16

Electrical Energy and Capacitance

Clicker Questions

Question L3.01

Description: Relating electrostatic potential to work.

Question

The electric potential at two points in space is: $V_1 = 200$ volts and $V_2 = 300$ volts. Which of the following statements is true for moving a point charge q from point 1 to 2?

A. The work done by an external agent to move q from point 1 to 2 is positive.

B. We can't determine the work done because we don't know the direction of V at the two points.

C. The work done by the electric force exerted on q in moving it from point 1 to 2 is $W = -q(100\ \text{V})$.

1. A
2. B
3. C
4. A and B
5. A and C
6. B and C
7. A, B, and C
8. None of the above

Commentary

Purpose: To link the concepts of electrostatic potential and work.

Discussion: The work done by an electric field on a moving charge is equal to the negative of the charge times the change in electric potential between the two points: $W = -q\Delta V$. $\Delta V = 100$ V, so statement C is correct.

Statement A is true only for positive charges, and we aren't told whether q is positive or negative. In particular, the work done on q by an external agent is $q\Delta V$, whether the charge is positive or negative. Also, the total work done on the charge is zero, so the work done by the electric force is $-q\Delta V$.

Statement B is nonsense: the electrostatic potential V doesn't have a direction. It's a scalar field. (The electric *field* **E** is a vector and has a direction.)

So, (3) is the best answer.

Key Points:

- The work done by an electric field on a moving charge is $W = -q\Delta V$.
- Whether the work done on a charge moving in an electric field is positive or negative depends on the sign of the charge, and whether you're talking about the work done by the field or by an external agent moving it in the field.
- Electric potential is a scalar, not a vector, field.

For Instructors Only

If any students include statement B (answers 2, 4, 6, or 7), it should be thoroughly discussed as it indicates a possible confusion between electric fields and electric potentials.

If students have difficulty understanding statement C, especially the presence of the minus sign, it may help to describe the situation in terms of electric field lines pointing from higher to lower potential, and charges moving with or against the field.

You can also invoke conservation of energy to explain the work done by the electric force. The total work done on the charge is zero, since its kinetic energy does not change. In moving a positive charge to a higher potential (think about pushing a boulder up a hill), positive work must be done by an external agent, so negative work must be done by the electric force, the only other force in the situation. In moving a negative charge to a higher potential, negative work must be done by an external agent, so positive work must be done by the electric force.

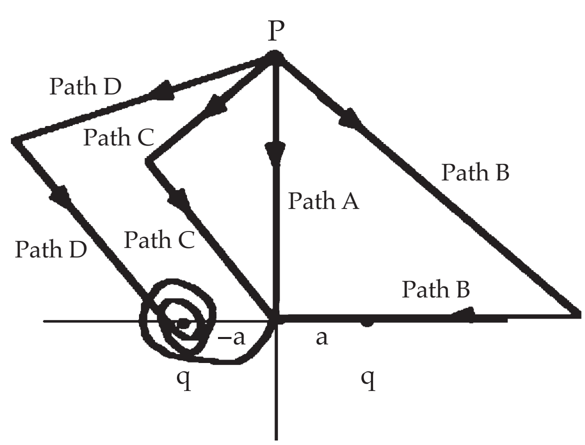
Note that, properly speaking, the charge "*q*" does not possess any electric potential energy. The system does.

Question L3.02

Description: Introducing the concept of "conservative force" for electrostatics.

Question

Two point charges are fixed on the *x*-axis. Imagine moving a positive charge from point P to the origin along the different paths shown in the diagram. For which path would you do the most work?



1. Path A
2. Path B
3. Path C
4. Path D
5. Cannot be determined
6. None of the above

Commentary

Purpose: To develop your understanding of what a *conservative* force is.

Discussion: The electrostatic force is *conservative*. That means moving a charge from one point in space to another takes the same amount of work no matter what path is followed. Thus, all four paths require the same work, so the best answer is (6), none of the above.

Because the electrostatic force is conservative, it is possible to define an electrostatic *potential,* and say that the work done on the charge is equal to the change in potential energy of the system. (In the same way, the gravitational force is conservative, so it takes the same amount of work to push an object to the top of a frictionless hill no matter what path up the hill is taken — an amount of work equal to the gravitational potential energy gained.)

Key Points:

- The electrostatic force is *conservative,* meaning the work required to move a charge between two points is path independent.
- We can define a potential for a conservative force, so that the work done is equal to the change in potential energy of the system.

For Instructors Only

As always, ask students who pick "none of the above" what answer they would pick if it were present. In this case, their response distinguishes those who are correct from incorrect reasons for selecting (6).

Exactly how this question is handled depends on when you use it. It can be employed relatively early as a way to introduce the electrostatic potential, or later to check whether students understand the relationship between work and potential.

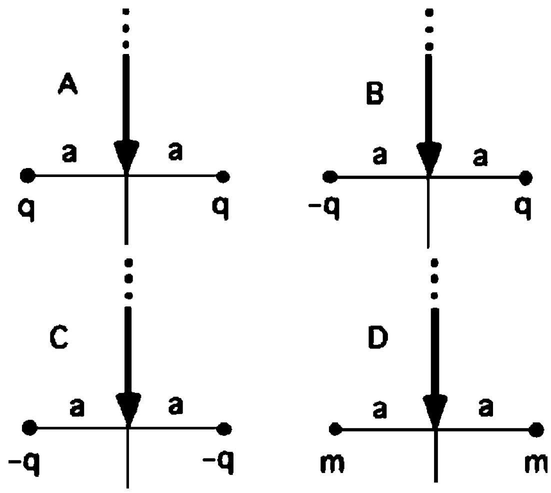
A good follow-up question is to ask how much work would be done if one of the fixed charges on the x-axis were negative.

Question L3.03

Description: Introducing work in an electrostatic context.

Question

For the following situations consider moving a positive charge from very far away to the origin along the y-axis. For which situation would you do the most work?



1. A
2. B
3. C
4. D
5. A and C
6. None of the above
7. Cannot be determined

Commentary

Purpose: To develop the concept of *work* in the context of simple charge configurations.

Discussion: According to the *work-energy theorem,* the work required to move a charge in an electric field is equal to the change in its electrostatic potential energy between the initial and final points. The potential energy of a charge "at infinity" is zero, so we only need to consider the potential energy at the final position for each configuration. The most work will be done for the situation in which the potential energy is largest. (We will assume that the charge is at rest when it starts and ends, so that there is no change in kinetic energy to worry about.)

The potential energy stored in the electrostatic interaction between two charges is kq_1q_2/r, where r is the distance between them. For the situations described, the work done to move the charge in to the origin is equal to the sum of the potential energy with each of the two charges shown, since the charge must be moved against the force exerted by each.

In case A, the total potential energy, and thus the work, will be positive. (The charge must be moved against a repulsive interaction.) In case B, the contribution from the two fixed charges will cancel since their signs are opposite, so the work required will be zero. (The net force on the moving charge will always be perpendicular to the direction of motion.) In case C, the potential energy and thus the work will be negative. (The charge must be moved "against" an attractive interaction.) So, case A requires more work to be done than cases B or C.

How about case D? In that situation, the two fixed "charges" do not have any charge, but instead have a mass. So, the work done to move the charge is done against a gravitational interaction rather than an electrostatic interaction. Gravitational interactions are always attractive, so the work required will be negative. Thus, case D cannot require more work than case A.

Key Points:

- The work required to change the position of a charge in an electrostatic field is equal to the change in electrostatic potential energy that occurs, assuming kinetic energy does not change (the *work-energy theorem*).
- The electrostatic potential energy of a point charge q_1 interacting with another point charge q_2 a distance r away is given by kq_1q_2/r.
- The electrostatic potential energy of a point charge interacting with a set of point charges is the sum of its potential energy due to each of those charges alone (*superposition*).
- Electrostatic potential energy can be positive or negative.

For Instructors Only

This is a useful question for beginning a discussion about electrostatic work, leading towards the introduction of the electric potential.

Some students who may select answer (7) on the grounds that they are not explicitly told the charges of the masses in case D are not incorrect, though they are not using all the information inherent in the question and its context to infer the intent of the question.

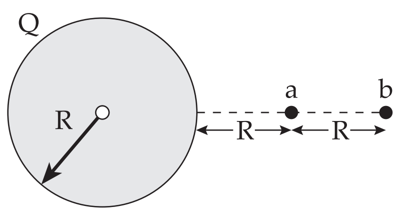
Students choosing answer (5) probably don't appreciate that electrostatic potential energy can be positive or negative, or are considering only the magnitude of the work.

Question L3.04

Description: Integrating energy conservation ideas with electrostatics.

Question

A uniform spherical volume distribution of charge has radius R and total charge Q. A point charge $-q$ is released from rest at point b, which is a distance $3R$ from the center of the distribution. When the point charge reaches a, which of the following is true regarding the total energy E?



1. $E_a = -E_b$
2. $E_a = -2E_b/3$
3. $E_a = -3E_b/2$
4. $E_a = -9E_b/4$
5. $E_a = E_b$
6. $E_a = 2E_b/3$
7. $E_a = 3E_b/2$
8. $E_a = 9E_b/4$
9. None of the above
10. Cannot be determined

Commentary

Purpose: To revisit energy conservation in the context of an electricity problem.

Discussion: Note that the question asks about the *total* energy of the system, not the *potential* energy. No non-conservative forces act on the system, so total energy must be conserved. Thus, the correct answer is (5).

When the point charge moves inward, the potential energy decreases (becomes more negative), offsetting the increase in kinetic energy as the point charge speeds up.

Key Points:

- Be careful not to confuse kinetic, potential, and total energy.
- Ideas you learned in mechanics, such as the work-energy theorem and conservation of energy principle, apply to electromagnetic systems as well.

For Instructors Only

Students often "pigeonhole" their learning, and don't think to apply ideas they learned in one course or topic area to later topics. Cross-topic questions like this one help to overcome that. Students answering (7) are likely providing the right answer to the wrong question, and answering for the potential rather than the total energy. Students selecting many of the other choices may be doing the same thing, but incorrectly.

Question L3.05

Description: Relating and distinguishing electric field and electric potential.

Question

True or false: it is possible to have the electric field be 0 at some point in space and the electric potential be nonzero at that same point.

1. True
2. False

Commentary

Purpose: To extend your understanding of the relationship between electric field and electric potential.

Discussion: We can prove that "it is possible" to have a zero electric field but nonzero potential at a point in space by finding an example of a charge configuration for which this is true. Consider two identical positive charges an equal distance on either side of the origin — say, a distance of $\pm a$ along the x-axis. The electric field at the origin is zero, because the fields due to the two charges cancels. However, it is nonzero everywhere else in space. It would take nonzero work to move another charge to the origin from infinity: along whichever path you push it, you would have to do positive work (on a positive charge) as you push it against the combined electric field of the two charges. Since the work required is equal to the change in potential energy of the system, and the potential is zero at infinity (by definition), the potential must be nonzero at the origin.

Another example is a hollow, uniformly charged spherical shell. The field is zero everywhere inside the shell. (This can be shown easily via Gauss's law, when you learn that.) However, the field is nonzero and pointing outward everywhere outside the shell (if it is positively charged), so again it takes nonzero work to move a point charge in from infinity and place it inside the shell, and we know the potential inside the sphere must be nonzero.

Key Points:

- To figure out if a statement is true or false, it sometimes helps to find an example.
- The electrostatic potential is not necessarily zero where the electric field is (and vice-versa).
- The electrostatic potential at a point in space is equal to the work required to move a charged point there from infinity (by any path), divided by the point's charge.

For Instructors Only

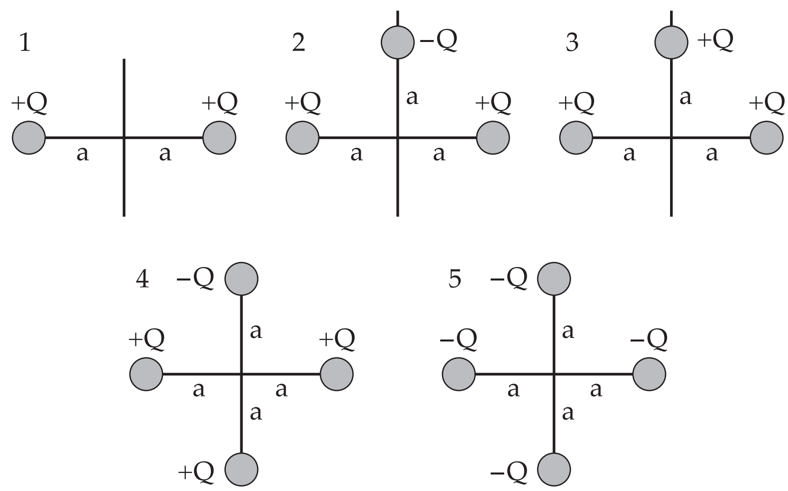
Before revealing which answer is correct, ask students choosing answer (1) to draw or describe a charge configuration satisfying the statement. (This is often sufficient to make them change their mind.) Ask students choosing (2) to prove it.

Question L3.06a

Description: Distinguishing the electric potential created by a charge distribution from the electric potential energy of the distribution.

Question

Which of the following charge distributions has the lowest potential energy?



Commentary

Purpose: To develop your intuition of and ability to reason with electrostatic potential energy.

Discussion: Rather than attempting to calculate potential energies for these situations or write algebraic expressions, you should try to reason your way to the answer as qualitatively as possible.

Cases 1, 3, and 5 are all assemblies of like charges (all positive or all negative), so assembling each would require positive work against electrostaic repulsion; thus, those cases must have positive potential energy.

Case 4 has zero potential energy. This can be seen by counting pairs: there are two pairs of positive charges separated by one diagonal "edge" (left charge with bottom charge, and right with bottom), and two pairs of opposite charges separated by that same distance (top with left, top with right). These four pairs will all have the same magnitude of potential energy, but the like-charge pairs will have positive energy and the opposite-charge pairs will have negative, so the total potential energy from these four parings will be zero. Likewise, there is one pair of like charges a distance $2a$ apart, and one pair of opposite charges the same distance apart, so the potential energy from these two pairs will cancel. Thus, the total potential energy of the configuration must be zero.

Another way to figure this out for Case 4 is to imagine constructing a pair of positive charges separated by $2a$. This will require some positive amount of work. Then, infinitely far from the first pair, assemble a positive-and-negative pair also separated by $2a$. This will require the same magnitude of work, but negative since the charges attract. Place the opposite-charge pair in the position shown along the y-axis; this requires no work. Now, move the positive pair into place, sliding it along the x-axis from infinity. This also requires no work, since every point along that axis is equidistant from the positive and negative charges of the first pair. Result: zero net work to construct the charge arrangement.

Case 2 must have negative potential energy, since it has two opposite-charge pairs with a relatively short separation (negative potential due to attractive interaction) and one like-charge pair with a larger separation (positive potential). So, Case 2 must have the least potential energy of all the configurations.

