Does the "Great Moderation" Matter for Classical Cycles? Monte Carlo Evidence

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March 1, 2008

Abstract

There has been substantial research regarding the causes and timing of the "Great Moderation" of volatility that occurred over the past two decades. Rather than debate the cause or exact timing of the moderation, this paper asks whether or not the moderation has had a significant impact on the measurement of the classical business cycles. Using a range of univariate business cycle models allowing for parameter instability, simple models are estimated for output in the U.S., U.K, and Australia. Monte Carlo estimation results show that the traditionally measured *classical business cycle* has essentially disappeared since the break in volatility. The results suggest that previously supported methods of business cycle measurement should be adjusted to account for the break in volatility. The estimation results also show that there has been a large change in the persistence of growth since the "Great Moderation" which if unaccounted for results in substantially different cyclical measurements.

1 Introduction

Since McConnell and Perez-Quiros (2000) estimated that there was a break in the volatility of GDP in the United States, the "Great Moderation" has been studied in depth with most research debating whether or not the cause was good policy, good luck, a widespread technology change, or something entirely different. While the ultimate cause of the downward change in volatility has not yet been decisively found, there is little debate that some

sort of moderation did actually occur. Whether or not a return to higher volatility can be prevented, or if this is even desired remains to be seen. This paper examines the existence of the Great Moderation in the context of the measurement of classical cycles for the U.S. the U.K, and Australia showing that the reduced volatility has a significant impact on the ability of traditionally measured classical cycle dating methods to detect changes in the business (or reference) cycle.[1] Ultimately, it appears that linear models have failed to continue being able to fit the reference cycle using dating algorithms with models that account for the Great Moderation. Another possible interpretation of the results is that the dating algorithms themselves are obsolete following the Great Moderation. Additionally, adaptation of statistical tests determining the fit of a model to the reference cycles such as those used by Cogley and Nason (1995) or Simkins (1994) is likely necessary given the failure of many models to fit the reference cycle's stylized facts presented by a dating algorithm.

This paper is not advocating that business cycles would be optimally modeled using the simple methods employed within (e.g., ARIMA(1,1,0) with switching variance), but questions whether or not the reference cycles measured by dating algorithms are flexible enough to test various models when faced with shifts in parameters. Generally, it would be optimal if all business cycle models such as the Hamilton regime-switching models in mean and variance could be tested against the cyclical nature of the underlying distribution. However, either the dating algorithms or models themselves have great difficulty at matching a moderating reference cycle.

Recent articles by Harding and Pagan (2002) Harding and Pagan (2003a) and Hess and Iwata (1997) have shown a revived interest in measuring the quality of business cycle models by using Monte Carlo methods to measure which models fit the data better with regard to the features of business cycles rather than simply looking at parametric statistics regarding fit and forecasting ability. McConnell and Perez-Quiros (2000) use a relatively simple macroeconomic model for output (ARIMA(1,1,0)) to show that a break in volatility occurred around the beginning of 1984. With the justification of a variance break, the authors adapted a Hamilton (1989) Markov-switching model in mean only to account for the downward break in volatility. More recently, there has been widespread study of the Great Moderation using VAR models with Markov-switching mean and volatility with very mixed results.

[1]The *traditional* method for dating the classical cycle using quarterly data can be attributed to Burns and Mitchell (1946), Harding and Pagan (2002), and Bry and Boschan (1971).

In a published debate between Harding and Pagan (2003b) and Hamilton (2003) the authors debate the merit of classical cycle measurement or model based estimation. While it seems that it would be ideal to use some parametric model to estimate the probability of a recession, the choice of the model is constrained to the current set of knowledge regarding the system. McConnell and Perez-Quiros (2000) show the Hamilton Markov-switching model in mean does not detect mild recessions like those since the Great Moderation unless the model is modified to switch in variance as well.

The issue at hand here is whether or not the classical cycle dating methods employed by Harding and Pagan (2002) or Hess and Iwata (1997) are useful in the context of the Great Moderation. Evidence presented here shows that simple ARIMA(1,1,0) models for output, which used to approximate the business cycle fairly well, have performed quite poorly when used on models allowing for breaks in variance.[2] Measuring the business cycle in output is complicated when the models are unable to produce business cycles which clearly still exist. Although, it may be the case that there has been a fundamental restructuring of how business cycles are recorded in the U.S., it seems unlikely that this state would persist indefinitely into the future, or has occurred in other countries experiencing the Great Moderation.

While fitting the classical business cycle is a challenge for any model, linear and non-linear alike, the results of this paper show that the classical business cycle is now almost currently undetectable in the U.S., U.K., and Australia when estimating models accounting for the "Great Moderation." In order to properly compare the models suggested by various authors, a suitable set of stylized facts regarding the actual economy must first be explored. After the stylized facts are presented, the estimation models are presented, and then simulated to test their performance at simulating the stylized facts.[3]

2 Methodology and Data

The methodology for determining turning points and cyclical characteristics depends on the break dates determined through estimation. Thus, it makes sense to first present the method of determining break dates. The data for the estimations and simulations are Gross Domestic Product in chained

[2]See Harding and Pagan (2002) and Hess and Iwata (1997) for examples of testing linear versus non-linear modeling methods.

