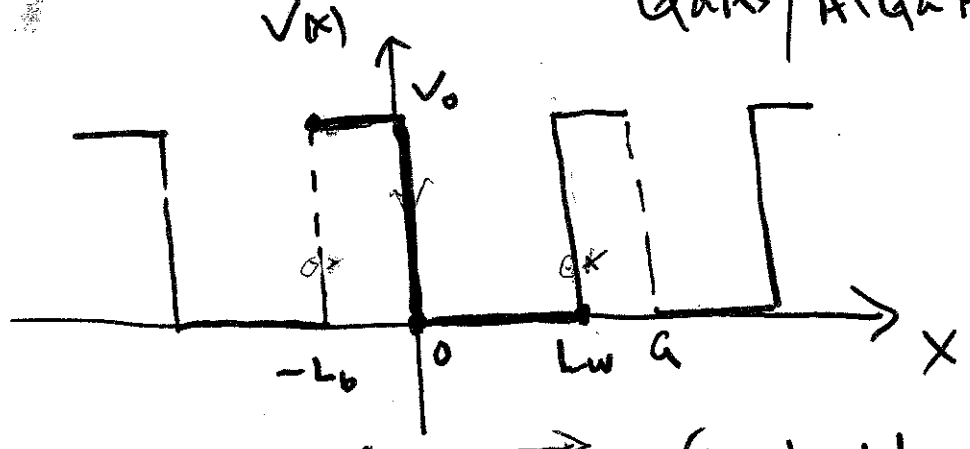
1D superlattice (semiconductor GaAs/AlGaAs)



for $0 < x < L_w$:

$$\varphi(x) = A e^{ik_w x} + B e^{-ik_w x}$$

$$k_w = \left(\frac{2m_w E}{\hbar^2}\right)^{1/2}$$

$x = 0^-, 0^+$

$x = L_w^-, L_w^+$ ($-L_b$)

for $-L_b < x < 0$:

$$\varphi(x) = C e^{k_b x} + D e^{-k_b x}$$

$$k_b = \left[\frac{2m_b(V_0 - E)}{\hbar^2}\right]^{1/2}$$

BC's:

$$\begin{cases} A + B = C + D & --(1) \quad \psi(0^-) = \psi(0^+) \\ ik_w(A - B) = k_b(C - D) & --(2) \quad \psi'(0^-) = \psi'(0^+) \end{cases}$$

$\psi(L_w^-) = \psi(L_w^+) = \psi(-L_b) e^{ika}$

$$A e^{ik_w L_w} + B e^{-ik_w L_w} = \left(C e^{-k_b L_b} + D e^{k_b L_b}\right) e^{ika} \quad (3)$$

$\psi'(L_w^-) = \psi'(-L_b) e^{ika}$

$$ik_w\left(A e^{ik_w L_w} - B e^{-ik_w L_w}\right) = k_b\left(C e^{-k_b L_b} - D e^{k_b L_b}\right) e^{ika} \quad (4)$$

[only wavefunctions used in $-L_b \le x \le L_w$ within the unit cell]

```
%while k<10
%k=k+1;
k=5
x=0;
while x<300
x=x+1;
y11=x-k^2;
y12=0:
y21=0;
y22=x-3*k^2;
y=[y11 y12;y12 y22];
%z=abs(det(y));
%if z<40
%a=x
%plot(k,a,'ro')
%hold on
%end
semilogy(x,abs(det(y)),'r+')
hold on
end
end
end
```