Key Points:

- Whenever possible, attempt to answer questions via qualitative reasoning rather than algebraic manipulations. You'll learn more, and be less likely to commit a math slip.
- The potential energy of a complex charge configuration can be thought of as the sum of potential energies of each pair of charges.
- The potential energy stored in the interaction between two like charges is positive; that stored between two opposite charges is negative.

For Instructors Only

Depending on their comfort level with potential energy and work, students may struggle with the qualitative arguments used here. We recommend spending the time necessary for them to fully grasp them (especially for Case 4); it will pay dividends later.

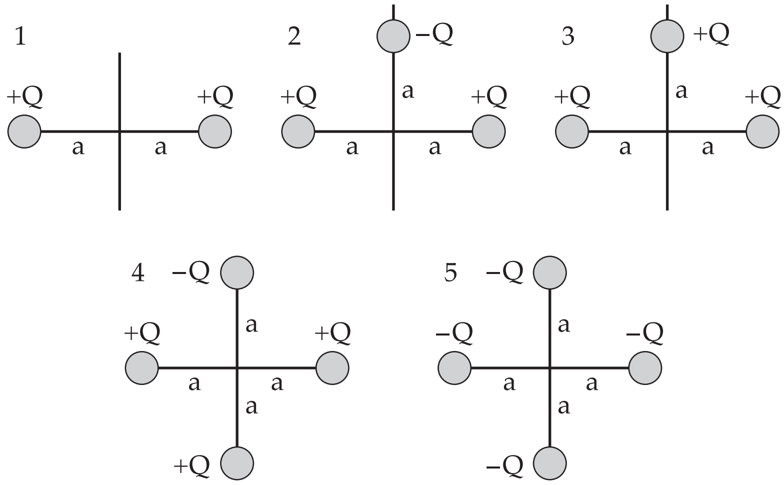
A good follow-up question is to ask students to order the cases according to increasing potential energy.

Question L3.06b

Description: Distinguishing the electric potential created by a charge distribution from the electric potential energy of the distribution.

Question

Which of the following charge distributions has the lowest electric potential at the origin?



Commentary

Purpose: To develop your ability to reason with electrostatic potential energy, and distinguish the potential energy of a charge configuration from the potential it creates in space around it.

Discussion: The electrostatic potential at some point due to an assembly of charges is the sum of the potentials due to each individual charge ("superposition"). Let's say the potential due to a positive charge $+Q$ a distance a away is U. The potential due to a negative charge the same distance away must be $-U$. So, distribution (1) has potential $2U$ at the origin, (2) has U, (3) has $3U$, (4) has $2U$, and (5) has $-4U$. Thus, distribution (5) creates the lowest electrostatic potential at the origin.

Key Points:

- The electric potential of a distribution of charges is the sum of the potentials due to each individual charge ("superposition").
- Potential is a scalar. It can be positive or negative, but has no direction.
- The potential created by a charge distribution is not the same as the potential energy stored *in* the distribution (i.e., the energy required to assemble it).

For Instructors Only

Juxtaposed with the previous question, this serves to highlight the difference between the potential energy stored in a distribution from the potential created by the distribution at some point.

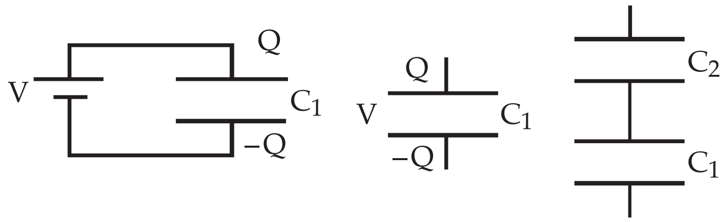
You may wish to stress that when potentials due to point charges are determined, a reference point at infinity is assumed.

Question N3.01

Description: Developing understanding of capacitors as circuit elements: series.

Question

A capacitor, C_1, is connected to a battery until charged, and then disconnected from the battery. A second capacitor, C_2, is connected in series to the first capacitor. What changes occur in capacitor C_1 after C_2 is connected as shown?



1. ΔV same, Q increases, U increases
2. ΔV same, Q decreases, U same
3. ΔV increases, Q decreases, U increases
4. ΔV decreases, Q same, U decreases
5. ΔV decreases, Q decreases, U decreases
6. None of the above
7. Cannot be determined

Commentary

Purpose: To improve your understanding of capacitors by reasoning about their behavior.

Discussion: Once the battery is disconnected, the charge on the capacitor's plates cannot change. When the second capacitor is connected in series to one of C_1's ends, the charge on C_1 still can't change. The charge on the lower plate can't go anywhere, and although some of the opposite charge on the upper plate could conceivably move to the connected plate of C_2, it won't because it is attracted to the charge on the lower plate. The charge on one plate of a capacitor will always be equal in magnitude and opposite in sign to the charge on the other. (All the electric field lines from one plate travel across the gap and terminate on the other, so the amount of charge must be the same.) So, Q remains the same.

Since the physical dimensions of C_1 don't change and no material enters or leaves its gap, its capacitance doesn't change. According to $C = Q/\Delta V$, if the capacitance and the charge are both constant, the potential difference ΔV must also be. The energy stored in a capacitor is $U = \frac{1}{2} C(\Delta V)^2$, so the stored energy remains the same as well.

All quantities remain the same, so the best answer is (6), "None of the above."

Key Points:

- The charge on one plate of a parallel-plate capacitor will always be equal in magnitude and opposite in sign to the charge on the other plate.
- The charge must stay the same on a disconnected capacitor: it has nowhere to go.
- A capacitor's capacitance depends on its physical construction (dimensions and materials), not on conditions such as the charge on its plates or the potential difference across its ends.

For Instructors Only

Despite the figure, students sometimes think the second capacitor is connected in parallel to the first rather than in series. This may be indicated by their choice of answer (5), though that is not the only possible reason for such a choice.

The issue most likely to need discussion is whether the charge on the upper plate of C_1 stays put, or distributes itself across the two now-connected plates of C_1 and C_2. The argument against the latter is outlined in the discussion, but may require elaboration, the drawing of field line diagrams, and the like.

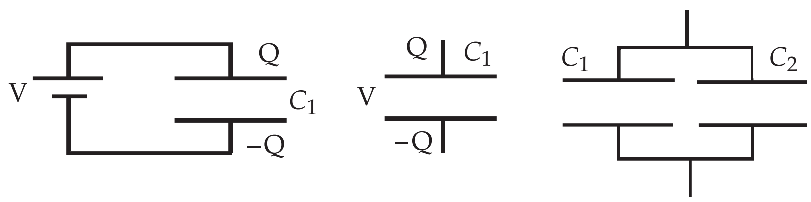
Depending upon the course it may be profitable to digress into a discussion of an ideal versus a non-ideal capacitor that can leak its charge.

Question N3.02

Description: Developing understanding of capacitors as circuit elements: parallel.

Question

A capacitor, C_1, is connected to a battery until charged, then disconnected from the battery. A second capacitor, C_2, is connected in parallel to the first capacitor. Which statements below are true?



1. Charge on C_1 decreases.
2. Total charge on C_1 and C_2 is the same as the original Q.
3. The total energy stored in both capacitors is the same as the original U stored in C_1.
4. The potential difference (voltage) across C_1 decreases.
5. All of the above
6. Only 1, 2, and 3 are true.
7. Only 1, 2, and 4 are true.
8. None of the above

Commentary

Purpose: To improve your understanding of capacitors by reasoning about their behavior.

Discussion: When capacitor C_1 is removed from the battery, it has a charge $+Q$ on one plate and $-Q$ on the other. When the second capacitor is connected in parallel, some of that charge will flow to the plates of the other capacitor. C_1 will now have a charge of Q_1 on it ($+Q_1$ on one plate and $-Q_1$ on the other), and C_2 will have a charge of Q_2, where $Q_1 + Q_2 = Q$. Thus, the charge on C_1 decreases (statements 1 and 2).

Since the physical construction of C_1 doesn't change, neither does its capacitance. Therefore, since $C = Q/\Delta V$, the potential difference ΔV across C_1 must decrease (statement 4).

The energy stored on a capacitor is $U = \frac{1}{2}C(\Delta V)^2 = Q^2/(2C)$. To make the analysis easier, let's assume $C_1 = C_2$. In that case, $Q_1 = Q_2 = \frac{1}{2}Q$. The original energy stored on C_1 was $U_i = Q^2/(2C_1)$, and the final energy stored on both capacitors is $U_f = Q_1^2/(2C_1) + Q_2^2/(2C_2) = Q^2/(4C_1)$, so $U_f = \frac{1}{2}U_i$. The total stored energy decreases. If the capacitances are not equal, the factor will not be $\frac{1}{2}$, but the energy will still decrease.

Thus, the best answer is (7).

(If you're wondering where the lost energy went, it was dissipated as heat by the wires as charge flowed between the capacitors. Wires are never perfectly conducting, and this is a case where even the very small resistance of "ideal" wires matters.)

Key Points:

- When dealing with capacitors, figure out what stays constant. When capacitors are isolated, charge remains constant. When capacitors are connected, charge can be shared. When a battery is connected, voltage remains constant.
- Event though total charge may stay the same, if it moves around, total stored energy can decrease.
- When charge spontaneously redistributes itself by flowing along a conductor, it decreases the total potential energy of the system.

For Instructors Only

Statement (3) is the most difficult for students to reason about. We simplified the discussion by assuming equal capacitances, but the general case can be proven (with additional algebra).

Question N3.01 serves as good preparation for this question. This question makes a good lead-in for introducing the equivalent capacitance of capacitors in parallel.

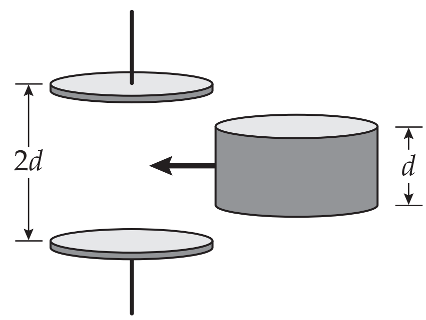
Some students may be ready for a discussion of how the energy was lost. Having them insert a resistance and determine the dissipated energy is instructive.

Question N3.03

Description: Understanding the mechanism of capacitance.

Question

Two parallel conducting plates form a capacitor. It is isolated and a charge Q is placed on it. A metal cylinder of length half the plate separation is then inserted between the plates.



How many of the quantities C, ΔV, Q, E, and U change?

Input a number from 0 to 5, or choose (9) for "impossible to determine."

Commentary

Purpose: To hone your understanding of capacitors and develop your analysis skills.

Discussion: The order in which we consider the quantities C, ΔV, Q, E, and U is critical to figuring out which change and which do not. It also helps to have a flexible view of what a "capacitor" is.

The arrangement is "isolated," which means it is not connected to anything (such as a battery or other power source, or another capacitor). This is useful, because it means that the charge on the outer plates cannot change. In other words, even though a cylinder has been inserted into the gap between the plates, no charge can flow onto or off the plates, because there are no conducting paths. This means that Q cannot change.

There can be no electric field inside a conductor. When a conductor is placed inside a region of space with an electric field, charge will flow within it such that positive and negative charge are induced on opposite faces of the cylinder, until the net electric field inside the conductor is zero. If we assume that $+Q$ is originally located on the upper plate of the capacitor and $-Q$ on the lower one, then $-Q$ will be induced on the upper surface of the cylinder and $+Q$ on its lower surface. Note that the "charge on the capacitor" still has not changed: it is still Q. Note also that this new arrangement is like having two capacitors in series, with the cylinder itself serving as the wire connecting the two capacitors.

The electric field E between the plates of a capacitor is uniform and depends only on the charge density on each plate. Thus, since the area and charge has not changed, the electric field E has not changed either. However, if we look at the entire gap, the electric field within the cylinder is now zero, where it was nonzero in that region before. In other words, in the gap but outside the cylinder the electric field has not changed; the electric field has changed only in the region now occupied by the cylinder.

The potential difference ΔV depends on two factors, the strength of the electric field E and the size of the gap between two plates. Since the electric field is constant, the original potential difference is $\Delta V = (E)(2d) = 2Ed$. The effective gap size changes from $2d$ to d when the cylinder is inserted, so ΔV will change as well.

Since the charge Q has not changed, but the potential difference ΔV has changed, the capacitance C has changed $\left(C = Q/\Delta V\right)$.

Since C has changed, but Q has not changed, the potential energy U stored in the capacitor has changed $\left(U = \frac{1}{2}Q^2/C\right)$. Note that using the alternative form for the potential energy, $U = \frac{1}{2}C(\Delta V)^2$, we cannot tell immediately if U has changed, since C has increased and ΔV has decreased. The other form is more useful, because only one of the quantities has changed (C).

Thus, only Q remains the same. The rest of the quantities change.

Key Points:

- When analyzing situations where something changes, start by identifying what quantities are held constant.
- The capacitance of a capacitor depends on what materials fill the gap between its plates.
- In a static (steady-state) situation, the electric field within a conducting material must be zero. If a conducting object is placed in an electric field, charge will rearrange itself on the object's surface so that the field becomes zero inside.
- The capacitance of a capacitor relates the charge on its plates to the potential difference across it via $C = Q/\Delta V$.
- The energy stored in a charged capacitor is given by $U = \frac{1}{2}C(\Delta V)^2$.

For Instructors Only

This is the first of two related questions. They are identical, except that in this question the capacitor is isolated, and in the next it is connected to a battery. (In the second, the cylinder is removed rather than being inserted.) By using both questions back-to-back, students will see how crucial the question of "what remains constant?" is.

The best answer is (4), since C, ΔV, E, and U all change. Only Q remains the same.

Note that many students could answer (4) without having the correct set of changing quantities. Thus, it is critical to focus less on the "correct" answer among the choices given and immediately look at each quantity individually. The answer histogram for this question does not tell you who understands it; it serves as a platform for discussion.

Some students might validly answer (3), if they say that E does not change because it does not change outside the cylinder (and they recognize that it becomes zero inside the cylinder). Others who answer (3) might not realize that the electric field must be zero inside the (conducting) cylinder.

It turns out that the potential energy goes down, which means negative work must be done to insert the cylinder. That is, once the cylinder is close to the capacitor, there will be a force on it pulling it into the gap. The person inserting it will need to exert a force opposite its displacement to complete the process slowly.

Students should be asked to indicate whether quantities go up or down. They might be surprised to learn, for instance, that the capacitance goes up and that the potential energy goes down.

