

22.33 (a) The angle of incidence at the first surface is $\theta_{1i} = 30°$, and the angle of refraction is



$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{glass}}\right)$$

$$= \sin^{-1}\left(\frac{1.0\sin 30°}{1.5}\right) = 19°$$

Also, $\alpha = 90° - \theta_{1r} = 71°$ and

$\beta = 180° - 60° - \alpha = 49°$

Therefore, the angle of incidence at the second surface is $\theta_{2i} = 90° - \beta = \boxed{41°}$. The angle of refraction at this surface is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{2i}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.5\sin 41°}{1.0}\right) = \boxed{77°}$$

(b) The angle of reflection at each surface equals the angle of incidence at that surface. Thus,

$$(\theta_1)_{reflection} = \theta_{1i} = \boxed{30°}, \text{ and } (\theta_2)_{reflection} = \theta_{2i} = \boxed{41°}$$

22.34 As light goes from a medium having a refractive index n_1 to a medium with refractive index $n_2 < n_1$, the critical angle is given the relation $\sin\theta_c = n_2/n_1$. Table 22.1 gives the refractive index for various substances at $\lambda_0 = 589$ nm.

(a) For fused quartz surrounded by air, $n_1 = 1.458$ and $n_2 = 1.00$, giving $\theta_c = \sin^{-1}(1.00/1.458) = \boxed{43.3°}$.

(b) In going from polystyrene ($n_1 = 1.49$) to air, $\theta_c = \sin^{-1}(1.00/1.49) = \boxed{42.2°}$.

(c) From sodium chloride ($n_1 = 1.544$) to air, $\theta_c = \sin^{-1}\left(1.00/1.544\right) = \boxed{40.4°}$.

22.35 When light is coming from a medium of refractive index n_1 into water ($n_2 = 1.333$), the critical angle is given by $\theta_c = \sin^{-1}(1.333/n_1)$.

(a) For fused quartz, $n_1 = 1.458$, giving $\theta_c = \sin^{-1}\left(1.333/1.458\right) = \boxed{66.1°}$.

(b) In going from polystyrene ($n_1 = 1.49$) to water, $\theta_c = \sin^{-1}\left(1.333/1.49\right) = \boxed{63.5°}$.

(c) From sodium chloride ($n_1 = 1.544$) to water, $\theta_c = \sin^{-1}\left(1.333/1.544\right) = \boxed{59.7°}$.

22.36 Using Snell's law, the index of refraction of the liquid is found to be

$$n_{\text{liquid}} = \frac{n_{air}\sin\theta_i}{\sin\theta_r} = \frac{(1.00)\sin 30.0°}{\sin 22.0°} = 1.33$$

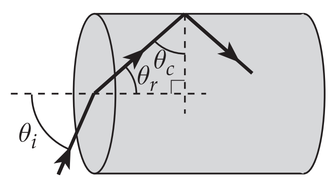
Thus, $\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{liquid}}}\right) = \sin^{-1}\left(\frac{1.00}{1.33}\right) = \boxed{48.5°}$

22.37 When light attempts to cross a boundary from one medium of refractive index n_1 into a new medium of refractive index $n_2 < n_1$, total internal reflection will occur if the angle of incidence exceeds the critical angle given by $\theta_c = \sin^{-1}\left(n_2/n_1\right)$.

(a) If $n_1 = 1.53$ and $n_2 = n_{\text{air}} = 1.00$, then $\theta_c = \sin^{-1}\left(\frac{1.00}{1.53}\right) = \boxed{40.8°}$

(b) If $n_1 = 1.53$ and $n_2 = n_{\text{water}} = 1.333$, then $\theta_c = \sin^{-1}\left(\frac{1.333}{1.53}\right) = \boxed{60.6°}$

22.38 The critical angle for this material in air is

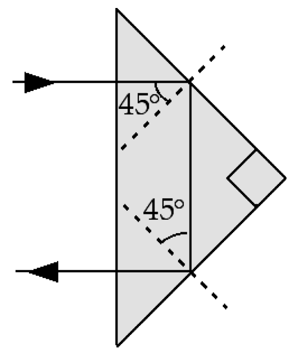


$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{pipe}}}\right) = \sin^{-1}\left(\frac{1.00}{1.36}\right) = 47.3°$$