$y_{21} = y_{12} = 0$

test function 3/25/10

$$\zeta = \begin{vmatrix} (x-k^2) & 0 \\ 0 & (x-3k^2) \end{vmatrix}$$

$$= (x-k^2)(x-3k^2) = 0$$

→ Fig. 1

$$Ak + B - C - D = 0 \quad \text{--- (1)}$$

$$k^2 Ak - k^2 B - k_b C + k_b D = 0 \quad \text{--- (2)}$$

$$Ak \quad B \quad C \quad D = 0 \quad \text{--- (3)}$$

$$Ak \quad B \quad C \quad D = 0 \quad \text{--- (4)}$$

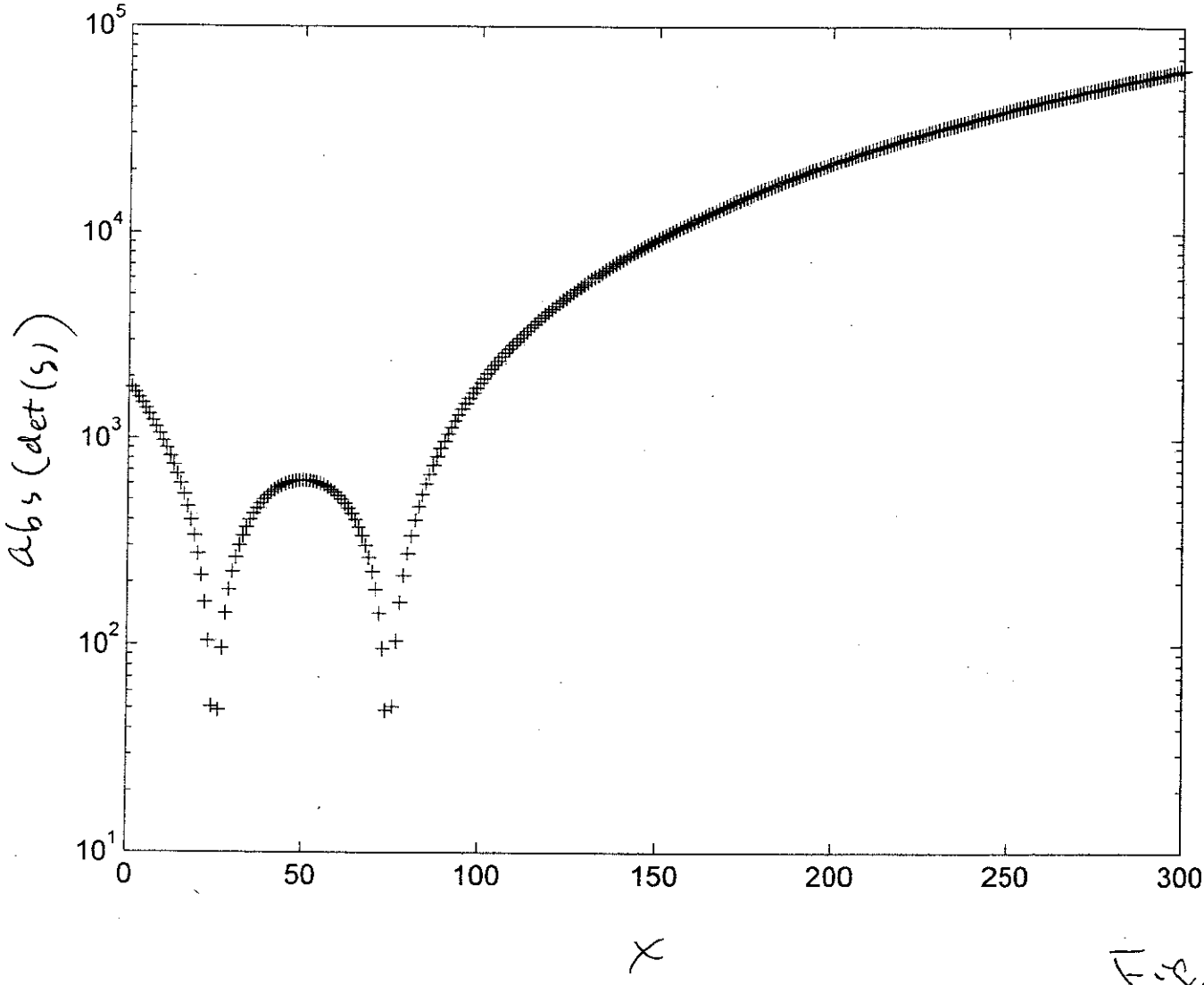


Fig. 1

```
k=0;
while k<10
k=k+1;
x=0;
while x<300
x=x+1;
y11=x-k^2;
y12=0;
y21=0;
y22=x-3*k^2;
y=[y11 y12;y12 y22];
z=abs(det(y));
if z<40
a=x
plot(k,a,'ro')
hold on
end
%semilogy(x,abs(det(y)))
%hold on
end
```