Additional Questions:

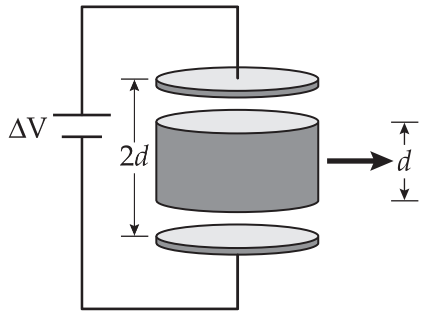
1. For the quantities that change, indicate whether each increases or decreases. Explain.
2. With the cylinder in place, the system is equivalent to two identical capacitors arranged in series, with the cylinder serving as a wire connecting the two capacitors. (a) What is the capacitance of each of the new capacitors, in terms of the original capacitance C? (b) What is the effective capacitance of two such capacitors arranged in series? (c) Does your answer to (b) make sense in terms of other details of this situation? Explain.
3. For the quantities that change, indicate the factor by which each changes. Explain.
4. What if the cylinder were made of a dielectric material instead?
5. Now that the cylinder is in place, what would happen if it is moved up or down within the gap? (Which quantities would change now? That is, do not compare this to the original, but compare this to the situation with the cylinder exactly in the middle of the gap.) Does it matter whether the cylinder is in contact with one of the plates?
6. The cylinder is moved so that it touches the upper plate. Then it is removed again and returned to the middle of the gap. What are the new values of C and ΔV as compared to the original values? (HINT: This is tricky, so don't rush to an answer.)

Question N3.04

Description: Understanding the mechanism of capacitance.

Question

Two parallel conducting plates form a capacitor. With a metal cylinder of length half the plate separation inserted between the plates, it is connected to a battery with potential ΔV. The cylinder is now removed.



How many of the quantities C, ΔV, Q, E, and U change?

Input a number from 0 to 5, or choose (9) for "impossible to determine."

Commentary

Purpose: To hone your understanding of capacitors and develop your analysis skills.

Discussion: The order in which we consider the various quantities C, ΔV, Q, E, and U matters; it's easier to determine whether some change after others have been figured out.

If the battery remains connected, it will maintain a constant potential difference ΔV across the capacitor. (That's what batteries do.) It will allow charge to flow to or from the plates as necessary, so Q is not *necessarily* constant.

What is the effect of the metal cylinder? It can have no electric field within it, so positive charge will build up on one end and negative charge on the other. Effectively, this will be like having a capacitor whose separation distance is d (or, equivalently, two capacitors in series whose separation distances add up to d). If the effective separation changes when the cylinder is removed, the capacitance C must change. (For a parallel-plate capacitor, $C = \epsilon_0 A/d$.) And if ΔV is constant but C changes, then Q must change according to $C = Q/\Delta V$.

The electric field within a capacitor is proportional to the charge density on the plates, so if Q changes, E must also. (We already know that E changes in the space formerly filled by the cylinder; now we know it changes in the rest of the capacitor, too.)

How about the stored energy U? Since $U = \frac{1}{2}C(\Delta V)^2$, that must change as well. So, C, Q, E, and U change, but ΔV does not. The best answer is therefore (4).

Key Points:

- When analyzing situations where something changes, start by identifying what quantities are held constant.
- The capacitance of a capacitor depends on what materials fill the gap between its plates.
- The capacitance of a capacitor relates the charge on its plates to the potential difference across it via $C = Q/\Delta V$.
- The energy stored in a charged capacitor is given by $U = \frac{1}{2}C(\Delta V)^2$.
- The capacitance of a parallel plate capacitor with an empty gap is $C = \epsilon_0 A/d$.

For Instructors Only

Because there are so many combinations in which 4 quantities change, students might select the right answer for the wrong reasons. Therefore, you should focus less on the "correctness" of the answer and more on exactly which quantities change.

Rather than discussing the correctness of the answers, use the distribution of student responses as a springboard to launch a discussion of the various quantities and how one can reason whether each changes. In addition to having students explain why they believe some quantity changes, you can ask them whether their reasoning suggests the quantity will increase or decrease.

Note that the potential energy U is going down, but we cannot determine from that alone whether the work done to remove the cylinder is positive or negative, since the battery absorbs energy. (It turns out that the battery absorbs twice as much energy as the capacitor loses, so positive work must be done to remove the cylinder.)

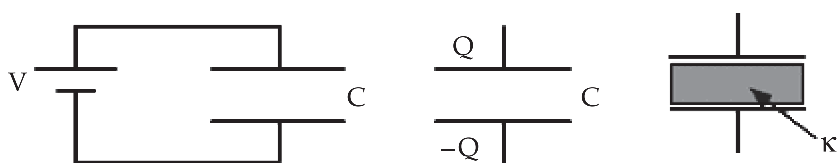
The pedagogic purpose of this question can be as much about learning to reason through a complicated, multi-variable question like this as to understand capacitors.

Question N3.05

Description: Understanding how dielectrics affect capacitors.

Question

A capacitor with capacitance C is connected to a battery until charged, then disconnected from the battery. A dielectric having constant κ is inserted in the capacitor. What changes occur in the charge, potential, and stored energy of the capacitor after the dielectric is inserted?



1. ΔV stays same, Q increases, U increases
2. ΔV stays same, Q decreases, U stays same
3. ΔV increases, Q decreases, U increases
4. ΔV decreases, Q stays same, U decreases
5. None of the above
6. Cannot be determined

Commentary

Purpose: To explore the effect of a dielectric on a capacitor, and check your understanding of the relationship between capacitance, potential difference, charge, and stored energy for a capacitor.

Discussion: Charge can neither leave nor appear on the plates of a disconnected capacitor, so Q must remain the same as the dielectric is inserted.

The effect of a dielectric is to increase the capacitance of a capacitor; that is why commercial capacitors are made with a dielectric-filled gap rather than an air gap. Since $C = Q/\Delta V$, if the charge remains constant and the capacitance increases, the potential difference must decrease.

The energy stored in a capacitor is given by $U = \frac{1}{2}C(\Delta V)^2$. This can be rewritten as $U = Q^2/2C$. So, if the capacitance increases while the charge remains constant, the energy stored must decrease. (This means that you do negative work on the dielectric slab to insert it — it gets “sucked in” — since there is no place else for the energy to go.)

Key Points:

- Filling the gap in a capacitor with dielectric material raises its capacitance.
- A capacitor’s capacitance, charge, and potential difference are related by $C = Q/\Delta V$.
- The energy stored in a capacitor is given by $U = \frac{1}{2}C(\Delta V)^2 = Q^2/2C$.

For Instructors Only

Depending on how you are fitting this question into your curriculum, you may wish to enter a discussion here about why and how a dielectric increases capacitance.

QUICK QUIZZES

1. (b). The field exerts a force on the electron, causing it to accelerate in the direction opposite to that of the field. In this process, electrical potential energy is converted into kinetic energy of the electron. Note that the electron moves to a region of higher potential, but because the electron has negative charge this corresponds to a decrease in the potential energy of the electron.

2. (a). The electron, a negatively charged particle, will move toward the region of higher electric potential. Because of the electron’s negative charge, this corresponds to a decrease in electrical potential energy.

3. (b). Charged particles always tend to move toward positions of lower potential energy. The electrical potential energy of a charged particle is $PE = qV$ and, for positively charged particles, this increases as V increases. Thus, a positively charged particle located at $x = A$ would move toward the left.

4. (d). For a negatively charged particle, the potential energy $(PE = qV)$ decreases as V increases. A negatively charged particle would oscillate around $x = B$, which is a position of minimum potential energy for negative charges.

5. (d). If the potential is zero at a point located a finite distance from charges, negative charges must be present in the region to make negative contributions to the potential and cancel positive contributions made by positive charges in the region.

6. (c). Both the electric potential and the magnitude of the electric field decrease as the distance from the charged particle increases. However, the electric flux through the balloon does not change because it is proportional to the total charge enclosed by the balloon, which does not change as the balloon increases in size.

7. (a). From the conservation of energy, the final kinetic energy of either particle will be given by

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + qV_i - qV_f = -q\left(V_f - V_i\right) = -q\left(\Delta V\right)$$

For the electron, $q = -e$ and $\Delta V = +1$ V, giving $KE_f = -\left(-e\right)\left(+1 \text{ V}\right) = +1$ eV.

For the proton, $q = +e$ and $\Delta V = -1$ V, so $KE_f = -\left(e\right)\left(-1 \text{ V}\right) = +1$ eV, the same as that of the electron.

8. (c). The battery moves negative charge from one plate and puts it on the other. The first plate is left with excess positive charge whose magnitude equals that of the negative charge moved to the other plate.

9. **(a)** C decreases. **(b)** Q stays the same. **(c)** E stays the same.

(d) ΔV increases. **(e)** The energy stored increases.

Because the capacitor is removed from the battery, charges on the plates have nowhere to go. Thus, the charge on the capacitor plates remains the same as the plates are pulled apart. Because $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q/A}{\epsilon_0}$, the electric field is constant as the plates are separated. Because $\Delta V = Ed$ and E does not change, ΔV increases as d increases. Because the same charge is stored at a higher potential difference, the capacitance has decreased. Because $\text{Energy stored} = Q^2/2C$ and Q stays the same while C decreases, the energy stored increases. The extra energy must have been transferred from somewhere, so work was done. This is consistent with the fact that the plates attract one another, and work must be done to pull them apart.

10. **(a)** C increases. **(b)** Q increases. **(c)** E stays the same.

(d) ΔV remains the same. **(e)** The energy stored increases.

The presence of a dielectric between the plates increases the capacitance by a factor equal to the dielectric constant. Since the battery holds the potential difference constant while the capacitance increases, the charge stored $\left(Q = C\Delta V\right)$ will increase. Because the potential difference and the distance between the plates are both constant, the electric field $\left(E = \Delta V/d\right)$ will stay the same. The battery maintains a constant potential difference. With ΔV constant while capacitance increases, the stored energy $\left(\text{Energy stored} = \frac{1}{2}C\left(\Delta V\right)^2\right)$ will increase.

11. (a). Increased random motions associated with an increase in temperature make it more difficult to maintain a high degree of polarization of the dielectric material. This has the effect of decreasing the dielectric constant of the material, and in turn, decreasing the capacitance of the capacitor.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The change in the potential energy of the proton is equal to the negative of the work done on it by the electric field. Thus,

$$\Delta PE = -W = -qE_x\left(\Delta x\right) = -\left(+1.6\times10^{-19} \text{ C}\right)\left(850 \text{ N/C}\right)\left(2.5 \text{ m} - 0\right) = -3.4\times10^{-16} \text{ J}$$

and (b) is the correct choice for this question.

2. Because electric forces are conservative, the kinetic energy gained is equal to the decrease in electrical potential energy, or

$$KE = -PE = -q(\Delta V) = -(-1\text{ e})(+1.0\times10^{4}\text{ V}) = +1.0\times10^{4}\text{ eV}$$

so the correct choice is (a).

3. From conservation of energy, $KE_f + PE_f = KE_i + PE_i$, or $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + qV_i - qV_f$

or $$v_f = \sqrt{v_i^2 + \frac{2q(V_i - V_f)}{m}}$$

$$= \sqrt{(6.20\times10^{5}\text{ m/s})^2 + \frac{2\left[2(1.60\times10^{-19}\text{ C})(1.50-4.00)\times10^{3}\text{ V}\right]}{6.63\times10^{-27}\text{ kg}}} = 3.78\times10^{5}\text{ m/s}$$

Thus, the correct answer is choice (b).

4. In a uniform electric field, the change in electric potential is $\Delta V = -E_x(\Delta x)$, giving

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{(V_f - V_i)}{(x_f - x_i)} = -\frac{(190\text{ V}-120\text{ V})}{(5.0\text{ m}-3.0\text{ m})} = -35\text{ V/m} = -35\text{ N/C}$$

and it is seen that the correct choice is (d).

5. With the given specifications, the capacitance of this parallel plate capacitor will be

$$C = \frac{\kappa \in_0 A}{d} = \frac{(1.00\times10^{2})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.0\text{ cm}^2)}{1.0\times10^{-3}\text{ m}}\left(\frac{1\text{ m}^2}{10^{4}\text{ cm}^2}\right)$$
$$= 8.85\times10^{-11}\text{ F} = 88.5\times10^{-12}\text{ F} = 88.5\text{ pF}$$

and the correct choice is (a).

6. The total potential at a point due to a set of point charges q_i is $V = \sum_i kq_i/r_i$, where r_i is the distance from the point of interest to the location of the charge q_i. Note that in this case, the point at the center of the circle is equidistant from the 4 point charges located on the rim of the circle. Note also that $q_2 + q_3 + q_4 = (+1.5 - 1.0 - 0.5)\ \mu\text{C} = 0$, so we have

$$V_{\text{center}} = \frac{k_e q_1}{r} + \frac{k_e q_2}{r} + \frac{k_e q_3}{r} + \frac{k_e q_4}{r} = \frac{k_e}{r}(q_1 + q_2 + q_3 + q_4) = \frac{k_e}{r}(q_1 + 0) = \frac{k_e q_1}{r} = V_1 = 4.5\times10^{4}\text{ V}$$

or the total potential at the center of the circle is just that due to the first charge alone, and the correct answer is choice (b).

7. In a series combination of capacitors, the equivalent capacitance is always less than any individual capacitance in the combination, meaning that choice (a) is false. Also, for a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination, making both choices (d) and (e) false. The potential difference across the capacitance C_i is $\Delta V_i = Q/C_i$, where Q is the common charge on each capacitor in the combination. Thus, the largest potential difference (voltage) appears across the capacitor with the *least* capacitance, making choice (b) the correct answer.

8. Keeping the capacitor connected to the battery means that the potential difference between the plates is kept at a constant value equal to the voltage of the battery. Since the capacitance of a parallel plate capacitor is $C = \kappa \in_0 A/d$, doubling the plate separation d, while holding other characteristics of the capacitor constant, means the capacitance will be decreased by a factor of 2.

The energy stored in a capacitor may be expressed as $U = \frac{1}{2}C(\Delta V)^2$, so when the potential difference ΔV is held constant while the capacitance is decreased by a factor of 2, the stored energy decreases by a factor of 2, making (c) the correct choice for this question.

9. When the battery is disconnected, there is no longer a path for charges to use in moving onto or off of the plates of the capacitor. This means that the charge Q is constant. The capacitance of a parallel plate capacitor is $C = \kappa \in_0 A/d$ and the dielectric constant is $\kappa \approx 1$ when the capacitor is air filled. When a dielectric with dielectric constant $\kappa = 2$ is inserted between the plates, the capacitance is doubled $(C_f = 2C_i)$. Thus, with Q constant, the potential difference between the plates, $\Delta V = Q/C$, is decreased by a factor of 2, meaning that choice (a) is a true statement. The electric field between the plates of a parallel plate capacitor is $E = \Delta V/d$ and decreases when ΔV decreases, making choice (e) false and leaving (a) as the only correct choice for this question.

