

CENTRIPETAL FORCE

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Objectives

- To study the main characteristics of rotational motion.
- To verify the formula for centripetal force.
- To verify Newton's second law for centripetal force.

Equipment

1. centripetal force apparatus
2. digital stopwatch
3. set of weights: 100 x 10, 50, 20 x 2, 10 g
4. ruler
5. balance

Theory

Uniform circular motion is motion with a constant speed on a circular path. This motion can be described by the period T – the time required to make one complete revolution around the circular path. A related value that characterizes the motion is the frequency f – the number of revolutions per unit of time. The two are related by

$$f = \frac{1}{T} \quad (1)$$

In SI the unit of frequency is Hz (hertz), which is one revolution per second. However, the angular frequency ω is often used in describing circular motion, where

$$\omega = 2\pi f = \frac{2\pi}{T} \quad (2)$$

The angular frequency ω is measured in radians per second.

The speed of circular motion v is equal to the circumference divided by the period

$$v = \frac{2\pi R}{T} = 2\pi Rf = R\omega \quad (3)$$

As already mentioned, the motion has a constant speed. The velocity of the motion is tangential to the orbit and is changing in direction only. This change results in a centripetal acceleration a_c directed to the center of the circle. The value of the centripetal acceleration is given by the formula

$$a_c = \frac{v^2}{R} = R\omega^2 \quad (4)$$

According to Newton's second law, the centripetal acceleration is created by a force – the centripetal force F_c – which is directed to the center of the circle (as is the centripetal acceleration) and is perpendicular to the velocity.

$$F_c = \frac{Mv^2}{R} = MR\omega^2 \quad (5)$$

Using the formula (2) the centripetal force can be rewritten as

$$F_c = 4\pi^2 MRf^2 = \frac{4\pi^2 MR}{T^2} \quad (6)$$

A centripetal force is not an extra force that occurs by itself. It is the resultant of some other force such as tension, gravity, friction, elasticity, electrical attraction, etc. which causes the object to move in a circular path.