Thus, $\theta_r = 90.0° - \theta_c = 42.7°$ and from Snell's law,

$$\theta_i = \sin^{-1}\left(\frac{n_{\text{pipe}}\sin\theta_r}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.36)\sin 42.7°}{1.00}\right) = \boxed{67.2°}$$

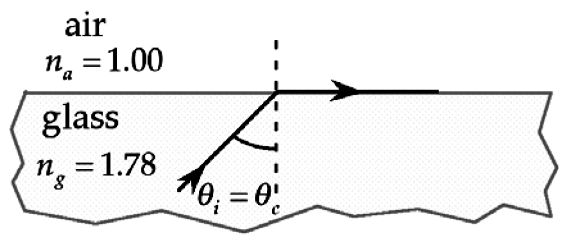
22.39 The angle of incidence at each of the shorter faces of the prism is 45°, as shown in the figure at the right. For total internal reflection to occur at these faces, it is necessary that the critical angle be less than 45°. With the prism surrounded by air, the critical angle is given by $\sin\theta_c = n_{\text{air}}/n_{\text{prism}} = 1.00/n_{\text{prism}}$, so it is necessary that



$$\sin\theta_c = \frac{1.00}{n_{\text{prism}}} < \sin 45°$$

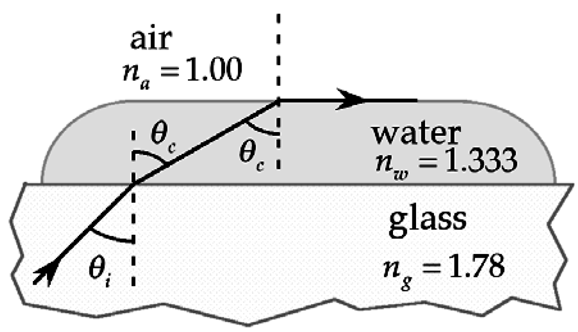
or $$n_{\text{prism}} > \frac{1.00}{\sin 45°} = \frac{1.00}{\sqrt{2}/2} = \boxed{\sqrt{2}}$$

22.40 (a) The minimum angle of incidence for which total internal reflection occurs is the critical angle. At the critical angle, the angle of refraction is 90°, as shown in the figure at the right. From Snell's law, $n_g \sin\theta_i = n_a \sin 90°$, the critical angle for the glass-air interface is found to be



$$\theta_i = \theta_c = \sin^{-1}\left(\frac{n_a \sin 90°}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{1.78}\right) = \boxed{34.2°}$$

(b) When the slab of glass has a layer of water on top, we want the angle of incidence at the water-air interface to equal the critical angle for that combination of media. At this angle, Snell's law gives



$$n_w \sin\theta_c = n_a \sin 90° = 1.00$$

and $$\sin\theta_c = 1.00/n_w$$

Now, considering the refraction at the glass-water interface, Snell's law gives $n_g \sin\theta_i = n_g \sin\theta_c$. Combining this with the result for $\sin\theta_c$ from above, we find the required angle of incidence in the glass to be

$$\theta_i = \sin^{-1}\left(\frac{n_w \sin\theta_c}{n_g}\right) = \sin^{-1}\left(\frac{n_w (1.00/n_w)}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{1.78}\right) = \boxed{34.2°}$$

(c) and (d) Observe in the calculation of part (b) that all the physical properties of the intervening layer (water in this case) canceled, and the result of part (b) is identical to that of part (a). This will always be true when the upper and lower surfaces of the intervening layer are parallel to each other. Neither the thickness nor the index of refraction of the intervening layer affects the result.

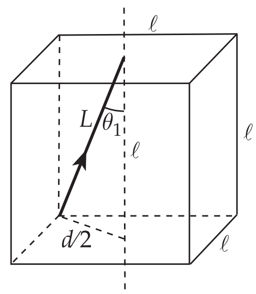
22.41 (a) Snell's law can be written as $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{v_1}{v_2}$. At the critical angle of incidence $(\theta_1 = \theta_c)$, the angle of refraction is 90° and Snell's law becomes $\sin\theta_c = \dfrac{v_1}{v_2}$. At the concrete-air boundary,

$$\theta_c = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{343\ \text{m/s}}{1\,850\ \text{m/s}}\right) = \boxed{10.7°}$$