10. Once the capacitor is disconnected from the battery, there is no path for charges to move onto or off of the plates, so the charges on the plates are constant, and choice (e) can be eliminated. The capacitance of a parallel plate capacitor is $C = \kappa\in_0 A/d$, so the capacitance decreases when the plate separation d is increased. With Q constant and C decreasing, the energy stored in the capacitor, $U = Q^2/2C$ increases, making choice (a) false and choice (b) true. The potential difference between the plates, $\Delta V = Q/C = Q \cdot d/\kappa\in_0 A$, increases and the electric field between the plates, $E = \Delta V/d = Q/\kappa\in_0 A$, is constant. This means that both choices (c) and (d) are false and leaves choice (b) as the only correct response.

11. Capacitances connected in parallel all have the same potential difference across them and the equivalent capacitance, $C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$, is larger than the capacitance of any one of the capacitors in the combination. Thus, choice (c) is a true statement. The charge on a capacitor is $Q = C(\Delta V)$, so with ΔV constant, but the capacitances different, the capacitors all store different charges that are proportional to the capacitances, making choices (a), (b), (d), and (e) all false. Therefore, (c) is the only correct answer.

12. For a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination. Also, the equivalent capacitance is always less than any individual capacitance in the combination. Therefore, choice (a) is true while choices (b) and (c) are both false. The potential difference across a capacitor is $\Delta V = Q/C$, so with Q constant, capacitors having different capacitances will have different potential differences across them, with the largest potential difference being across the capacitor with the smallest capacitance. This means that choice (d) is false and choice (e) is true. Thus, both choices (a) and (e) are true statements.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Changing the area will change the capacitance and maximum charge but not the maximum voltage. The question does not allow you to increase the plate separation. You can increase the maximum operating voltage by inserting a material with higher dielectric strength between the plates.

4. Electric potential V is a measure of the potential energy per unit charge. Electrical potential energy, $PE = QV$, gives the energy of the total charge Q.

6. A sharp point on a charged conductor would produce a large electric field in the region near the point. An electric discharge could most easily take place at the point.

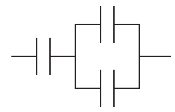
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series - $C_{\text{eq}} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} \right)^{-1}$

All three capacitors in parallel - $C_{\text{eq}} = C_1 + C_2 + C_3$

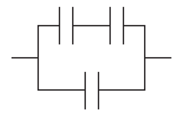
One capacitor in series with a parallel combination of the other two:

$$C_{\text{eq}} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3} \right)^{-1}, \; C_{\text{eq}} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2} \right)^{-1}, \; C_{\text{eq}} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1} \right)^{-1}$$



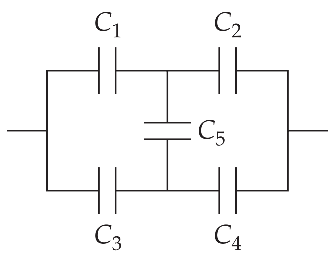
One capacitor in parallel with a series combination of the other two:

$$C_{\text{eq}} = \left(\frac{C_1 C_2}{C_1 + C_2} \right) + C_3, \; C_{\text{eq}} = \left(\frac{C_3 C_1}{C_3 + C_1} \right) + C_2, \; C_{\text{eq}} = \left(\frac{C_2 C_3}{C_2 + C_3} \right) + C_1$$



10. Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, the charge rapidly recombines, leaving the capacitor uncharged.

12. All connections of capacitors are not simple combinations of series and parallel circuits. As an example of such a complex circuit, consider the network of five capacitors C_1, C_2, C_3, C_4, and C_5 shown below.



This combination cannot be reduced to a simple equivalent by the techniques of combining series and parallel capacitors.

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

PROBLEM SOLUTIONS

16.1 (a) Because the electron has a negative charge, it experiences a force in the direction opposite to the field and, when released from rest, will move in the negative x direction. The work done on the electron by the field is

$$W = F_x(\Delta x) = (qE_x)\Delta x = (-1.60 \times 10^{-19} \text{ C})(375 \text{ N/C})(-3.20 \times 10^{-2} \text{ m}) = \boxed{1.92 \times 10^{-18} \text{ J}}$$

(b) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

$$\Delta PE = -W = \boxed{-1.92 \times 10^{-18} \text{ J}}$$

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(c) Since the Coulomb force is a conservative force, conservation of energy gives $\Delta KE + \Delta PE = 0$, or $KE_f = m_e v_f^2/2 - \Delta PE = 0 - \Delta PE$, and

$$v_f = \sqrt{\frac{-2(\Delta PE)}{m_e}} = \sqrt{\frac{-2(-1.92\times10^{-18}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = \boxed{2.05\times10^{6}\text{ m/s in the }-x\text{ direction}}$$

16.2 (a) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

$$\Delta PE = -W = -\left[qE_x(\Delta x) + qE_y(\Delta y)\right]$$
$$= -\left[q(0)\Delta x + (5.40\times10^{-6}\text{ C})(+327\text{ N/C})(-32.0\times10^{-2}\text{ m})\right] = \boxed{+5.65\times10^{-4}\text{ J}}$$

(b) The change in the electrical potential is the change in electric potential energy per unit charge, or

$$\Delta V = \frac{\Delta PE}{q} = \frac{+5.65\times10^{-4}\text{ J}}{+5.40\times10^{-6}\text{ C}} = \boxed{+105\text{ V}}$$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{\text{input}} = -W_{\text{field}} = -(-\Delta PE_e) = +q(\Delta V)$$

$$= (1.60\times10^{-19}\text{ C})\left(+90\times10^{-3}\ \frac{\text{J}}{\text{C}}\right) = \boxed{1.4\times10^{-20}\text{ J}}$$

16.4 $\Delta PE_e = q(\Delta V) = q(V_f - V_i)$, so $q = \dfrac{\Delta PE_e}{V_f - V_i} = \dfrac{-1.92\times10^{-17}\text{ J}}{+60.0\text{ J/C}} = \boxed{-3.20\times10^{-19}\text{ C}}$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\,000\text{ J/C}}{1.5\times10^{-2}\text{ m}} = \boxed{1.7\times10^{6}\text{ N/C}}$

16.6 Since potential difference is work per unit charge $\Delta V = \dfrac{W}{q}$, the work done is

$$W = q(\Delta V) = (3.6\times10^{5}\text{ C})(+12\text{ J/C}) = \boxed{4.3\times10^{6}\text{ J}}$$

16.7 (a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600\text{ J/C}}{5.33\times10^{-3}\text{ m}} = \boxed{1.13\times10^{5}\text{ N/C}}$

(b) $F = |q|E = (1.60\times10^{-19}\text{ C})(1.13\times10^{5}\text{ N/C}) = \boxed{1.80\times10^{-14}\text{ N}}$

(c) $W = F\cdot s\cos\theta$

$$= (1.80\times10^{-14}\text{ N})\left[(5.33-2.90)\times10^{-3}\text{ m}\right]\cos 0° = \boxed{4.38\times10^{-17}\text{ J}}$$

16.8 (a) Using conservation of energy, $\Delta KE + \Delta PE = 0$, with $KE_f = 0$ since the particle is "stopped," we have

$$\Delta PE = -\Delta KE = -\left(0 - \frac{1}{2}m_e v_i^2\right) = +\frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(2.85\times10^{7}\text{ m/s}\right)^2 = +3.70\times10^{-16}\text{ J}$$

The required stopping potential is then

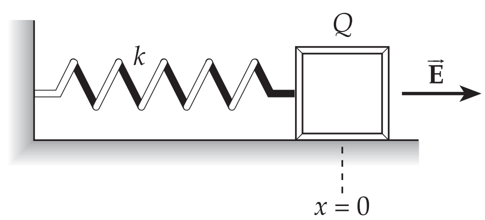
$$\Delta V = \frac{\Delta PE}{q} = \frac{+3.70\times10^{-16}\text{ J}}{-1.60\times10^{-19}\text{ C}} = -2.31\times10^{3}\text{ V} = \boxed{-2.31\text{ kV}}$$

(b) Being more massive than electrons, protons traveling at the same initial speed will have more initial kinetic energy and $\boxed{\text{require a greater magnitude stopping potential}}$.

(c) Since $\Delta V_{\text{stopping}} = \frac{\Delta PE}{q} = \frac{-\Delta KE}{q} = \frac{-mv^2/2}{q}$, the ratio of the stopping potential for a proton to that for an electron having the same initial speed is

$$\frac{\Delta V_p}{\Delta V_e} = \frac{-m_p v_i^2/2(+e)}{-m_e v_i^2/2(-e)} = \boxed{-m_p/m_e}$$

16.9 (a) Use conservation of energy



$$\left(KE + PE_s + PE_e\right)_f = \left(KE + PE_s + PE_e\right)_i$$

or $$\Delta(KE) + \Delta(PE_s) + \Delta(PE_e) = 0$$

$\Delta(KE) = 0$ since the block is at rest at both beginning and end.

$\Delta(PE_s) = \frac{1}{2}kx_{\max}^2 - 0$, where $x_{\max}$ is the maximum stretch of the spring.

$\Delta(PE_e) = -W = -(QE)x_{\max}$

Thus, $0 + \frac{1}{2}kx_{\max}^2 - (QE)x_{\max} = 0$, giving

$$x_{\max} = \frac{2QE}{k} = \frac{2\left(35.0\times10^{-6}\text{ C}\right)\left(4.86\times10^{4}\text{ V/m}\right)}{78.0\text{ N/m}} = 4.36\times10^{-2}\text{ m} = \boxed{4.36\text{ cm}}$$

(b) At equilibrium, $\Sigma F = F_s + F_e = 0$, or $-kx_{eq} + QE = 0$

Therefore, $$x_{eq} = \frac{QE}{k} = \frac{1}{2}x_{\max} = \boxed{2.18\text{ cm}}$$

The amplitude is the distance from the equilibrium position to each of the turning points $(\text{at } x = 0 \text{ and } x = 4.36\text{ cm})$, so $\boxed{A = 2.18\text{ cm} = x_{\max}/2}$.

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(c) From conservation of energy, $\Delta(KE)+\Delta(PE_s)+\Delta(PE_e)=0+\frac{1}{2}kx_{\max}^2+Q(\Delta V)=0$. Since $x_{\max}=2A$, this gives

$$\Delta V=-\frac{kx_{\max}^2}{2Q}=-\frac{k(2A)^2}{2Q} \quad \text{or} \quad \boxed{\Delta V=-\frac{2kA^2}{Q}}$$

16.10 Using $\Delta y=v_{0y}t+\frac{1}{2}a_y t^2$ for the full flight gives

$$0=v_{0y}t+\frac{1}{2}a_y t^2, \quad \text{or} \quad a_y=\frac{-2v_{0y}}{t}$$

Then, using $v_y^2=v_{0y}^2+2a_y(\Delta y)$ for the upward part of the flight gives

$$(\Delta y)_{\max}=\frac{0-v_{0y}^2}{2a_y}=\frac{-v_{0y}^2}{2\left(-2v_{0y}/t\right)}=\frac{v_{0y}\,t}{4}=\frac{(20.1\text{ m/s})(4.10\text{ s})}{4}=20.6\text{ m}$$

From Newton's second law, $a_y=\frac{\Sigma F_y}{m}=\frac{-mg-qE}{m}=-\left(g+\frac{qE}{m}\right)$. Equating this to the earlier result gives $a_y=-\left(g+\frac{qE}{m}\right)=\frac{-2v_{0y}}{t}$, so the electric field strength is

$$E=\left(\frac{m}{q}\right)\left[\frac{2v_{0y}}{t}-g\right]=\left(\frac{2.00\text{ kg}}{5.00\times10^{-6}\text{ C}}\right)\left[\frac{2(20.1\text{ m/s})}{4.10\text{ s}}-9.80\text{ m/s}^2\right]=1.95\times10^3\text{ N/C}$$

Thus, $(\Delta V)_{\max}=(\Delta y_{\max})E=(20.6\text{ m})(1.95\times10^3\text{ N/C})=4.02\times10^4\text{ V}=\boxed{40.2\text{ kV}}$

16.11 (a) $$V_A=\frac{k_e q}{r_A}=\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.250\times10^{-2}\text{ m}}=\boxed{-5.75\times10^{-7}\text{ V}}$$

(b) $$V_B=\frac{k_e q}{r_B}=\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.750\times10^{-2}\text{ m}}=\boxed{-1.92\times10^{-7}\text{ V}}$$

$$\Delta V=V_B-V_A=-1.92\times10^{-7}\text{ V}-\left(-5.75\times10^{-7}\text{ V}\right)=\boxed{+3.83\times10^{-7}\text{ V}}$$

(c) The original electron will be repelled by the negatively charged particle which suddenly appears at point *A*. Unless the electron is fixed in place, it will move in the opposite direction, away from points *A* and *B*, thereby lowering the potential difference between these points.

16.12 (a) At the origin, the total potential is

$$V_{\text{origin}}=\frac{kq_1}{r_1}+\frac{kq_2}{r_2}$$
$$=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\left[\frac{4.50\times10^{-6}\text{ C}}{1.25\times10^{-2}\text{ m}}+\frac{(-2.24\times10^{-6}\text{ C})}{1.80\times10^{-2}\text{ m}}\right]=\boxed{2.12\times10^6}\text{V}$$

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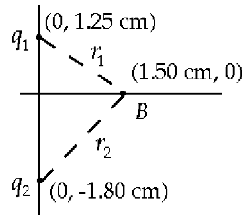
(b) At point B located at $(1.50 \text{ cm}, 0)$, the needed distances are



$$r_1 = \sqrt{|x_B - x_1|^2 + |y_B - y_1|^2} = \sqrt{(1.50 \text{ cm})^2 + (1.25 \text{ cm})^2} = 1.95 \text{ cm}$$

and

$$r_2 = \sqrt{|x_B - x_2|^2 + |y_B - y_2|^2} = \sqrt{(1.50 \text{ cm})^2 + (1.80 \text{ cm})^2} = 2.34 \text{ cm}$$

giving

$$V_B = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = (8.99 \times 10^9 \text{ N} \cdot \text{m/C}^2)\left[\frac{4.50 \times 10^{-6} \text{ C}}{1.95 \times 10^{-2} \text{ m}} + \frac{(-2.24 \times 10^{-6} \text{ C})}{2.34 \times 10^{-2} \text{ m}}\right] = \boxed{1.21 \times 10^6 \text{ V}}$$

16.13 (a) Calling the $2.00 \ \mu\text{C}$ charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{8.00 \times 10^{-6} \text{ C}}{0.060\,0 \text{ m}} + \frac{4.00 \times 10^{-6} \text{ C}}{0.030\,0 \text{ m}} + \frac{2.00 \times 10^{-6} \text{ C}}{\sqrt{(0.060\,0)^2 + (0.030\,0)^2} \text{ m}}\right)$$

$$V = \boxed{2.67 \times 10^6 \text{ V}}$$

(b) Replacing 2.00×10^{-6} C by -2.00×10^{-6} C in part (a) yields

$$V = \boxed{2.13 \times 10^6 \text{ V}}$$

16.14 $W = q(\Delta V) = q(V_f - V_i)$, and

$V_f = 0$ since the $8.00 \ \mu\text{C}$ is infinite distance from other charges.

$$V_i = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{2.00 \times 10^{-6} \text{ C}}{0.030\,0 \text{ m}} + \frac{4.00 \times 10^{-6} \text{ C}}{\sqrt{(0.030\,0)^2 + (0.060\,0)^2} \text{ m}}\right)$$

$$= 1.135 \times 10^6 \text{ V}$$

Thus, $W = (8.00 \times 10^{-6} \text{ C})(0 - 1.135 \times 10^6 \text{ V}) = \boxed{-9.08 \text{ J}}$

16.15 (a) $V = \sum_i \frac{k_e q_i}{r_i}$

$$= \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{5.00 \times 10^{-9} \text{ C}}{0.175 \text{ m}} - \frac{3.00 \times 10^{-9} \text{ C}}{0.175 \text{ m}}\right) = \boxed{103 \text{ V}}$$

(b) $PE = \frac{k_e q_i q_2}{r_{12}}$

$$= \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\frac{\left(5.00 \times 10^{-9} \text{ C}\right)\left(-3.00 \times 10^{-9} \text{ C}\right)}{0.350 \text{ m}} = \boxed{-3.85 \times 10^{-7} \text{ J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges (that is, bring them up to a state of zero potential energy).