→ Fig. 2

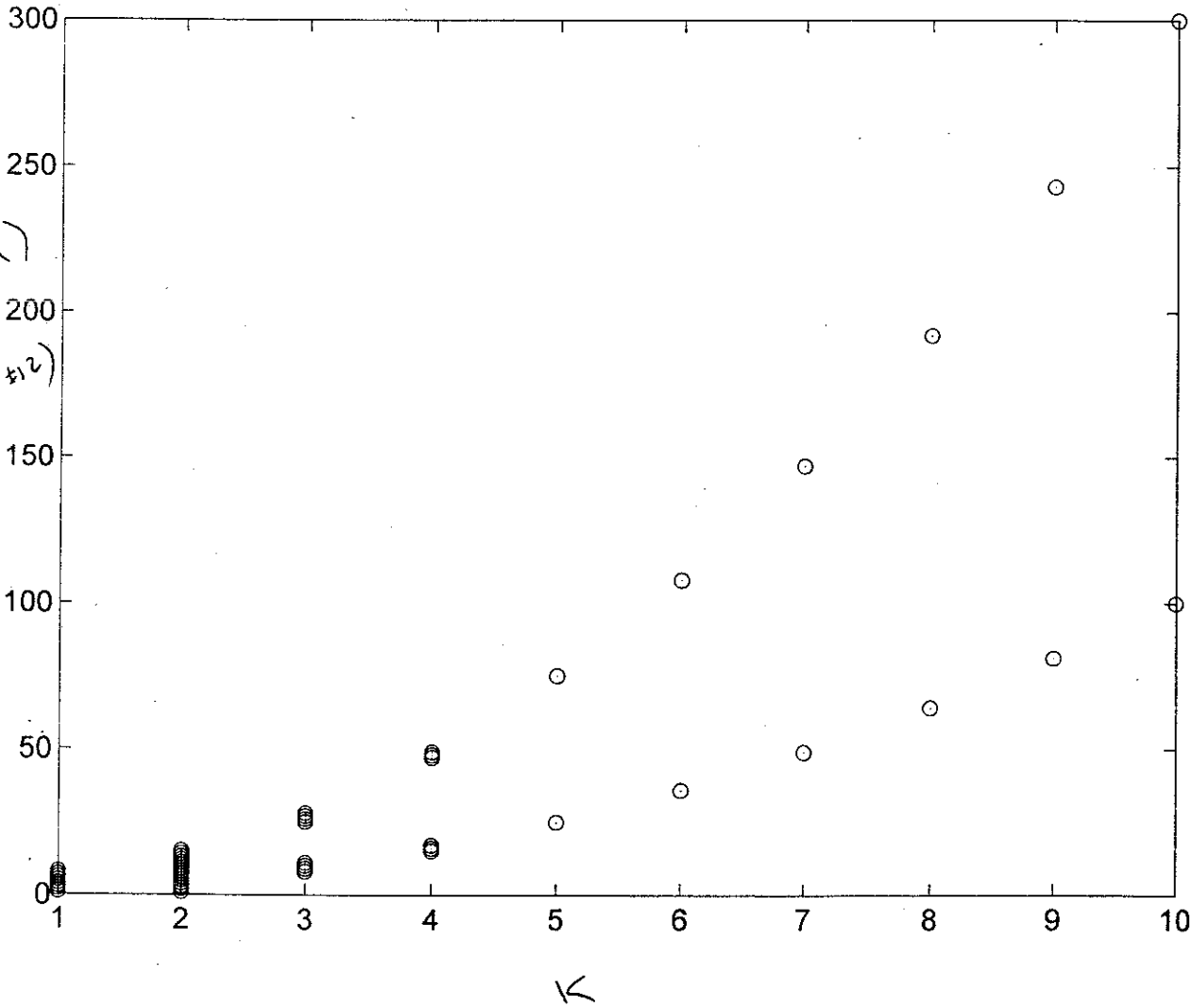