(b) Sound can be totally reflected only if it is initially traveling in the slower medium. Hence, at the concrete-air boundary, the sound must be traveling in $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

22.42 The sketch at the right shows a light ray entering at the painted corner of the cube and striking the center of one of the three unpainted faces of the cube. The angle of incidence at this face is the angle θ_1 in the triangle shown. Note that one side of this triangle is half the diagonal of a face and is given by



$$\frac{d}{2} = \frac{\sqrt{\ell^2 + \ell^2}}{2} = \frac{\ell}{\sqrt{2}}$$

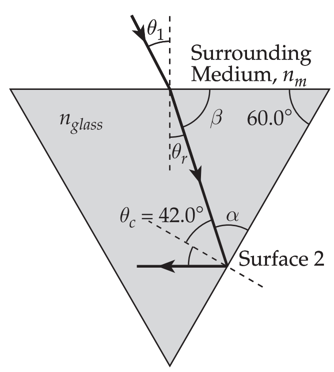
Also, the hypotenuse of this triangle is $L = \sqrt{\ell^2 + \left(\dfrac{d}{2}\right)^2} = \sqrt{\ell^2 + \dfrac{\ell^2}{2}} = \ell\sqrt{\dfrac{3}{2}}$

Thus, $\sin\theta_1 = \dfrac{d/2}{L} = \dfrac{\ell/\sqrt{2}}{\ell\left(\sqrt{3}/\sqrt{2}\right)} = \dfrac{1}{\sqrt{3}}$

For total internal reflection at this face, it is necessary that

$$\sin\theta_1 \geq \sin\theta_c = \frac{n_{\text{air}}}{n_{\text{cube}}} \quad \text{or} \quad \frac{1}{\sqrt{3}} \geq \frac{1.00}{n} \quad \text{giving} \quad \boxed{n \geq \sqrt{3}}$$

22.43 If $\theta_c = 42.0°$ at the boundary between the prism glass and the surrounding medium, then $\sin\theta_c = \dfrac{n_2}{n_1}$ gives



$$\frac{n_m}{n_{\text{glass}}} = \sin 42.0°$$

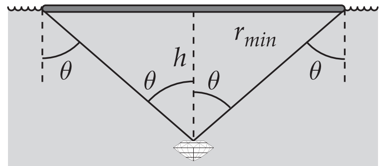
From the geometry shown in the figure at the right,

$$\alpha = 90.0° - 42.0° = 48.0°,\ \beta = 180° - 60.0° - \alpha = 72.0°$$

and $\theta_r = 90.0° - \beta = 18.0°$. Thus, applying Snell's law at the first surface gives

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\theta_r}{n_m}\right) = \sin^{-1}\left(\frac{\sin\theta_r}{n_m/n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{\sin 18.0°}{\sin 42.0°}\right) = \boxed{27.5°}$$

22.44 The circular raft must cover the area of the surface through which light from the diamond could emerge. Thus, it must form the base of a cone (with apex at the diamond) whose half angle is θ, where θ is greater than or equal to the critical angle.



The critical angle at the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.00}{1.333}\right) = 48.6°$$

Thus, the minimum diameter of the raft is

$$2r_{\min} = 2h\tan\theta_{\min} = 2h\tan\theta_c = 2(2.00\text{ m})\tan 48.6° = \boxed{4.54\text{ m}}$$

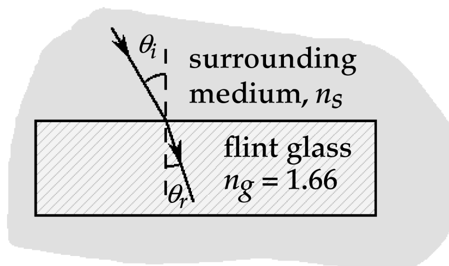
22.45 At the air-ice boundary, Snell's law gives the angle of refraction in the ice as

$$\theta_{1r} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_{1i}}{n_{\text{ice}}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 30.0°}{1.309}\right) = 22.5°$$

Since the sides of the ice layer are parallel, the angle of incidence at the ice-water boundary is $\theta_{2i} = \theta_{1r} = 22.5°$. Then, from Snell's law, the angle of refraction in the water is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{\text{ice}}\sin\theta_{2i}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{(1.309)\sin 22.5°}{1.333}\right) = \boxed{22.0°}$$