16.16 The potential at distance $r = 0.300$ m from a charge $Q = +9.00 \times 10^{-9}$ C is

$$V = \frac{k_e Q}{r} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(9.00 \times 10^{-9} \text{ C}\right)}{0.300 \text{ m}} = +270 \text{ V}$$

Thus, the work required to carry a charge $q = 3.00 \times 10^{-9}$ C from infinity to this location is

$$W = qV = \left(3.00 \times 10^{-9} \text{ C}\right)\left(+270 \text{ V}\right) = \boxed{8.09 \times 10^{-7} \text{ J}}$$

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as

$$r_3 = \sqrt{\left(4.00 \text{ cm}\right)^2 - \left(1.00 \text{ cm}\right)^2} = \sqrt{15} \text{ cm} = \sqrt{15} \times 10^{-2} \text{ m}$$

Then, the potential at the midpoint of the base is $V = \sum_i k_e q_i / r_i$, or

$$V = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{\left(-7.00 \times 10^{-9} \text{ C}\right)}{0.010\,0 \text{ m}} + \frac{\left(-7.00 \times 10^{-9} \text{ C}\right)}{0.010\,0 \text{ m}} + \frac{\left(+7.00 \times 10^{-9} \text{ C}\right)}{\sqrt{15} \times 10^{-2} \text{ m}}\right)$$

$$= -1.10 \times 10^4 \text{ V} = \boxed{-11.0 \text{ kV}}$$

16.18 Outside the spherical charge distribution, the potential is the same as for a point charge at the center of the sphere,

$V = k_e Q/r$, where $Q = 1.00 \times 10^{-9}$ C

Thus, $\Delta(PE_e) = q(\Delta V) = -ek_e Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

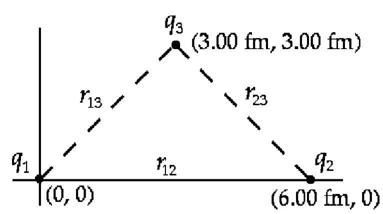
and from conservation of energy, $\Delta(KE) = -\Delta(PE_e)$,

or $\frac{1}{2}m_e v^2 - 0 = -\left[-ek_e Q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right]$. This gives $v = \sqrt{\frac{2k_e Qe}{m_e}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$, or

$$v = \sqrt{\frac{2\left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.00 \times 10^{-9} \text{ C}\right)\left(1.60 \times 10^{-19} \text{ C}\right)}{9.11 \times 10^{-31} \text{ kg}}\left(\frac{1}{0.020\,0 \text{ m}} - \frac{1}{0.030\,0 \text{ m}}\right)}$$

$$v = \boxed{7.25 \times 10^6 \text{ m/s}}$$

16.19 (a) When the charge configuration consists of only the two protons (q_1 and q_2 in the sketch), the potential energy of the configuration is



$$PE_a = \frac{k_e q_1 q_2}{r_{12}} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{6.00 \times 10^{-15} \text{ m}}$$

or $PE_a = \boxed{3.84 \times 10^{-14} \text{ J}}$

(b) When the alpha particle (q_3 in the sketch) is added to the configuration, there are three distinct pairs of particles, each of which possesses potential energy. The total potential energy of the configuration is now

$$PE_b = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}} = PE_a + 2\left(\frac{k_e\left(2e^2\right)}{r_{13}}\right)$$

where use has been made of the facts that $q_1 q_3 = q_2 q_3 = e(2e) = 2e^2$ and

$r_{13} = r_{23} = \sqrt{(3.00 \text{ fm})^2 + (3.00 \text{ fm})^2} = 4.24 \text{ fm} = 4.24 \times 10^{-15}$ m. Also, note that the first term in this computation is just the potential energy computed in part (a). Thus,

$$PE_b = PE_a + \frac{4k_e e^2}{r_{13}}$$

$$= 3.84 \times 10^{-14} \text{ J} + \frac{4\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{4.24 \times 10^{-15} \text{ m}} = \boxed{2.55 \times 10^{-13} \text{ J}}$$

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(c) If we start with the three-particle system of part (b) and allow the alpha particle to escape to infinity [thereby returning us to the two-particle system of part (a)], the change in electric potential energy will be

$$\Delta PE = PE_a - PE_b = 3.84 \times 10^{-14}\ \text{J} - 2.55 \times 10^{-13}\ \text{J} = \boxed{-2.17 \times 10^{-13}\ \text{J}}$$

(d) Conservation of energy, $\Delta KE + \Delta PE = 0$, gives the speed of the alpha particle at infinity in the situation of part (c) as $m_\alpha v_\alpha^2/2 - 0 = -\Delta PE$, or

$$v_\alpha = \sqrt{\frac{-2(\Delta PE)}{m_\alpha}} = \sqrt{\frac{-2(-2.17 \times 10^{-13}\ \text{J})}{6.64 \times 10^{-27}\ \text{kg}}} = \boxed{8.08 \times 10^6\ \text{m/s}}$$

(e) When, starting with the three-particle system, the two protons are both allowed to escape to infinity, there will be no remaining pairs of particles and hence no remaining potential energy. Thus, $\Delta PE = 0 - PE_b = -PE_b$, and conservation of energy gives the change in kinetic energy as $\Delta KE = -\Delta PE = +PE_b$. Since the protons are identical particles, this increase in kinetic energy is split equally between them giving

$$KE_{\text{proton}} = \frac{1}{2} m_p v_p^2 = \frac{1}{2}(PE_b)$$

or $$v_p = \sqrt{\frac{PE_b}{m_p}} = \sqrt{\frac{2.55 \times 10^{-13}\ \text{J}}{1.67 \times 10^{-27}\ \text{kg}}} = \boxed{1.24 \times 10^7\ \text{m/s}}$$

16.20 (a) If a proton and an alpha particle, initially at rest 4.00 fm apart, are released and allowed to recede to infinity, the final speeds of the two particles will differ because of the difference in the masses of the particles. Thus, attempting to solve for the final speeds by use of conservation of energy alone leads to a situation of having $\boxed{\text{one equation with two unknowns}}$, and does not permit a solution.

(b) In the situation described in part (a) above, one can obtain a second equation with the two unknown final speeds by using $\boxed{\text{conservation of linear momentum}}$. Then, one would have two equations which could be solved simultaneously both unknowns.

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(c) From conservation of energy: $\left[\left(\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_p v_p^2\right) - 0\right] + \left[0 - \frac{k_e q_\alpha q_p}{r_i}\right] = 0$

or $\quad m_\alpha v_\alpha^2 + m_p v_p^2 = \frac{2k_e q_\alpha q_p}{r_i} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.20\times10^{-19}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{4.00\times10^{-15}\ \text{m}}$

yielding $\quad m_\alpha v_\alpha^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$ **[1]**

From conservation of linear momentum,

$$m_\alpha v_\alpha + m_p v_p = 0 \qquad \text{or} \qquad |v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p \qquad \textbf{[2]}$$

Substituting Equation [2] into Equation [1] gives

$$m_\alpha\left(\frac{m_p}{m_\alpha}\right)^2 v_p^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J} \quad \text{or} \quad \left(\frac{m_p}{m_\alpha}+1\right)m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$$

and

$$v_p = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(m_p/m_\alpha+1\right)m_p}} = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(1.67\times10^{-27}/6.64\times10^{-27}+1\right)\left(1.67\times10^{-27}\ \text{kg}\right)}} = \boxed{1.05\times10^7\ \text{m/s}}$$

Then, Equation [2] gives the final speed of the alpha particle as

$$|v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p = \left(\frac{1.67\times10^{-27}\ \text{kg}}{6.64\times10^{-27}\ \text{kg}}\right)\left(1.05\times10^7\ \text{m/s}\right) = \boxed{2.64\times10^6\ \text{m/s}}$$

16.21 $\quad V = \frac{k_e Q}{r} \quad$ so

$$r = \frac{k_e Q}{V} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-9}\ \text{C}\right)}{V} = \frac{71.9\ \text{V}\cdot\text{m}}{V}$$

For $V = 100\ \text{V},\ 50.0\ \text{V, and } 25.0\ \text{V}$, $\boxed{r = 0.719\ \text{m},\ 1.44\ \text{m, and } 2.88\ \text{m}}$

The radii are $\boxed{\text{inversely proportional}}$ to the potential.

16.22 By definition, the work required to move a charge from one point to any other point on an equipotential surface is zero. From the definition of work, $W = (F\cos\theta)\cdot s$, the work is zero only if $s = 0$ or $F\cos\theta = 0$. The displacement s cannot be assumed to be zero in all cases. Thus, one must require that $F\cos\theta = 0$. The force F is given by $F = qE$ and neither the charge q nor the field strength E can be assumed to be zero in all cases. Therefore, the only way the work can be zero in all cases is if $\cos\theta = 0$. But if $\cos\theta = 0$, then $\theta = 90°$ or the force (and hence the electric field) must be perpendicular to the displacement s (which is tangent to the surface). That is, the field must be perpendicular to the equipotential surface at all points on that surface.

16.23 From conservation of energy, $(KE+PE_e)_f=(KE+PE_e)_i$, which gives

$$0+\frac{k_eQq}{r_f}=\frac{1}{2}m_\alpha v_i^2+0 \quad \text{or} \quad r_f=\frac{2\,k_eQq}{m_\alpha v_i^2}=\frac{2\,k_e(79e)(2e)}{m_\alpha v_i^2}$$

$$r_f=\frac{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2}=\boxed{2.74\times10^{-14}\ \text{m}}$$

16.24 (a) The distance from any one of the corners of the square to the point at the center is one half the length of the diagonal of the square, or

$$r=\frac{diagonal}{2}=\frac{\sqrt{a^2+a^2}}{2}=\frac{a\sqrt{2}}{2}=\frac{a}{\sqrt{2}}$$

Since the charges have equal magnitudes and are all the same distance from the center of the square, they make equal contributions to the total potential. Thus,

$$V_{\text{total}}=4V_{\substack{\text{single}\\ \text{charge}}}=4\,\frac{k_eQ}{r}=4\,\frac{k_eQ}{a/\sqrt{2}}=\boxed{4\sqrt{2}k_e\,\frac{Q}{a}}$$

(b) The work required to carry charge q from infinity to the point at the center of the square is equal to the increase in the electric potential energy of the charge, or

$$W=PE_{\text{center}}-PE_\infty=qV_{\text{total}}-0=q\left(4\sqrt{2}k_e\,\frac{Q}{a}\right)=\boxed{4\sqrt{2}k_e\,\frac{qQ}{a}}$$

16.25 (a) $C=\epsilon_0\,\dfrac{A}{d}=\left(8.85\times10^{-12}\ \dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\dfrac{\left(1.0\times10^6\ \text{m}^2\right)}{(800\ \text{m})}=\boxed{1.1\times10^{-8}\ \text{F}}$

(b) $Q_{\max}=C(\Delta V)_{\max}=C(E_{\max}d)$

$$=\left(1.11\times10^{-8}\ \text{F}\right)\left(3.0\times10^6\ \text{N/C}\right)(800\ \text{m})=\boxed{27\ \text{C}}$$

16.26 (a) $C=\dfrac{Q}{\Delta V}=\dfrac{27.0\ \mu\text{C}}{9.00\ \text{V}}=\boxed{3.00\ \mu\text{F}}$

(b) $Q=C(\Delta V)=(3.00\ \mu\text{F})(12.0\ \text{V})=\boxed{36.0\ \mu\text{C}}$

16.27 (a) The capacitance of this air filled (dielectric constant, $\kappa=1.00$) parallel plate capacitor is

$$C=\frac{k\,\epsilon_0\,A}{d}=\frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.30\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-3}\ \text{m}}=1.36\times10^{-12}\ \text{F}=\boxed{1.36\ \text{pF}}$$

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(b) $Q = C(\Delta V) = (1.36\times10^{-12}\ \text{F})(12.0\ \text{V}) = 1.63\times10^{-11}\ \text{C} = 16.3\times10^{-12}\ \text{C} = \boxed{16.3\ \text{pC}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{12.0\ \text{V}}{1.50\times10^{-3}\ \text{m}} = \boxed{8.00\times10^{3}\ \text{V/m}} = \boxed{8.00\times10^{3}\ \text{N/C}}$

16.28 (a) $Q = C(\Delta V) = (4.00\times10^{-6}\ \text{F})(12.0\ \text{V}) = 48.0\times10^{-6}\ \text{C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C(\Delta V) = (4.00\times10^{-6}\ \text{F})(1.50\ \text{V}) = 6.00\times10^{-6}\ \text{C} = \boxed{6.00\ \mu\text{C}}$

16.29 (a) $E = \dfrac{\Delta V}{d} = \dfrac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = 1.11\times10^{4}\ \text{V/m} = \boxed{11.1\ \text{kV/m}}$ directed toward the negative plate

(b) $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(7.60\times10^{-4}\ \text{m}^2)}{1.80\times10^{-3}\ \text{m}}$

$= 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$

(c) $Q = C(\Delta V) = (3.74\times10^{-12}\ \text{F})(20.0\ \text{V}) = 7.47\times10^{-11}\ \text{C} = \boxed{74.7\ \text{pC}}$ on one plate and $\boxed{-74.7\ \text{pC}}$ on the other plate.