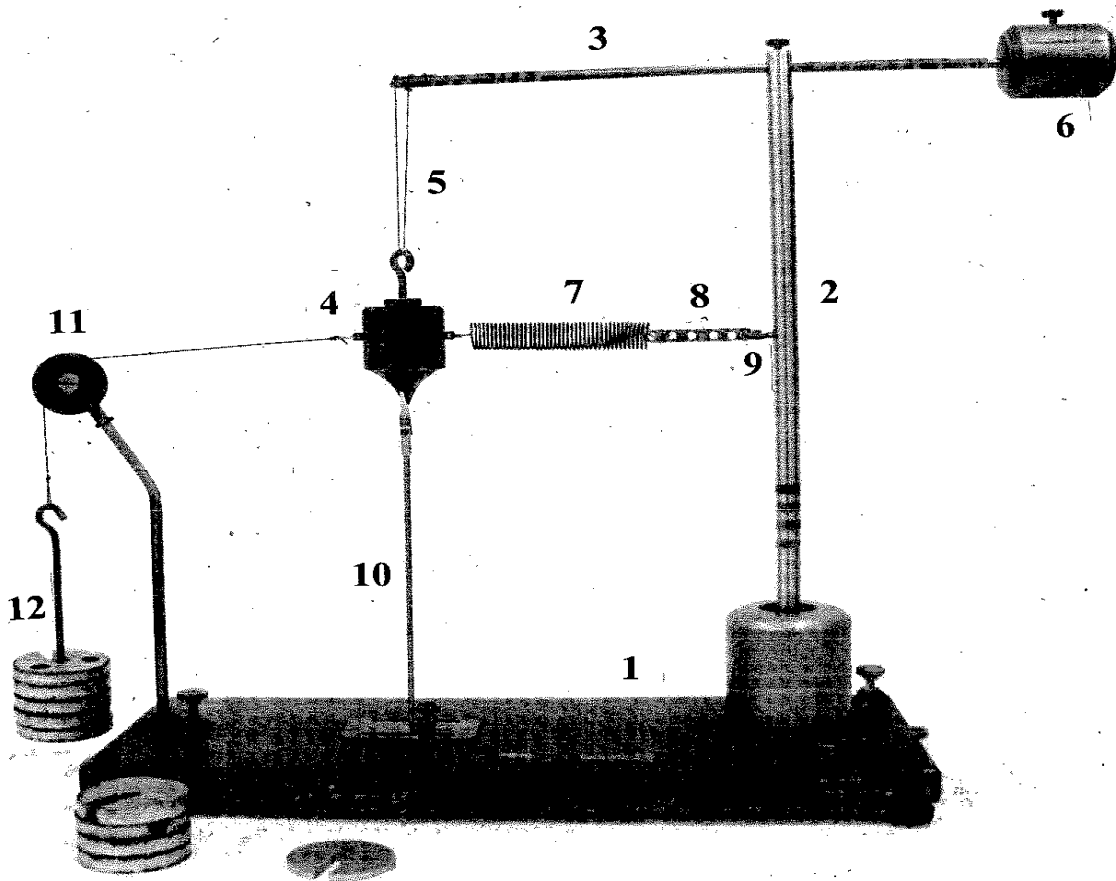


Fig. 1. Centripetal force apparatus. 1. Base with adjacent level screw 2. Shaft 3. Arm 4. Rotating mass 5. String 6. Counterbalance 7. Spring 8. Metal strip with holes 9. Hook on the shaft 10. Pointer with resilient tine 11. Pulley 12. Hanger with weights

The special apparatus shown in Fig. 1 is used in the experiment. On a horizontal base, a vertical shaft (2) is supported by a bearing that allows the shaft to rotate with low friction. A horizontal sliding arm (3) is fixed on the top of the shaft. On one side of the arm a mass M (4) is suspended on a string; on the other side is a counterbalance (6). The

mass M is attached to the shaft by a spring, whose tension can be varied by hooking the other end of the spring into one of a set of holes on a metal strip (8). There is a movable long vertical pointer (10) attached to the base to identify the radius of rotation of the mass M . The pointer has a resilient tine (a flat spring) on the upper end that projects upward toward the cone on the mass. The shaft can be rotated by spinning the knurling located near the bottom. The shaft is to be twirled just fast enough so that the tip on the mass M lines up and touches the tine on the pointer. The centripetal force is supplied by the spring (7) attached to the side of the rotating mass M . The free-body diagram for this case is given in Fig. 2(a).