22.46 When light coming from the surrounding medium is incident on the surface of the glass slab, Snell's law gives $n_g \sin\theta_r = n_s \sin\theta_i$, or



$$\sin\theta_r = (n_s/n_g)\sin\theta_i$$

(a) If $\theta_i = 30.0°$ and the surrounding medium is water ($n_s = 1.333$), the angle of refraction is

$$\theta_r = \sin^{-1}\left[\frac{1.333\sin(30.0°)}{1.66}\right] = \boxed{23.7°}$$

(b) From Snell's law given above, we see that as $n_s \to n_g$ we have $\sin\theta_r \to \sin\theta_i$, or the angle or refraction approaches the angle of incidence, $\boxed{\theta_r \to \theta_i = 30.0°}$.

(c) If $n_s > n_g$, then $\sin\theta_r = (n_s/n_g)\sin\theta_i > \sin\theta_i$, or $\boxed{\theta_r > \theta_i}$.

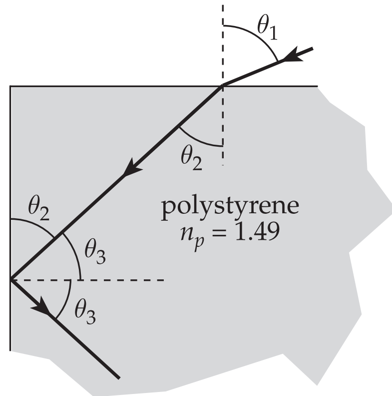
22.47 From Snell's law, $n_g \sin\theta_r = n_s \sin\theta_i$, where n_g is the refractive index of the glass and n_s is that of the surrounding medium. If $n_g = 1.52$ (crown glass), $n_s = 1.333$ (water), and $\theta_r = 19.6°$, the angle of incidence must have been

$$\theta_i = \sin^{-1}\left[\frac{n_g\sin\theta_r}{n_s}\right] = \sin^{-1}\left[\frac{(1.52)\sin 19.6°}{1.333}\right] = 22.5°$$

From the law of reflection, the angle of reflection for any light reflecting from the glass surface as the light is incident on the glass will be $\theta_{\text{reflection}} = \theta_i = \boxed{22.5°}$.

22.48 (a) For polystyrene surrounded by air, total internal reflection at the left vertical face requires that

$$\theta_3 \ge \theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_p}\right) = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$



From the geometry shown in the figure at the right,

$$\theta_2 = 90.0° - \theta_3 \le 90.0° - 42.2° = 47.8°$$

Thus, use of Snell's law at the upper surface gives

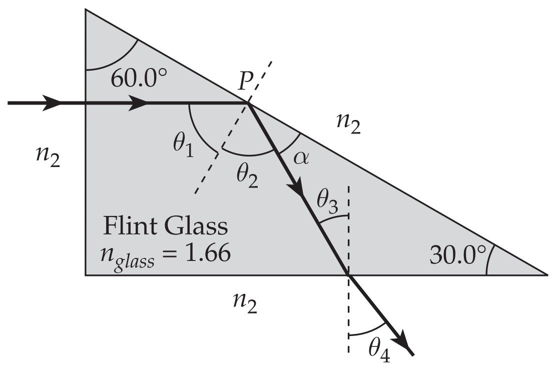
$$\sin\theta_1 = \frac{n_p \sin\theta_2}{n_{\text{air}}} \le \frac{(1.49)\sin 47.8°}{1.00} = 1.10$$

so it is seen that $\boxed{\text{any angle of incidence} \le 90°}$ at the upper surface will yield total internal reflection at the left vertical face.

(b) Repeating the steps of part (a) with the index of refraction of air replaced by that of water yields $\theta_3 \ge 63.5°$, $\theta_2 \le 26.5°$, $\sin\theta_1 \le 0.499$, and $\theta_1 \le \boxed{30.0°}$.

(c) Total internal reflection is $\boxed{\text{not possible since } n_{\text{polystyrene}} < n_{\text{carbon disulfide}}}$

22.49 (a) From the geometry of the figure at the right, observe that $\theta_1 = 60.0°$. Also, from the law of reflection, $\theta_2 = \theta_1 = 60.0°$. Therefore, $\alpha = 90.0° - \theta_2 = 30.0°$, and $\theta_3 + 90.0° = 180 - \alpha - 30.0°$ or $\theta_3 = 30.0°$.