16.30 $C = \dfrac{\epsilon_0 A}{d}$, so

$$d = \frac{\epsilon_0 A}{C} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(21.0\times10^{-12}\ \text{m}^2)}{60.0\times10^{-15}\ \text{F}} = 3.10\times10^{-9}\ \text{m}$$

$$d = (3.10\times10^{-9}\ \text{m})\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = \boxed{31.0\ \text{Å}}$$

16.31 (a) Assuming the capacitor is air-filled $(\kappa = 1)$, the capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.200\ \text{m}^2)}{3.00\times10^{-3}\ \text{m}} = \boxed{5.90\times10^{-10}\ \text{F}}$$

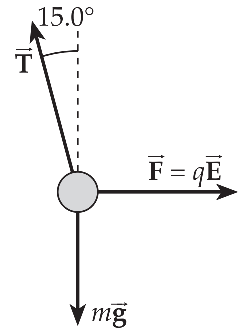
(b) $Q = C(\Delta V) = (5.90\times10^{-10}\ \text{F})(6.00\ \text{V}) = \boxed{3.54\times10^{-9}\ \text{C}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{6.00\ \text{V}}{3.00\times10^{-3}\ \text{m}} = \boxed{2.00\times10^{3}\ \text{V/m}} = \boxed{2.00\times10^{3}\ \text{N/C}}$

(d) $\sigma = \dfrac{Q}{A} = \dfrac{3.54\times10^{-9}\ \text{C}}{0.200\ \text{m}^2} = \boxed{1.77\times10^{-8}\ \text{C/m}^2}$

(e) Increasing the distance separating the plates decreases the capacitance, the charge stored, and the electric field strength between the plates. This means that all of the $\boxed{\text{previous answers will be decreased}}$.

16.32 $\Sigma F_y = 0 \Rightarrow T\cos 15.0° = mg$ or $T = \dfrac{mg}{\cos 15.0°}$



$\Sigma F_x = 0 \Rightarrow qE = T\sin 15.0° = mg\tan 15.0°$

or $E = \dfrac{mg\tan 15.0°}{q}$

$$\Delta V = Ed = \frac{mgd\tan 15.0°}{q}$$

$$\Delta V = \frac{\left(350\times10^{-6}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(0.040\ 0\text{ m}\right)\tan 15.0°}{30.0\times10^{-9}\text{ C}} = 1.23\times10^3\text{ V} = \boxed{1.23\text{ kV}}$$

16.33 (a) Capacitors in a series combination store the same charge, $Q = C_{eq}(\Delta V)$, where C_{eq} is the equivalent capacitance and ΔV is the potential difference maintained across the series combination. The equivalent capacitance for the given series combination is

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$, or $C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$, giving

$$C_{eq} = \frac{(2.50\ \mu\text{F})(6.25\ \mu\text{F})}{2.50\ \mu\text{F} + 6.25\ \mu\text{F}} = 1.79\ \mu\text{F}$$

so the charge stored on each capacitor in the series combination is

$$Q = C_{eq}(\Delta V) = (1.79\ \mu\text{F})(6.00\text{ V}) = \boxed{10.7\ \mu\text{C}}$$

(b) When connected in parallel, each capacitor has the same potential difference, $\Delta V = 6.00$ V, maintained across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu$F: $Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\text{ V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu$F: $Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\text{ V}) = \boxed{37.5\ \mu\text{C}}$

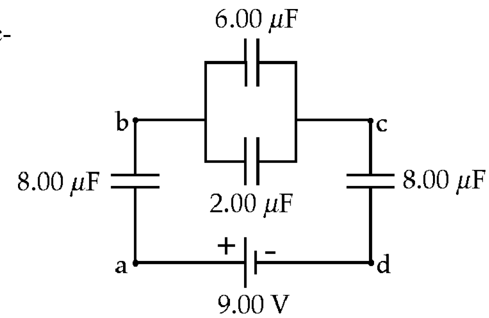
16.34 (a) When connected in series, the equivalent capacitance is $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$, or

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{(4.20\ \mu\text{F})(8.50\ \mu\text{F})}{4.20\ \mu\text{F} + 8.50\ \mu\text{F}} = \boxed{2.81\ \mu\text{F}}$$

(b) When connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 = 4.20\ \mu\text{F} + 8.50\ \mu\text{F} = \boxed{12.7\ \mu\text{F}}$$

16.35 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance, $C_{bc} = 2.00\ \mu F + 6.00\ \mu F = 8.00\ \mu F$. Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore



$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu F}$$

giving $C_{eq} = \dfrac{8.00\ \mu F}{3} = \boxed{2.67\ \mu F}$

(b) The charge stored on each capacitor in the series combination is

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu F)(9.00\ V) = 24.0\ \mu C$$

Then, note that $\Delta V_{bc} = \dfrac{Q_{bc}}{C_{bc}} = \dfrac{24.0\ \mu C}{8.00\ \mu F} = 3.00\ V$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b: $Q_8 = Q_{ab} = \boxed{24.0\ \mu C}$

On the 8.00 μF between c and d: $Q_8 = Q_{cd} = \boxed{24.0\ \mu C}$

On the 2.00 μF between b and c: $Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu F)(3.00\ V) = \boxed{6.00\ \mu C}$

On the 6.00 μF between b and c: $Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu F)(3.00\ V) = \boxed{18.0\ \mu C}$

(c) Note that $\Delta V_{ab} = Q_{ab}/C_{ab} = 24.0\ \mu C/8.00\ \mu F = 3.00\ V$, and that $\Delta V_{cd} = Q_{cd}/C_{cd} = 24.0\ \mu C/8.00\ \mu F = 3.00\ V$. We earlier found that $\Delta V_{bc} = 3.00\ V$, so we conclude that the potential difference across each capacitor in the circuit is

$$\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ V}$$

16.36 $C_{parallel} = C_1 + C_2 = 9.00\ pF \Rightarrow C_1 = 9.00\ pF - C_2$ **[1]**

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\ pF$$

Thus, using Equation [1], $C_{series} = \dfrac{(9.00\ pF - C_2)C_2}{(9.00\ pF - C_2) + C_2} = 2.00\ pF$, which reduces to

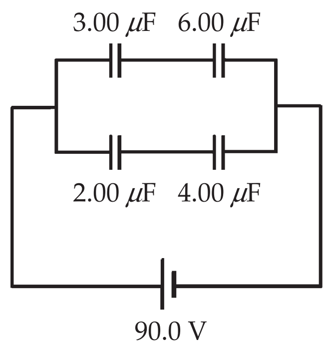
$$C_2^2 - (9.00\ pF)C_2 + 18.0\ (pF)^2 = 0, \quad \text{or} \quad (C_2 - 6.00\ pF)(C_2 - 3.00\ pF) = 0$$

Therefore, either $C_2 = 6.00$ pF and, from Equation [1], $C_1 = 3.00$ pF

or $C_2 = 3.00$ pF and $C_1 = 6.00$ pF.

We conclude that the two capacitances are $\boxed{3.00\ pF \text{ and } 6.00\ pF}$.

16.37 (a) The equivalent capacitance of the series combination in the upper branch is



$$\frac{1}{C_{\text{upper}}} = \frac{1}{3.00\ \mu\text{F}} + \frac{1}{6.00\ \mu\text{F}} = \frac{2+1}{6.00\ \mu\text{F}}$$

or $C_{\text{upper}} = 2.00\ \mu\text{F}$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} = \frac{2+1}{4.00\ \mu\text{F}} \quad \text{or} \quad C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$\text{C}_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. The charge stored on each capacitor in the series combination in the upper branch is

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\ \text{V}) = \boxed{180\ \mu\text{C}}$$

and the charge stored on each capacitor in the series combination in the lower branch is

$$Q_3 = Q_6 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\ \text{V}) = \boxed{120\ \mu\text{C}}$$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}} \qquad \Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}} \qquad \Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

16.38 (a) The equivalent capacitance of the series combination in the rightmost branch of the circuit is

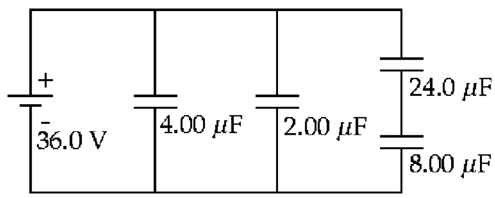


Figure P16.38

$$\frac{1}{C_{\text{right}}} = \frac{1}{24.0\ \mu\text{F}} + \frac{1}{8.00\ \mu\text{F}} = \frac{1+3}{24.0\ \mu\text{F}}$$

or $C_{\text{right}} = \boxed{6.00\ \mu\text{F}}$

(b) The equivalent capacitance of the three capacitors now connected in parallel with each other and with the battery is

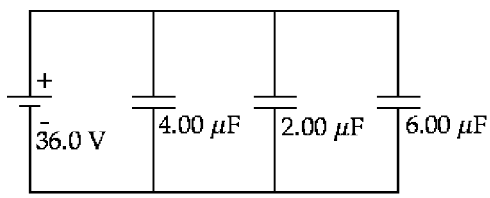


Diagram 1

$$C_{\text{eq}} = 4.00\ \mu\text{F} + 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.0\ \mu\text{F}}$$

(c) The total charge stored in this circuit is

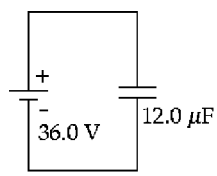


Diagram 2

$$Q_{\text{total}} = C_{\text{eq}}(\Delta V) = (12.0\ \mu\text{F})(36.0\ \text{V})$$

or $Q_{\text{total}} = \boxed{432\ \mu\text{C}}$

(d) The charges on the three capacitors shown in Diagram 1 are:

$$Q_4 = C_4(\Delta V) = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$$

$$Q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72\ \mu\text{C}}$$

$$Q_{\text{right}} = C_{\text{right}}(\Delta V) = (6.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{216\ \mu\text{C}}$$

$\boxed{\text{Yes.}}$ $Q_4 + Q_2 + Q_{\text{right}} = Q_{\text{total}}$ as it should.

(e) The charge on each capacitor in the series combination in the rightmost branch of the original circuit (Figure P16.38) is

$$Q_{24} = Q_8 = Q_{\text{right}} = \boxed{216\ \mu\text{C}}$$

(f) $\Delta V_{24} = \dfrac{Q_{24}}{C_{24}} = \dfrac{216\ \mu\text{C}}{24.0\ \mu\text{F}} = \boxed{9.00\ \text{V}}$

(g) $\Delta V_8 = \dfrac{Q_8}{C_8} = \dfrac{216\ \mu\text{C}}{8.00\ \mu\text{F}} = \boxed{27.0\ \text{V}}$ Note that $\Delta V_8 + \Delta V_{24} = \Delta V = 36.0\ \text{V}$ as it should.

16.39

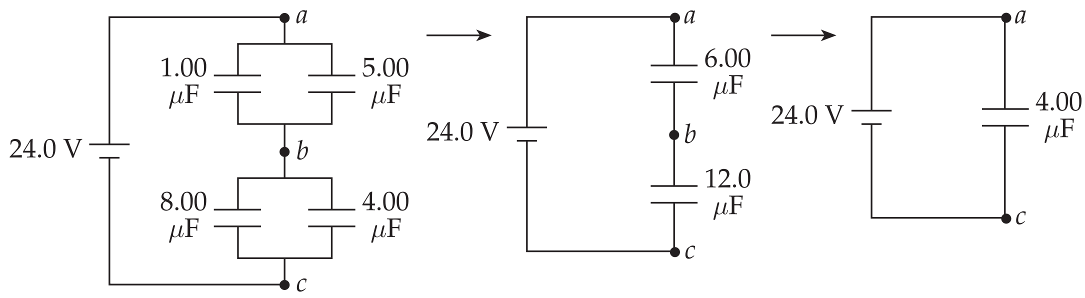


Figure 1 Figure 2 Figure 3

The circuit may be reduced in steps as shown above.

Using Figure 3, $Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\ \text{V} - 16.0\ \text{V} = 8.00\ \text{V}$

Finally, using Figure 1, $Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\ \text{V}) = \boxed{16.0\ \mu\text{C}}$

$Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}$, $\quad Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}}$

and $Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$

16.40 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_{10} = (10.0\ \mu\text{F})(12.0\ \text{V}) = 120\ \mu\text{C} \text{ and } Q_x = C_x(0) = 0$$

After the capacitors are connected in parallel, the potential difference across each is $\Delta V' = 3.00$ V, and the total charge of $Q = Q_{10} + Q_x = 120\ \mu\text{C}$ is divided between the two capacitors as

$$Q'_{10} = (10.0\ \mu\text{F})(3.00\ \text{V}) = 30.0\ \mu\text{C} \text{ and}$$

$$Q'_x = Q - Q'_{10} = 120\ \mu\text{C} - 30.0\ \mu\text{C} = 90.0\ \mu\text{C}$$

Thus, $C_x = \dfrac{Q'_x}{\Delta V'} = \dfrac{90.0\ \mu\text{C}}{3.00\ \text{V}} = \boxed{30.0\ \mu\text{F}}$

16.41 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25\times10^{3}\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00\times10^{3}\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) When the two capacitors are connected in parallel, the equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 25.0\ \mu\text{F} + 40.0\ \mu\text{F} = 65.0\ \mu\text{F}$.

Since the negative plate of one was connected to the positive plate of the other, the total charge stored in the parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00\times10^{3}\ \mu\text{C} - 1.25\times10^{3}\ \mu\text{C} = 750\ \mu\text{C}$$

The potential difference across each capacitor of the parallel combination is

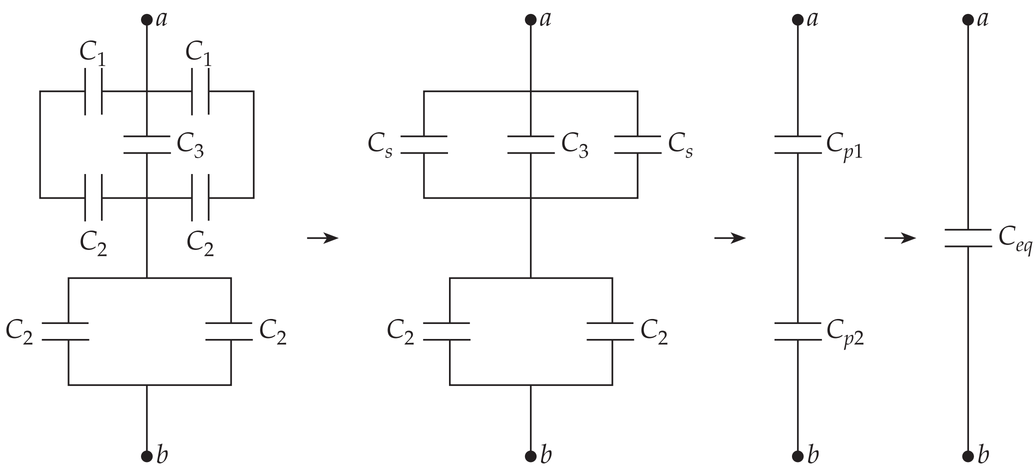
$$\Delta V = \frac{Q}{C_{\text{eq}}} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

and the final charge stored in each capacitor is

$$Q'_{25} = C_1(\Delta V) = (25.0\ \mu\text{F})(11.5\ \text{V}) = \boxed{288\ \mu\text{C}}$$

and $Q'_{40} = Q - Q'_{25} = 750\ \mu\text{C} - 288\ \mu\text{C} = \boxed{462\ \mu\text{C}}$

16.42 (a) The original circuit reduces to a single equivalent capacitor in the steps shown below.