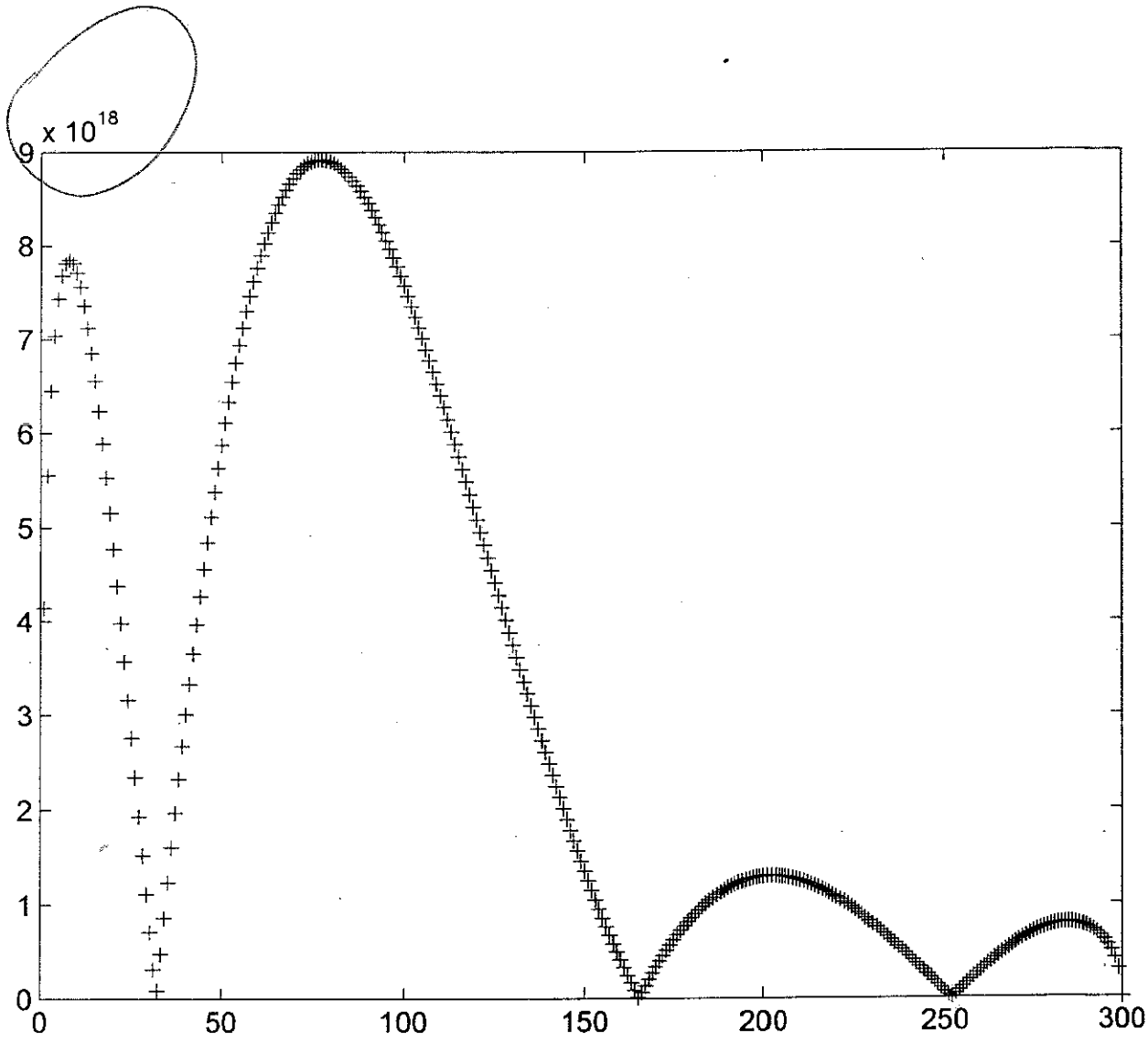