Then, since the prism is immersed in water $(n_2 = 1.333)$, Snell's law gives

$$\theta_4 = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\theta_3}{n_2}\right) = \sin^{-1}\left(\frac{(1.66)\sin 30.0°}{1.333}\right) = \boxed{38.5°}$$

(b) For refraction to occur at point P, it is necessary that $\theta_c > \theta_1$.

Thus, $\theta_c = \sin^{-1}\left(\frac{n_2}{n_{\text{glass}}}\right) > \theta_1$, which gives

$$n_2 > n_{\text{glass}}\sin\theta_1 = (1.66)\sin 60.0° = \boxed{1.44}$$

22.50 Applying Snell's law to this refraction gives $n_{\text{glass}}\sin\theta_2 = n_{\text{air}}\sin\theta_1$

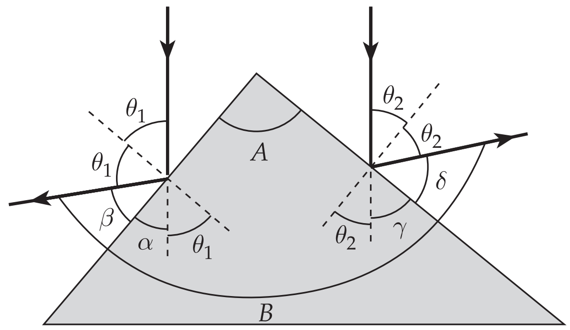
If $\theta_1 = 2\theta_2$, this becomes

$$n_{\text{glass}}\sin\theta_2 = \sin(2\theta_2) = 2\sin\theta_2\cos\theta_2 \;\text{ or }\; \cos\theta_2 = \frac{n_{\text{glass}}}{2}$$

Then, the angle of incidence is

$$\theta_1 = 2\theta_2 = 2\cos^{-1}\left(\frac{n_{\text{glass}}}{2}\right) = 2\cos^{-1}\left(\frac{1.56}{2}\right) = \boxed{77.5°}$$

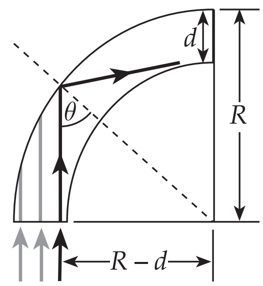
22.51 In the figure at the right, observe that $\beta = 90° - \theta_1$ and $\alpha = 90° - \theta_1$. Thus, $\beta = \alpha$.



Similarly, on the right side of the prism, $\delta = 90° - \theta_2$ and $\gamma = 90° - \theta_2$, giving $\delta = \gamma$.

Next, observe that the angle between the reflected rays is $B = (\alpha + \beta) + (\gamma + \delta)$, so $B = 2(\alpha + \gamma)$. Finally, observe that the left side of the prism is sloped at angle α from the vertical, and the right side is sloped at angle γ. Thus, the angle between the two sides is $A = \alpha + \gamma$, and we obtain the result $B = 2(\alpha + \gamma) = \boxed{2A}$.

22.52 (a) Observe in the sketch at the right that a ray originally traveling along the inner edge will have the smallest angle of incidence when it strikes the outer edge of the fiber in the curve. Thus, if this ray is totally internally reflected, all of the others are also totally reflected.



For this ray to be totally internally reflected it is necessary that

$$\theta \geq \theta_c \quad \text{or} \quad \sin\theta \geq \sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1}{n}$$

But, $\sin\theta = \dfrac{R-d}{R}$, so we must have $\dfrac{R-d}{R} \geq \dfrac{1}{n}$

which simplifies to $\boxed{R \geq nd/(n-1)}$

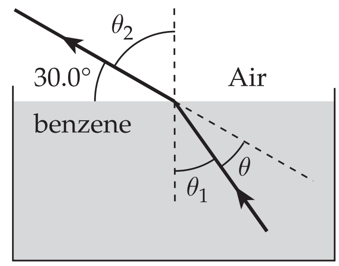
(b) As $d \to 0$, $R \to 0$. This is $\boxed{\text{reasonable}}$ behavior.