$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}} + \frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\ \mu\text{F}) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2(10.0\ \mu\text{F}) = 20.0\ \mu\text{F}$$

$$C_{\text{eq}} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

continued on next page

(b) The total charge stored between points *a* and *b* is

$$Q_{\text{total}} = C_{\text{eq}}(\Delta V)_{ab} = (6.04\ \mu\text{F})(60.0\ \text{V}) = 362\ \mu\text{C}$$

Then, looking at the third figure, observe that the charges of the series capacitors of that figure are $Q_{p1} = Q_{p2} = Q_{\text{total}} = 362\ \mu$C. Thus, the potential difference across the upper parallel combination shown in the second figure is

$$(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362\ \mu\text{C}}{8.66\ \mu\text{F}} = 41.8\ \text{V}$$

Finally, the charge on C_3 is

$$Q_3 = C_3(\Delta V)_{p1} = (2.00\ \mu\text{F})(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$

16.43 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_1 = (1.00\ \mu\text{F})(10.0\ \text{V}) = 10.0\ \mu\text{C} \quad \text{and} \quad Q_2 = (2.00\ \mu\text{F})(0) = 0$$

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

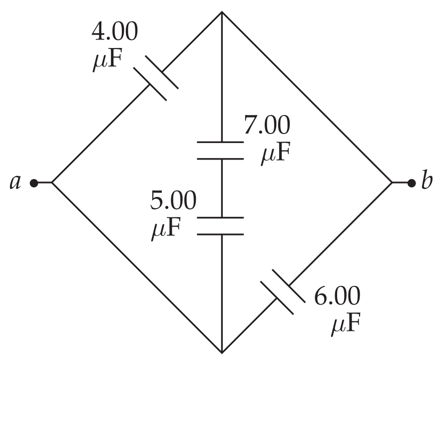
$$\Delta V = \frac{Q_1'}{1.00\ \mu\text{F}} = \frac{Q_2'}{2.00\ \mu\text{F}} \quad \text{or} \quad Q_2' = 2Q_1' \qquad \textbf{[1]}$$

From conservation of charge, $Q_1' + Q_2' = Q_1 + Q_2 = 10.0\ \mu\text{C}$. Then, substituting from Equation [1], this becomes

$$Q_1' + 2Q_1' = 10.0\ \mu\text{C}, \quad \text{giving} \quad Q_1' = \boxed{\frac{10}{3}\ \mu\text{C}}$$

Finally, from Equation [1], $$Q_2' = \boxed{\frac{20}{3}\ \mu\text{C}}$$

16.44 Recognize that the $7.00\ \mu$F and the $5.00\ \mu$F of the center branch are connected in series. The total capacitance of that branch is



$$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

Then recognize that this capacitor, the $4.00\ \mu$F capacitor, and the $6.00\ \mu$F capacitor are all connected in parallel between points *a* and *b*. Thus, the equivalent capacitance between points *a* and *b* is

$$C_{\text{eq}} = 4.00\ \mu\text{F} + 2.92\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.9\ \mu\text{F}}$$

16.45 Energy stored $= \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(4.50\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{3.24\times10^{-4}\text{ J}}$

16.46 (a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{eq}} = \frac{1}{18.0\ \mu\text{F}} + \frac{1}{36.0\ \mu\text{F}} = \frac{2+1}{36.0\ \mu\text{F}} \quad \text{or} \quad \boxed{C_{eq} = 12.0\ \mu\text{F}}$$

When this series combination is connected to a 12.0-V battery, the total stored energy is

$$\text{Total energy stored} = \frac{1}{2}C_{eq}(\Delta V)^2 = \frac{1}{2}(12.0\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{8.64\times10^{-4}\text{ J}}$$

(b) The charge stored on each of the two capacitors in the series combination is

$$Q_1 = Q_2 = Q_{total} = C_{eq}(\Delta V) = (12.0\ \mu\text{F})(12.0\text{ V}) = 144\ \mu\text{C} = 1.44\times10^{-4}\text{ C}$$

and the energy stored in each of the individual capacitors is

$$\text{Energy stored in } C_1 = \frac{Q_1^2}{2C_1} = \frac{(1.44\times10^{-4}\text{ C})^2}{2(18.0\times10^{-6}\text{ F})} = \boxed{5.76\times10^{-4}\text{ J}}$$

and $$\text{Energy stored in } C_2 = \frac{Q_2^2}{2C_2} = \frac{(1.44\times10^{-4}\text{ C})^2}{2(36.0\times10^{-6}\text{ F})} = \boxed{2.88\times10^{-4}\text{ J}}$$

Energy stored in C_1 + Energy stored in $C_2 = 5.76\times10^{-4}\text{ J} + 2.88\times10^{-4}\text{ J} = 8.64\times10^{-4}\text{ J}$, which is the same as the total stored energy found in part (a). This must be true $\boxed{\text{if the computed equivalent capacitance is truly equivalent to the original combination}}$.

(c) If C_1 and C_2 had been connected in parallel rather than in series, the equivalent capacitance would have been $C_{eq} = C_1 + C_2 = 18.0\ \mu\text{F} + 36.0\ \mu\text{F} = 54.0\ \mu\text{F}$. If the total energy stored $\left[\frac{1}{2}C_{eq}(\Delta V)^2\right]$ in this parallel combination is to be the same as was stored in the original series combination, it is necessary that

$$\Delta V = \sqrt{\frac{2(\text{Total energy stored})}{C_{eq}}} = \sqrt{\frac{2(8.64\times10^{-4}\text{ J})}{54.0\times10^{-6}\text{ F}}} = \boxed{5.66\text{ V}}$$

Since the two capacitors in parallel have the same potential difference across them, the energy stored in the individual capacitors $\left[\frac{1}{2}C(\Delta V)^2\right]$ is directly proportional to their capacitances. $\boxed{\text{The larger capacitor, } C_2\text{, stores the most energy in this case.}}$

16.47 (a) The energy initially stored in the capacitor is

$$(\text{Energy stored})_1 = \frac{Q_i^2}{2C_i} = \frac{1}{2}C_i(\Delta V)_i^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$$

(b) When the capacitor is disconnected from the battery, the stored charge becomes isolated with no way off the plates. Thus, the charge remains constant at the value Q_i as long as the capacitor remains disconnected. Since the capacitance of a parallel plate capacitor is $C = \kappa\epsilon_0 A/d$, when the distance d separating the plates is doubled, the capacitance is decreased by a factor of 2 $\left(\text{i.e., } C_f = C_i/2 = 1.50\ \mu\text{F}\right)$. The stored energy (with Q unchanged) becomes

$$(\text{Energy stored})_2 = \frac{Q_i^2}{2C_f} = \frac{Q_i^2}{2(C_i/2)} = 2\left(\frac{Q_i^2}{2C_f}\right) = 2(\text{Energy stored})_1 = \boxed{108\ \mu\text{J}}$$

(c) When the capacitor is reconnected to the battery, the potential difference between the plates is reestablished at the original value of $\Delta V = (\Delta V)_i = 6.00$ V, while the capacitance remains at $C_f = C_i/2 = 1.50\ \mu$F. The energy stored under these conditions is

$$(\text{Energy stored})_3 = \frac{1}{2}C_f(\Delta V)_i^2 = \frac{1}{2}(1.50\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{27.0\ \mu\text{J}}$$

16.48 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ \text{C})(1.00\times10^8\ \text{V})}{200} = 2.50\times10^7\ \text{J}$$

Thus, if m is the mass of water boiled away,

$W = m\left[c(\Delta T) + L_v\right]$ becomes

$$2.50\times10^7\ \text{J} = m\left[\left(4186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 30.0^\circ\text{C}) + 2.26\times10^6\ \text{J/kg}\right]$$

giving $$m = \frac{2.50\times10^7\ \text{J}}{2.55\ \text{J/kg}} = \boxed{9.79\ \text{kg}}$$

16.49 (a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa\,\epsilon_0\,A/d}{\epsilon_0\,A/d} = \kappa \qquad \text{and} \qquad \frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0\ \text{V}}{25.0\ \text{V}} = 3.40$$

Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$, and the material is probably $\boxed{\text{nylon}}$ (see Table 16.1).

(b) If the dielectric only partially filled the space between the plates, leaving the remaining space air-filled, the equivalent dielectric constant would be somewhere between $\kappa = 1.00$ (air) and $\kappa = 3.40$. The resulting potential difference would then lie somewhere between $(\Delta V)_i = 85.0$ V and $(\Delta V)_f = 25.0$ V.

16.50 (a) The capacitance of the capacitor while air-filled is

$$C_0 = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-2}\ \text{m}} = 1.48\times10^{-12}\ \text{F} = 1.48\ \text{pF}$$

The original charge stored on the plates is

$$Q_0 = C_0\left(\Delta V\right)_0 = \left(1.48\times10^{-12}\text{F}\right)\left(2.50\times10^{2}\ \text{V}\right) = 370\times10^{-12}\ \text{C} = \boxed{370\ \text{pC}}$$

Since distilled water is an insulator, introducing it between the isolated capacitor plates does not allow the charge to change. Thus, the final charge is $Q_f = \boxed{370\ \text{pC}}$.

(b) After immersion distilled water $\left(\kappa = 80 — \text{see Table 16.1}\right)$, the new capacitance is

$$C_f = \kappa C_0 = (80)(1.48\ \text{pF}) = \boxed{118\ \text{pF}}$$

and the new potential difference is $\left(\Delta V\right)_f = \dfrac{Q_f}{C_f} = \dfrac{370\ \text{pC}}{118\ \text{pF}} = \boxed{3.14\ \text{V}}$.

(c) The energy stored in a capacitor is: Energy stored $= Q^2/2C$. Thus, the change in the stored energy due to immersion in the distilled water is

$$\Delta E = \frac{Q_f^2}{2C_f} - \frac{Q_0^2}{2C_i} = \left(\frac{Q_0^2}{2}\right)\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{\left(370\times10^{-12}\ \text{C}\right)^2}{2}\left(\frac{1}{118\times10^{-12}\ \text{F}} - \frac{1}{1.48\times10^{-12}\ \text{F}}\right)$$

$$= -4.57\times10^{-8}\ \text{J} = -45.7\times10^{-9}\ \text{J} = \boxed{-45.7\ \text{nJ}}$$

16.51 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{(2.1)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(175\times10^{-4}\ \text{m}^2\right)}{0.040\,0\times10^{-3}\ \text{m}}$$

$$C = 8.13\times10^{-9}\ \text{F} = \boxed{8.13\ \text{nF}}$$

(b) For Teflon®, the dielectric strength is $E_{\max} = 60.0\times10^{6}\ \text{V/m}$, so the maximum voltage is

$$V_{\max} = E_{\max} d = \left(60.0\times10^{6}\ \text{V/m}\right)\left(0.040\,0\times10^{-3}\ \text{m}\right)$$

$$V_{\max} = 2.40\times10^{3}\ \text{V} = \boxed{2.40\ \text{kV}}$$

16.52 Before the capacitor is rolled, the capacitance of this parallel plate capacitor is

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{\kappa \epsilon_0 \left(w\times L\right)}{d}$$

where A is the surface area of one side of a foil strip. Thus, the required length is

$$L = \frac{C\cdot d}{\kappa \epsilon_0 w} = \frac{\left(9.50\times10^{-8}\ \text{F}\right)\left(0.025\,0\times10^{-3}\ \text{m}\right)}{(3.70)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.00\times10^{-2}\ \text{m}\right)} = \boxed{1.04\ \text{m}}$$

16.53 (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = \dfrac{4\pi r^3}{3}$, the radius is $r = \left[\dfrac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \in_0 A}{d}$

$$= \frac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^{5}}$$

16.54 Since the capacitors are in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 = \frac{\in_0 A_1}{d} + \frac{\in_0 A_2}{d} + \frac{\in_0 A_3}{d} = \frac{\in_0 \left(A_1 + A_2 + A_3\right)}{d}$$

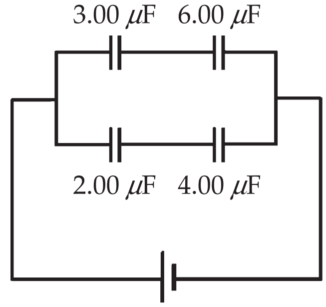
or $C_{eq} = \boxed{\dfrac{\in_0 A}{d}\ \text{where}\ A = A_1 + A_2 + A_3}$

16.55 Since the capacitors are in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\in_0 A} + \frac{d_2}{\in_0 A} + \frac{d_3}{\in_0 A} = \frac{d_1 + d_2 + d_3}{\in_0 A}$$

or $C_{eq} = \boxed{\dfrac{\in_0 A}{d}\ \text{where}\ d = d_1 + d_2 + d_3}$

16.56 (a) Please refer to the solution of Problem 16.37, where the following results were obtained:



$C_{eq} = 3.33\ \mu\text{F} \quad Q_3 = Q_6 = 180\ \mu\text{C} \quad Q_2 = Q_4 = 120\ \mu\text{C}$

The total energy stored in the full circuit is then

$$(\text{Energy stored})_{\text{total}} = \frac{1}{2}C_{eq}(\Delta V)^2 = \frac{1}{2}\left(3.33\times10^{-6}\ \text{F}\right)(90.0\ \text{V})^2$$
$$= 1.35\times10^{-2}\ \text{J} = 13.5\times10^{-3}\ \text{J} = \boxed{13.5\ \text{mJ}}$$

continued on next page

(b) The energy stored in each individual capacitor is

For 2.00 μF: $\left(\text{Energy stored}\right)_2 = \dfrac{Q_2^2}{2C_2} = \dfrac{\left(120\times10^{-6}\text{ C}\right)^2}{2\left(2.00\times10^{-6}\text{ F}\right)} = 3.60\times10^{-3}\text{ J} = \boxed{3.60\text{ mJ}}$

For 3.00 μF: $\left(\text{Energy stored}\right)_3 = \dfrac{Q_3^2}{2C_3} = \dfrac{\left(180\times10^{-6}\text{ C}\right)^2}{2\left(3.00\times10^{-6}\text{ F}\right)} = 5.40\times10^{-3}\text{ J} = \boxed{5.40\text{ mJ}}$

For 4.00 μF: $\left(\text{Energy stored}\right)_4 = \dfrac{Q_4^2}{2C_4} = \dfrac{\left(120\times10^{-6}\text{ C}\right)^2}{2\left(4.00\times10^{-6}\text{ F}\right)} = 1.80\times10^{-3}\text{ J} = \boxed{1.80\text{ mJ}}$

For 6.00 μF: $\left(\text{Energy stored}\right)_6 = \dfrac{Q_6^2}{2C_6} = \dfrac{\left(180\times10^{-6}\text{ C}\right)^2}{2\left(6.00\times10^{-6}\text{ F}\right)} = 2.70\times10^{-3}\text{ J} = \boxed{2.70\text{ mJ}}$

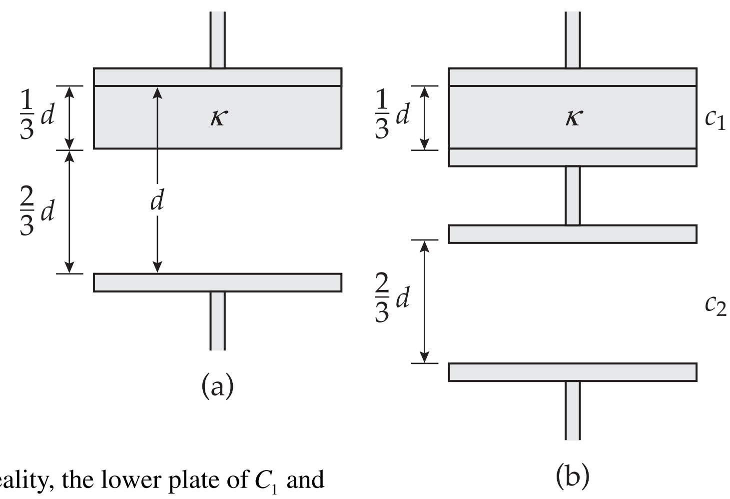
(c) The total energy stored in the individual capacitors is

$$\text{Energy stored} = \left(3.60+5.40+1.80+2.70\right)\text{ mJ} = \boxed{13.5\text{ mJ}} = \left(\text{Energy stored}\right)_{\text{total}}$$

Thus, the sums of the energies stored in the individual capacitors equals the total energy stored by the system.