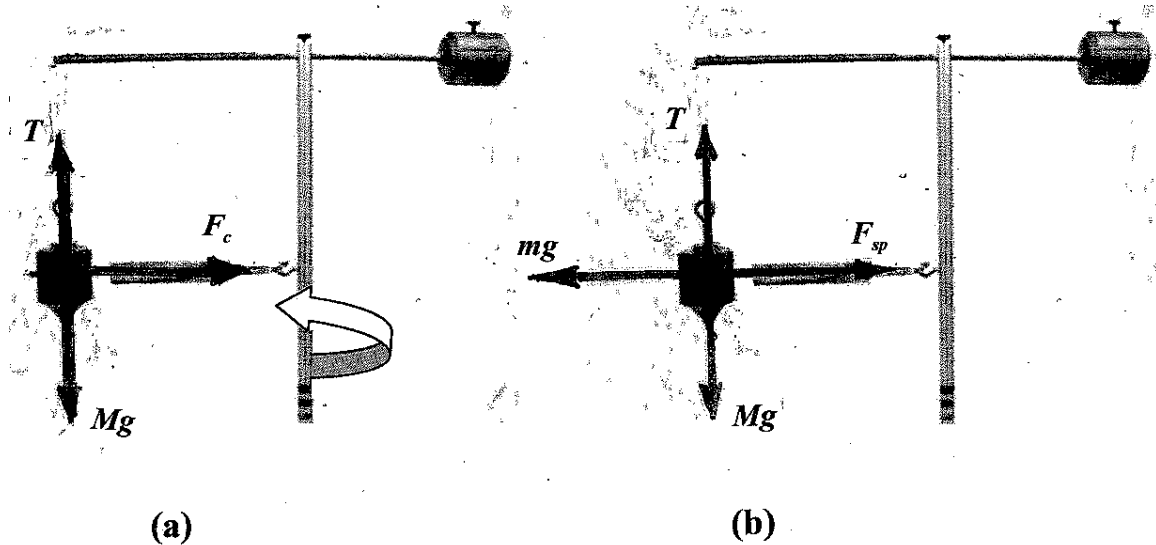


Fig. 2. Free-body diagrams for (a) dynamic and (b) static measurements.

The resultant of two forces, the weight of the mass M and the tension of the suspended string (5), equals zero if they are on the same vertical line, so that the only remaining force is the centripetal force F_c , which produces the acceleration of the mass M . This force can be measured in a static state, without rotation, as shown in Fig.2(b). The same value of the spring force F_{sp} , which equals to the centripetal force F_c , can be compared with a weight of a hanger on which weights can be loaded, and which is connected by a string through a pulley to the other side of the rotation mass M . The load mass is adjusted so that the pointer and the cone again line up. In this case the value of the force equals

$$F_{sp} = mg \quad (7)$$

where m is mass of the hanger together with the load mass. The force mg is equal to the spring tension, which is equal to the centripetal force holding the mass at the same radius when it is rotating.

Procedure

Part 1. Dependence of the frequency and speed of rotation on the centripetal force at constant radius

1. Level the base by using the thumb screws in the base to set the arm of the apparatus in the indifferent equilibrium.
2. Release the spring from the mass M and adjust the position of the arm, so that the cone of the mass M is lined up with the pointer. Measure the mass M and the radius of rotation R , which is the distance between the center of the shaft and the pointer. Write down the results in Table 1.
3. Connect the spring to the hook on the shaft and to the mass M . Try to rotate the shaft. **Be careful. Watch out that your head does not get hit by the rotating parts.** The rotation should be just fast enough that the cone of the mass M hits the resilient tine (the flat spring) on the pointer to produce a sound. Keep the rotation steady so that the sound of the cone hitting the tine is regular. With the digital stopwatch measure the time t of 20 revolutions and write down the result in the data sheet.
4. Stop the rotation and connect the string of the hanger over the pulley to the opposite side of the mass M . Adjust the hanger weight so that the pointer and the cone again line up. Write down the value of the mass m , the total mass of the hanger plus the load mass, in the data sheet.
5. Change the force applied by the spring by attaching it to a different hole in the metal strip. Repeat steps 3 and 4.
6. Repeat step 5 for several different holes in the metal strip.

Part 2. Dependence of the period of rotation on the mass at constant radius

1. Keep the same set up as in Part 1. Write down the values of rotating mass M and the radius in Table 2. Connect the spring to any hole on the metal strip.
2. Rotate the shaft to hear the the mass cone hitting the pointer with regularity, as before. Measure the time t of 20 revolutions, and write down the result in Table 2.
3. Stop the rotation. Connect the string of the hanger over the pulley to the opposite side of mass M and adjust the hanger weight so that the pointer and the cone again line up. Write down the mass m , the total mass of the hanger plus the load mass, in the data sheet.
4. Add 100 g to the rotating mass M , tighten it with a nut, and write down the new total rotating mass M in Table 2. Repeat step 2.
5. Repeat step 4 with an additional 100 g of the rotating mass. Note that it is not necessary to repeat step 3, the static measurement of the force, because the radius and the tension of the spring are the same.