As n increases, $R_{\text{min}} = \dfrac{nd}{n-1} = \dfrac{d}{1-1/n}$ decreases. This is $\boxed{\text{reasonable}}$ behavior.

As $n \to 1$, R_{min} increases. This is $\boxed{\text{reasonable}}$ behavior.

(c) $$R_{\text{min}} = \frac{nd}{n-1} = \frac{(1.40)(100\ \mu\text{m})}{1.40-1} = \boxed{350\ \mu\text{m}}$$

22.53 Consider light which leaves the lower end of the wire and travels parallel to the wire while in the benzene. If the wire appears straight to an observer looking along the dry portion of the wire, this ray from the lower end of the wire must enter the observers eye as he sights along the wire. Thus, the ray must refract and travel parallel to the wire in air. The angle of refraction is then $\theta_2 = 90.0° - 30.0° = 60.0°$. From Snell's law, the angle of incidence was

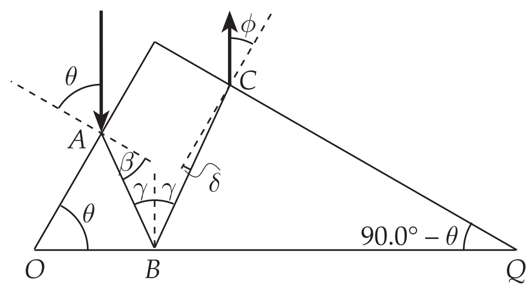


$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_2}{n_{\text{benzene}}}\right)$$

$$= \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.50}\right) = 35.3°$$

and the wire is bent by angle $\theta = 60.0° - \theta_1 = 60.0° - 35.3° = \boxed{24.7°}$.

22.54 From the sketch at the right, observe that the angle of incidence at A is the same as the prism angle at point O. Given that $\theta = 60.0°$, application of Snell's law at point A gives



$$1.50\sin\beta = (1.00)\sin 60.0° \quad \text{or} \quad \beta = 35.3°$$

From triangle AOB, we calculate the angle of incidence and reflection, γ, at point B:

$$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180° \quad \text{or} \quad \gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$$

Now, we find the angle of incidence at point C using triangle BCQ:

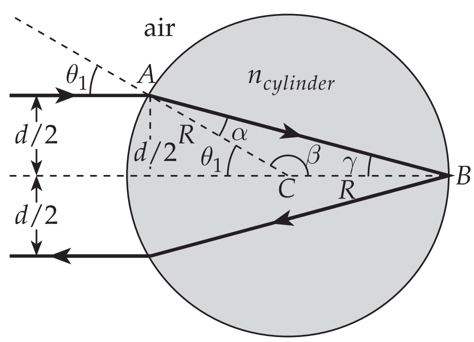
$$(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$$

or $\delta = 90.0° - (\theta + \gamma) = 90.0° - 84.7° = 5.26°$

Finally, application of Snell's law at point C gives $(1.00)\sin\phi = (1.50)\sin(5.26°)$

or $\phi = \sin^{-1}(1.50\sin 5.26°) = \boxed{7.91°}$

22.55 The path of a light ray during a reflection and/or refraction process is always reversible. Thus, if the emerging ray is parallel to the incident ray, the path which the light follows through this cylinder must be symmetric about the center line as shown at the right.

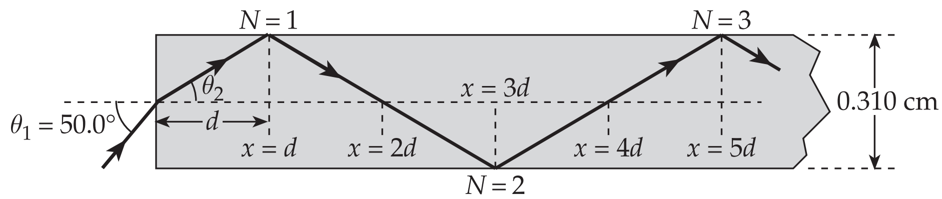


Thus, $\theta_1 = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$

Triangle ABC is isosceles, so $\gamma = \alpha$ and $\beta = 180° - \alpha - \gamma = 180° - 2\alpha$. Also, $\beta = 180° - \theta_1$, which gives $\alpha = \theta_1/2 = 15.0°$. Then, from applying Snell's law at point A,

$$n_{\text{cylinder}} = \frac{n_{\text{air}}\sin\theta_1}{\sin\alpha} = \frac{(1.00)\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