16.57 In the absence of a dielectric, the capacitance of the parallel plate capacitor is

$$C_0 = \frac{\epsilon_0 A}{d}$$



With the dielectric inserted, it fills one-third of the gap between the plates as shown in sketch (a) at the right. We model this situation as consisting of a pair of capacitors, C_1 and C_2, connected in series as shown in sketch (b) at the right. In reality, the lower plate of C_1 and the upper plate of C_2 are one and the same, consisting of the lower surface of the dielectric shown in sketch (a). The capacitances in the model of sketch (b) are given by:

$$C_1 = \frac{\kappa \epsilon_0 A}{d/3} = \frac{3\kappa \epsilon_0 A}{d} \qquad \text{and} \qquad C_2 = \frac{\epsilon_0 A}{2d/3} = \frac{3\epsilon_0 A}{2d}$$

and the equivalent capacitance of the series combination is

$$\frac{1}{C_{eq}} = \frac{d}{3\kappa \epsilon_0 A} + \frac{2d}{3\epsilon_0 A} = \left(\frac{1}{\kappa}+2\right)\left(\frac{d}{3\epsilon_0 A}\right) = \left(\frac{2\kappa+1}{\kappa}\right)\frac{d}{3\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{d}{\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{1}{C_0}$$

and $\boxed{C_{eq} = \left(\dfrac{3\kappa}{2\kappa+1}\right)C_0}$

16.58 For the parallel combination: $C_p = C_1 + C_2$ which gives $C_2 = C_p - C_1$ **[1]**

For the series combination: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ or $\frac{1}{C_2} = \frac{1}{C_s} - \frac{1}{C_1} = \frac{C_1 - C_s}{C_s C_1}$

Thus, we have $C_2 = \frac{C_s C_1}{C_1 - C_s}$ and equating this to Equation [1] above gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \quad \text{or} \quad C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

We write this result as : $C_1^2 - C_p C_1 + C_p C_s = 0$

and use the quadratic formula to obtain $\boxed{C_1 = \frac{1}{2} C_p \pm \sqrt{\frac{1}{4} C_p^2 - C_p C_s}}$

Then, Equation [1] gives $\boxed{C_2 = \frac{1}{2} C_p \mp \sqrt{\frac{1}{4} C_p^2 - C_p C_s}}$

16.59 The charge stored on the capacitor by the battery is

$$Q = C(\Delta V)_1 = C(100\ \text{V})$$

This is also the total charge stored in the parallel combination when this charged capacitor is connected in parallel with an uncharged $10.0\text{-}\mu\text{F}$ capacitor. Thus, if $(\Delta V)_2$ is the resulting voltage across the parallel combination, $Q = C_p(\Delta V)_2$ gives

$$C(100\ \text{V}) = (C + 10.0\ \mu\text{F})(30.0\ \text{V}) \quad \text{or} \quad (70.0\ \text{V})C = (30.0\ \text{V})(10.0\ \mu\text{F})$$

and $C = \left(\frac{30.0\ \text{V}}{70.0\ \text{V}}\right)(10.0\ \mu\text{F}) = \boxed{4.29\ \mu\text{F}}$

16.60 (a) The $1.0\text{-}\mu C$ is located 0.50 m from point P, so its contribution to the potential at P is

$$V_1 = k_e \frac{q_1}{r_1} = \left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1.0 \times 10^{-6}\ \text{C}}{0.50\ \text{m}}\right) = \boxed{1.8 \times 10^4\ \text{V}}$$

(b) The potential at P due to the $-2.0\text{-}\mu C$ charge located 0.50 m away is

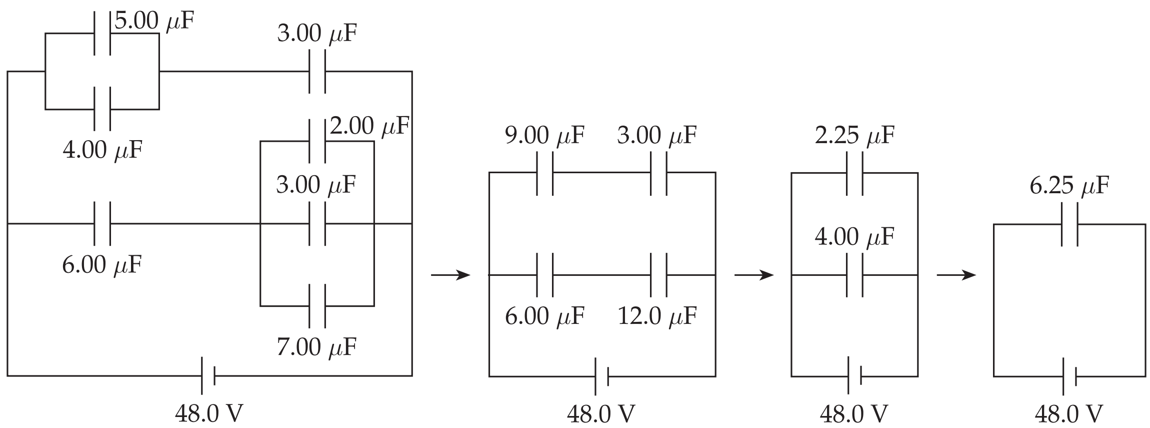
$$V_2 = k_e \frac{q_2}{r_2} = \left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-2.0 \times 10^{-6}\ \text{C}}{0.50\ \text{m}}\right) = \boxed{-3.6 \times 10^4\ \text{V}}$$

(c) The total potential at point P is $V_P = V_1 + V_2 = (+1.8 - 3.6) \times 10^4\ \text{V} = \boxed{-1.8 \times 10^4\ \text{V}}$

(d) The work required to move a charge $q = 3.0\ \mu\text{C}$ to point P from infinity is

$$W = q\Delta V = q(V_P - V_\infty) = \left(3.0 \times 10^{-6}\ \text{C}\right)\left(-1.8 \times 10^4\ \text{V} - 0\right) = \boxed{-5.4 \times 10^{-2}\ \text{J}}$$

16.61 The stages for the reduction of this circuit are shown below.



Thus, $C_{eq} = \boxed{6.25\ \mu\text{F}}$

16.62 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by $V = \dfrac{k_e q}{r}$, where r is the distance from the center of that surface and q is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge $+Q$ and the outer shell having radius b and charge $-Q$, the total electric potential is given by

$$V = V_{\substack{\text{due to}\\ \text{inner shell}}} + V_{\substack{\text{due to}\\ \text{outer shell}}} = \frac{k_e Q}{r} + \frac{k_e(-Q)}{b} = k_e Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

The potential difference between the two shells is therefore

$$\Delta V = V\big|_{r=a} - V\big|_{r=b} = k_e Q\left(\frac{1}{a} - \frac{1}{b}\right) - k_e Q\left(\frac{1}{b} - \frac{1}{b}\right) = k_e Q\left(\frac{b-a}{ab}\right)$$

The capacitance of this device is given by

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

(b) When $b >> a$, then $b - a \approx b$. Thus, in the limit as $b \to \infty$, the capacitance found above becomes

$$C \to \frac{ab}{k_e(b)} = \frac{a}{k_e} = \boxed{4\pi \in_0 a}$$

16.63 The energy stored in a charged capacitor is $W = \frac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(300\text{ J})}{30.0\times10^{-6}\text{ F}}} = 4.47\times10^{3}\text{ V} = \boxed{4.47\text{ kV}}$$

16.64 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\Delta V}\right)\left(\frac{\Delta V}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$

16.65 The charges initially stored on the capacitors are

$$Q_1 = C_1(\Delta V)_i = (6.0\ \mu\text{F})(250\ \text{V}) = 1.5\times 10^3\ \mu\text{C}$$

and $$Q_2 = C_2(\Delta V)_i = (2.0\ \mu\text{F})(250\ \text{V}) = 5.0\times 10^2\ \mu\text{C}$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive plate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5\times 10^3\ \mu\text{C} - 5.0\times 10^2\ \mu\text{C} = 1.0\times 10^3\ \mu\text{C}$$

The equivalent capacitance of the parallel combination is $C_{\text{eq}} = C_1 + C_2 = 8.0\ \mu\text{F}$. Thus, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q}{C_{\text{eq}}} = \frac{1.0\times 10^3\ \mu\text{C}}{8.0\ \mu\text{F}} = 125\ \text{V}$$

and the final charge on each capacitor is

$$Q_1' = C_1(\Delta V)' = (6.0\ \mu\text{F})(125\ \text{V}) = 750\ \mu\text{C} = \boxed{0.75\ \text{mC}}$$

and $$Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(125\ \text{V}) = 250\ \mu\text{C} = \boxed{0.25\ \text{mC}}$$

16.66 The energy required to melt the lead sample is

$$\begin{aligned} W &= m\left[c_{\text{Pb}}(\Delta T) + L_f\right] \\ &= (6.00\times 10^{-6}\ \text{kg})\left[(128\ \text{J/kg}\cdot{}^\circ\text{C})(327.3^\circ\text{C} - 20.0^\circ\text{C}) + 24.5\times 10^3\ \text{J/kg}\right] \\ &= 0.383\ \text{J} \end{aligned}$$

The energy stored in a capacitor is $W = \frac{1}{2}C(\Delta V)^2$, so the required potential difference is

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(0.383\ \text{J})}{52.0\times 10^{-6}\ \text{F}}} = \boxed{121\ \text{V}}$$

16.67 When excess charge resides on a spherical surface that is far removed from any other charge, this excess charge is uniformly distributed over the spherical surface, and the electric potential at the surface is the same as if all the excess charge were concentrated at the center of the spherical surface.

In the given situation, we have two charged spheres, initially isolated from each other, with charges and potentials of: $Q_1 = +6.00\ \mu\text{C}$, $V_1 = k_e Q_1/R_1$ where $R_1 = 12.0$ cm, $Q_2 = -4.00\ \mu\text{C}$, and $V_2 = k_e Q_2/R_2$ with $R_2 = 18.0$ cm.

When these spheres are then connected by a long conducting thread, the charges are redistributed (yielding charges of Q_1' and Q_2', respectively) until the two surfaces come to a common potential $\left(V_1' = kQ_1'/R_1 = V_2' = kQ_2'/R_2\right)$. When equilibrium is established, we have:

From conservation of charge: $Q_1' + Q_2' = Q_1 + Q_2 \quad \Rightarrow \quad Q_1' + Q_2' = +2.00\ \mu\text{C}$ **[1]**

From equal potentials: $\dfrac{kQ_1'}{R_1} = \dfrac{kQ_2'}{R_2} \quad \Rightarrow \quad Q_2' = \left(\dfrac{R_2}{R_1}\right)Q_1' \quad \text{or} \quad Q_2' = 1.50\,Q_1'$ **[2]**

Substituting Equation [2] into [1] gives: $Q_1' = \dfrac{+2.00\ \mu\text{C}}{2.50} = \boxed{0.800\ \mu\text{C}}$

Then, Equation [2] gives: $Q_2' = 1.50(0.800\ \mu\text{C}) = \boxed{1.20\ \mu\text{C}}$

16.68 The electric field between the plates is directed downward with magnitude

$$\left|E_y\right| = \frac{\Delta V}{d} = \frac{100\ \text{V}}{2.00\times 10^{-3}\ \text{m}} = 5.00\times 10^{4}\ \text{N/m}$$

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

$$a_y = \frac{F_y}{m_e} = \frac{qE_y}{m_e} = \frac{\left(-1.60\times 10^{-19}\ \text{C}\right)\left(-5.00\times 10^{4}\ \text{N/m}\right)}{9.11\times 10^{-31}\ \text{kg}} = +8.78\times 10^{15}\ \text{m/s}^2$$

(a) At the closest approach to the bottom plate, $v_y = 0$. Thus, the vertical displacement from point O is found from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$\Delta y = \frac{0-\left(v_0 \sin\theta_0\right)^2}{2a_y} = \frac{-\left[-\left(5.6\times 10^{6}\ \text{m/s}\right)\sin 45°\right]^2}{2\left(8.78\times 10^{15}\ \text{m/s}^2\right)} = -0.89\ \text{mm}$$

The minimum distance above the bottom plate is then

$$d = \frac{D}{2} + \Delta y = 1.00\ \text{mm} - 0.89\ \text{mm} = \boxed{0.11\ \text{mm}}$$

(b) The time for the electron to go from point O to the upper plate is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$+1.00\times 10^{-3}\ \text{m} = \left[-\left(5.6\times 10^{6}\ \frac{\text{m}}{\text{s}}\right)\sin 45°\right]t + \frac{1}{2}\left(8.78\times 10^{15}\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

Solving for t gives a positive solution of $t = 1.11\times 10^{-9}$ s. The horizontal displacement from point O at this time is

$$\Delta x = v_{0x}t = \left[\left(5.6\times 10^{6}\ \text{m/s}\right)\cos 45°\right]\left(1.11\times 10^{-9}\ \text{s}\right) = \boxed{4.4\ \text{mm}}$$