Part 3. Dependence of the radius on the velocity at constant centripetal force

1. Set the pointer at the closest position to the shaft, i.e., at the smallest radius. Release the spring from the mass M and adjust the position of the arm (3) so that the cone of the mass M is lined up with the pointer. Measure the mass M and the radius of rotation R , as before. Write down the results in Table 3.
2. Connect the spring to the hook on the shaft and to the rotating mass M . Connect the string of the hanger to the opposite side of the rotation mass M , and adjust the hanger weight so that the pointer and the cone again line up. Write down the mass m , the total mass of the hanger plus the load mass, in the data sheet.
3. Disconnect the hanger string. Rotate the shaft to hear the mass cone hitting the pointer, as before. Measure the time t of 20 revolutions, and write down the result in Table 3.
4. Connect the spring to the hole on the metal strip closest to the shaft. Then connect the rotating mass M to the string of the hanger with the previously measured mass. Set the arm so that the suspending string is vertical and adjust the pointer in the position to line it up with the cone. Measure the radius and write down the result in Table 3.
5. Disconnect the hanger string. Rotate the shaft to hear the mass cone hitting the pointer. Measure the time t of 20 revolutions and write down the result in Table 3.
6. Repeat steps 4 and 5 for each hole of the metal strip.

Computation and Data Analysis

1. For each measurement of the time t of 20 revolutions calculate the period T , the time of one revolution, and the square of the period where it is necessary.
2. From each static measurement calculate F_{sp} from equation (7).
3. Using equations (1) and (3) calculate the frequency and velocity for each trial as well as their squares.
4. Using equations (5) and (6) calculate the theoretical value of the centripetal force F_c and for Table 1 compare it to static measurements F_{sp} .
5. From the data of Table 1 plot graphs of F_{sp} vs. f^2 and F_{sp} vs. v^2 . Find the slope for each graph. Calculate the theoretical value of the slope and compare to the slope measured from the graph.
6. From data of Table 2 plot the graph T^2 vs. M . Find the slope from the graph. Calculate the theoretical value of the slope and compare it to the slope from the graph.
7. From data of Table 3 plot the graph v^2 vs. R . Find the slope from the graph. Calculate the theoretical value of the slope and compare it to the slope from the graph.

Questions

1. How can a body move at constant speed and still be considered to be accelerated?

2. A car is driving with constant speed around a circular track. (a) Is its velocity constant? (b) Is the magnitude of its acceleration constant? (c) Is the direction of its acceleration constant?
3. What would be the effect if the suspended point on the arm of the mass and the pointer are not on the same vertical line in the experiment?
4. Explain the difference between centripetal and centrifugal force.

Student's name _____

Date _____

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Table 1. Dependence of the frequency and speed of rotation on the centripetal force at constant radius

$M =$ _____ kg $R =$ _____ m

trial	t of 20 rev, s	T ,s	f , Hz	f^2 , Hz^2	v , m/s	v^2 , (m/s) ²	m , kg	F_{sp} , N	F_c , N	% diff.
1										
2										
3										
4										
5										
6										

Slope of the graph F_{sp} vs. $f^2 =$ _____ kg m	Theoretical slope = _____ kg m	% diff = _____ %
Slope of the graph F_{sp} vs. $v^2 =$ _____ kg/m	Theoretical slope = _____ kg/m	% diff = _____ %

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Table 2. Dependence of the period of rotation on the mass at constant radius

$R =$ _____ m $m =$ _____ kg $F_{sp} =$ _____ N

Trial	M , kg	t of 20 rev, s	T , s	T^2 , s ²	F_c , N
1					
2					
3					
					Average $F_c =$ _____ N

Slope of graph T^2 vs. $M =$ _____ s ² /kg	Theoretical slope = _____ s ² /kg
	% diff = _____ %

Table 3. Dependence of the radius on the velocity at constant centripetal force

$M =$ _____ kg $m =$ _____ kg $F_{sp} =$ _____ N

Trial	t of 20 rev, s	T , s	R , m	v , m/s	v^2 , (m/s) ²	F_c , N
1						
2						
3						
4						
5						
					Average $F_c =$ _____ N	

Slope of graph v^2 vs. $R =$ _____ m/s ²	Theoretical slope = _____ m/s ²
	% diff = _____ %