22.56



The angle of refraction as the light enters the left end of the slab is

$$\theta_2 = \sin^{-1}\left(\frac{n_{air}\sin\theta_1}{n_{slab}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 50.0°}{1.48}\right) = 31.2°$$

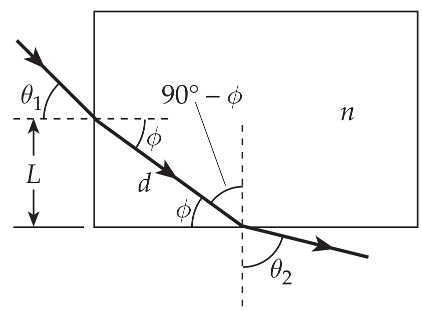
Observe from the figure that the first reflection occurs at $x = d$, the second reflection is at $x = 3d$, the third is at $x = 5d$, and so forth. In general, the N^{th} reflection occurs at $x = (2N-1)d$, where

$$d = \frac{(0.310 \text{ cm}/2)}{\tan\theta_2} = \frac{0.310 \text{ cm}}{2\tan 31.2°} = 0.256 \text{ cm}$$

Therefore, the number of reflections made before reaching the other end of the slab at $x = L = 42$ cm is found from $L = (2N-1)d$ to be

$$N = \frac{1}{2}\left(\frac{L}{d}+1\right) = \frac{1}{2}\left(\frac{42 \text{ cm}}{0.256 \text{ cm}}+1\right) = 82.5 \quad \text{or} \quad \boxed{\text{82 complete reflections}}$$

22.57 (a) If $\theta_1 = 45.0°$, application of Snell's law at the point where the beam enters the plastic block gives



$$(1.00)\sin 45.0° = n\sin\phi \qquad \textbf{[1]}$$

Application of Snell's law at the point where the beam emerges from the plastic, with $\theta_2 = 76.0°$, gives

$$n\sin(90° - \phi) = (1.00)\sin 76° \quad \text{or} \quad (1.00)\sin 76° = n\cos\phi \qquad \textbf{[2]}$$

Dividing Equation [1] by Equation [2], we obtain

$$\tan\phi = \frac{\sin 45.0°}{\sin 76°} = 0.729 \quad \text{and} \quad \phi = 36.1°$$

Thus, from Equation [1], $n = \dfrac{\sin 45.0°}{\sin\phi} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) Observe from the figure above that $\sin\phi = L/d$. Thus, the distance the light travels inside the plastic is $d = L/\sin\phi$, and if $L = 50.0 \text{ cm} = 0.500 \text{ m}$, the time required is

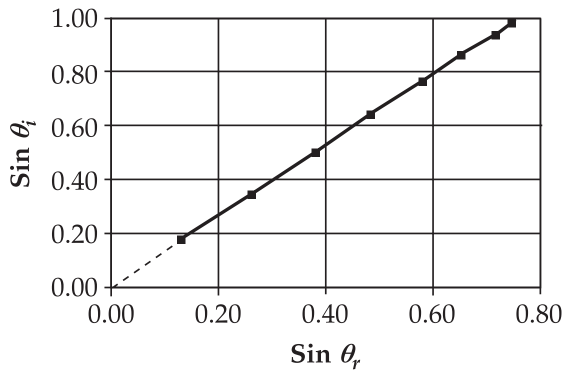
$$\Delta t = \frac{d}{v} = \frac{L/\sin\phi}{c/n} = \frac{nL}{c\sin\phi} = \frac{1.20(0.500 \text{ m})}{(3.00\times10^8 \text{ m/s})\sin 36.1°} = 3.40\times10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}$$

22.58 Snell's law would predict that $n_{\text{air}} \sin\theta_i = n_{\text{water}} \sin\theta_r$, or since $n_{\text{air}} = 1.00$,

$$\sin\theta_i = n_{\text{water}} \sin\theta_r$$

Comparing this equation to the equation of a straight line, $y = mx + b$, shows that if Snell's law is valid, a graph of $\sin\theta_i$ versus $\sin\theta_r$ should yield a straight line that would pass through the origin if extended and would have a slope equal to n_{water}.