Fig. 2

```
% this m.file is to calculate the band structure of
% 1D superlattice
% The following is a set of constants used
%
i=sqrt(-1);             % define imaginary unit
m0=9.1e-31;             % free electron mass, kg
hbar=1.05459e-34;       % Planck constant/2pi in unit of J.s
mw=0.067*m0;                    % electron effective mass in well
mb=0.1*m0;              % electron effective mass in barrier
mev=1.6e-22;            % energy conversion factor
v0=300*mev;             % barrier height
angs=1e-10;             % length conversion factor
lw=100*angs;            % well width
lb=20*angs;             % barrier width
a=lb+lw;                % period, be considered as spacing bet ions
%
% 1st. for a given k value
k=pi/a/50;
% for given k, you can plot the abs(det(z)) vs e and find out
% where are the roots, where z is a matrix by 4 boundary conditions
e=0;
while e<300
e=e+1;
kw=sqrt(2*mw*e*mev/hbar/hbar);
kb=sqrt(2*mb*(v0-e*mev)/hbar/hbar);
z11=1;
z12=1;
z13=-1;
z14=-1;
z21=i*kw;
z22=-i*kw;
z23=-kb;
z24=kb;
z31=exp(i*kw*lw);
z32=exp(-i*kw*lw);
z33=-exp(-kb*lb+i*k*a);
z34=-exp(kb*lb+i*k*a);
z41=i*kw*exp(i*kw*lw);
z42=-i*kw*exp(-i*kw*lw);
z43=-kb*exp(-kb*lb+i*k*a);
z44=kb*exp(kb*lb+i*k*a);
x=[z11 z12 z13 z14;z21 z22 z23 z24;z31 z32 z33 z34;z41 z42 z43 z44];
dx=abs(det(x));
plot(e,dx,'r+')
hold on
end
```

→ Fig. 3



σ_{bs} (cm²/sr)

E(mev)

Fig. 3

```
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% 1D superlattice
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m0=9.1e-31;        % free electron mass, kg
hbar=1.05459e-34;     % Planck constant/2pi in unit of J.s
mw=0.067*m0;       % electron effective mass in well
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v0=300*mev;     % barrier height
angs=1e-10;     % length conversion factor
lw=100*angs;       % well width
lb=20*angs;     % barrier width
a=lb+lw;     % period, be considered as spacing bet ions
%
% 1st, for a given k value
k=pi/a/50;
while k<pi/a
   k=k+pi/a/50;
% for given k, you can plot the abs(det(z)) vs e and find out
% where are the roots, where z is a matrix by 4 boundary conditions
e=0;
while e<300
e=e+1;
kw=sqrt(2*mw*e*mev/hbar/hbar);
kb=sqrt(2*mb*(v0-e*mev)/hbar/hbar);
z11=1;
z12=1;
z13=-1;
z14=-1;
z21=i*kw;
z22=-i*kw;
z23=-kb;
z24=kb;
z31=exp(i*kw*lw);
z32=exp(-i*kw*lw);
z33=-exp(-kb*lb+i*k*a);
z34=-exp(kb*lb+i*k*a);
z41=i*kw*exp(i*kw*lw);
z42=-i*kw*exp(-i*kw*lw);
z43=-kb*exp(-kb*lb+i*k*a);
z44=kb*exp(kb*lb+i*k*a);
x=[z11 z12 z13 z14;z21 z22 z23 z24;z31 z32 z33 z34;z41 z42 z43 z44];
dx=abs(det(x));
if dx<2e17 <-- note the log of dx
   ee=e;
plot(k,ee,'r+') --> Fig. 4
hold on
end
end
end
```