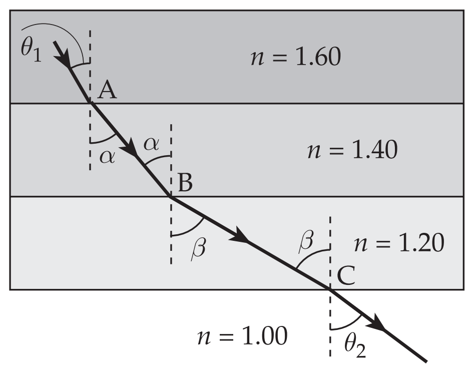
θ_i (deg)	θ_r (deg)	$\sin\theta_i$	$\sin\theta_r$
10.0	7.50	0.174	0.131
20.0	15.1	0.342	0.261
30.0	22.3	0.500	0.379
40.0	28.7	0.643	0.480
50.0	35.2	0.766	0.576
60.0	40.3	0.866	0.647
70.0	45.3	0.940	0.711
80.0	47.7	0.985	0.740



The straightness of the graph line and the fact that its extension passes through the origin demonstrates the validity of Snell's law. Using the end points of the graph line to calculate its slope gives the value of the index of refraction of water as

$$n_{\text{water}} = slope = \frac{0.985 - 0.174}{0.740 - 0.131} = 1.33$$

22.59 Applying Snell's law at points A, B, and C gives



$$1.40 \sin\alpha = 1.60 \sin\theta_1 \qquad \textbf{[1]}$$

$$1.20 \sin\beta = 1.40 \sin\alpha \qquad \textbf{[2]}$$

and $$1.00 \sin\theta_2 = 1.20 \sin\beta \qquad \textbf{[3]}$$

Combining Equations [1], [2], and [3] yields

$$\sin\theta_2 = 1.60 \sin\theta_1 \qquad \textbf{[4]}$$

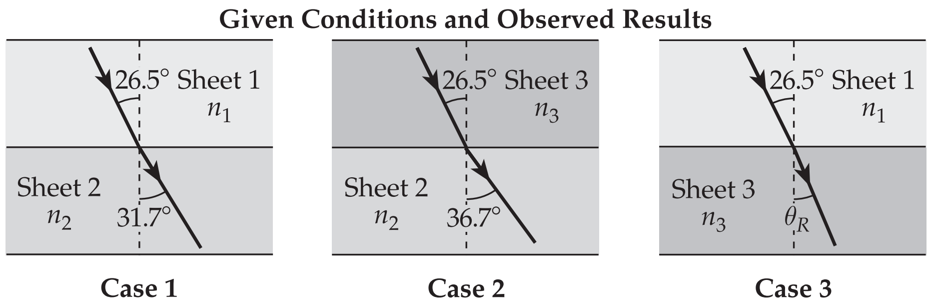
Note that Equation [4] is exactly what Snell's law would yield if the second and third layers of this "sandwich" were ignored. This will always be true if the surfaces of all the layers are parallel to each other.

(a) If $\theta_1 = 30.0°$, then Equation [4] gives $\theta_2 = \sin^{-1}(1.60 \sin 30.0°) = \boxed{53.1°}$.

(b) At the critical angle of incidence on the lowest surface, $\theta_2 = 90.0°$. Then, Equation [4] gives

$$\theta_1 = \sin^{-1}\left(\frac{\sin\theta_2}{1.60}\right) = \sin^{-1}\left(\frac{\sin 90.0°}{1.60}\right) = \boxed{38.7°}$$

22.60



For the first placement, Snell's law gives $n_2 = \dfrac{n_1 \sin 26.5°}{\sin 31.7°}$

In the second placement, application of Snell's law yields

$$n_3 \sin 26.5° = n_2 \sin 36.7° = \left(\frac{n_1 \sin 26.5°}{\sin 31.7°}\right)\sin 36.7°, \quad \text{or} \quad n_3 = \frac{n_1 \sin 36.7°}{\sin 31.7°}$$

Finally, using Snell's law in the third placement gives

$$\sin\theta_R = \frac{n_1 \sin 26.5°}{n_3} = \left(n_1 \sin 26.5°\right)\left(\frac{\sin 31.7°}{n_1 \sin 36.7°}\right) = 0.392$$

and $\theta_R = \boxed{23.1°}$