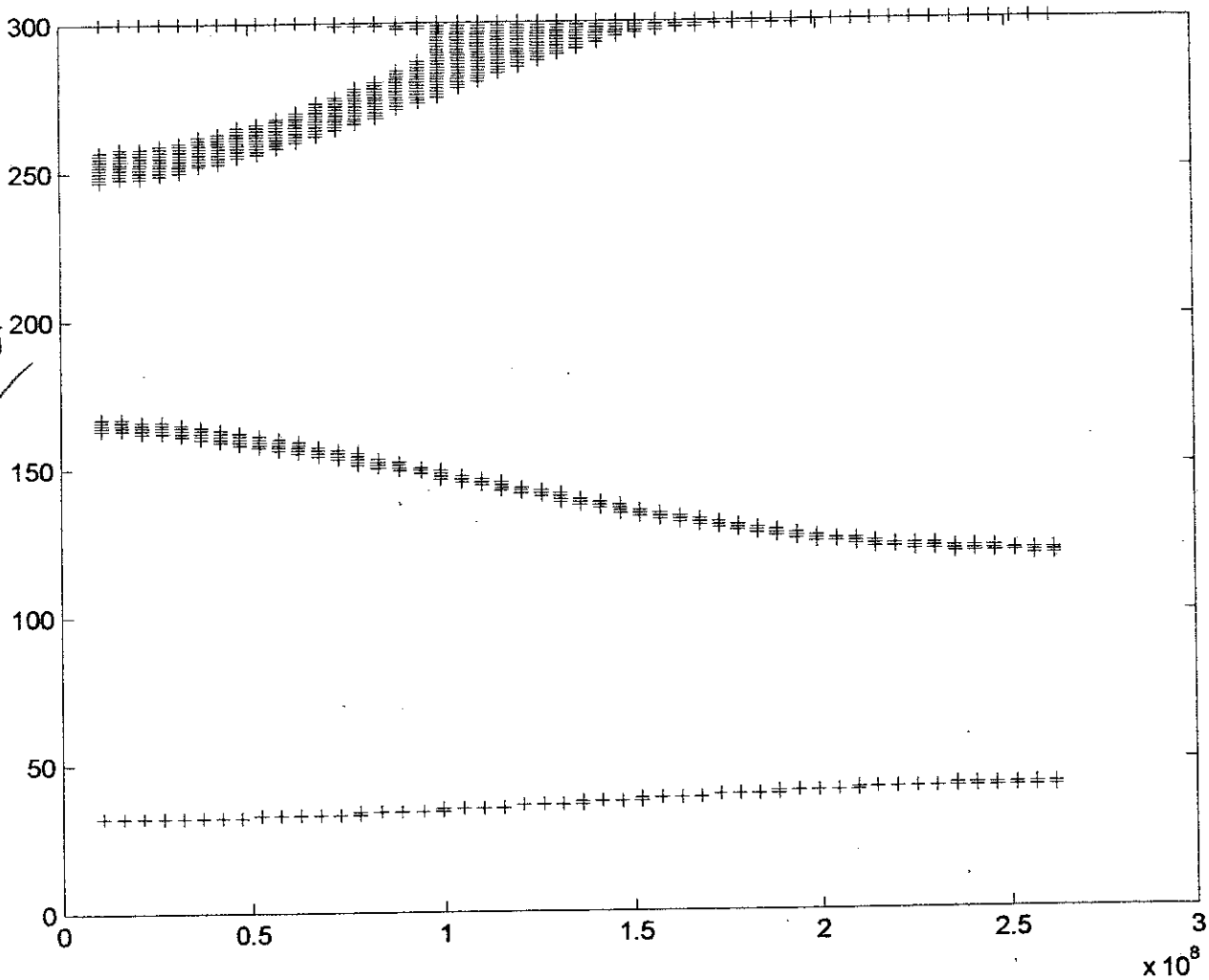

$K_m = \frac{\pi}{120 A}$

$= \frac{\pi}{120 \times 10^{-10}\ m} = \frac{\pi}{120} \times 10^{10}\ m^{-1}$

$= \frac{\pi}{120} \times 10^{8}\ cm^{-1}$

Fig. 4