[3]Business cycle turning points can also be estimated using non-linear methods as shown in Chauvet and Potter (2001) and Chauvet and Hamilton (2005)

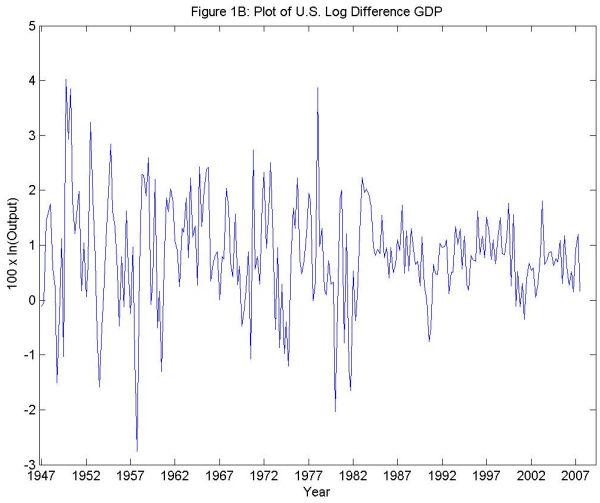

Figure 1: US Growth Rates

dollars for the U.S. (1947Q1-2007Q4), U.K. (1955Q1-2007Q3), and Australia (1959Q3-2007Q1). The volatility of each of these countries varies a great deal across nations, and it is apparent from Figures 1 to 3 that their growth rates have moderated quite abruptly over time.

As can be seen, and has been documented thoroughly, each of the countries studied here has experienced a "Great Moderation" of its own. The variety of models that have been used to fit each of these series has displayed that the volatility breaks are likely to have occurred almost concurrently, however there are a range of estimations of the exact date of each country's moderation. Work by Sensier and van Dijk (2004) estimates the different possible confidence intervals for the break dates in volatility showing that many experienced breaks in output volatility at approximately the same time.

2.1 Break Tests and Parameter Estimation

Following the work by McConnell and Perez-Quiros (2000) I use Generalized Method of Moments to simultaneously estimate the following system (and variants listed below) and endogenously determine the break dates for the

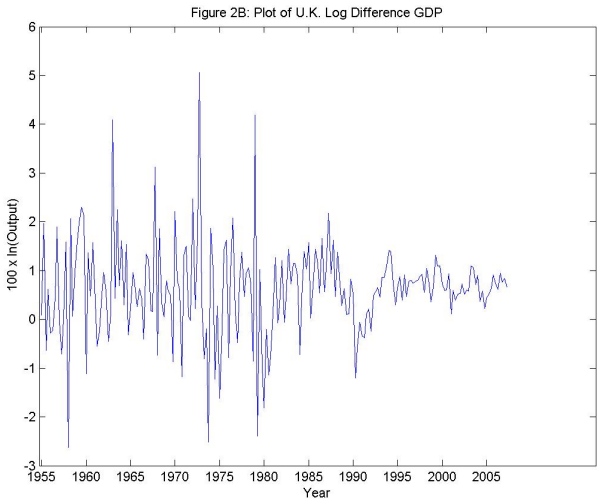

Figure 2: UK Growth Rates

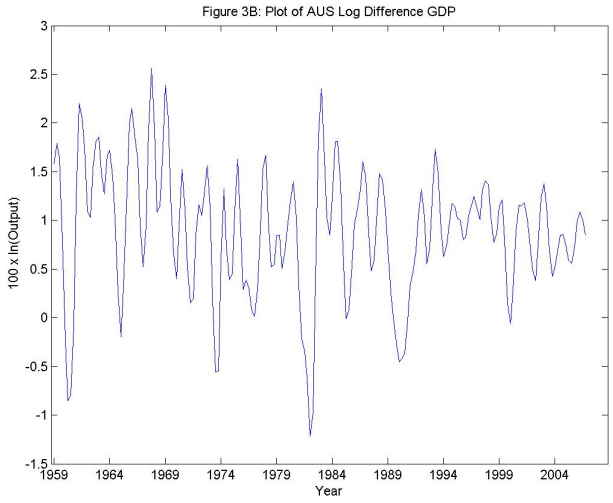

Figure 3: Australian Growth Rates

variance (and other parameters).[4]

$$\Delta y_t = \phi_o + \phi_1 \Delta y_{t-1} + \epsilon_t \tag{1}$$

$$\sqrt{\frac{\pi}{2}} |\epsilon_t| = D_{1t}\alpha_1 + D_{2t}\alpha_2 + \mu_t \tag{2}$$

The instruments used in the systems are the constant, lagged growth, and the dummy variables indicating the regime.[5] Each model was estimated over the range of possible break dates (T) between T_1=0.15 and T_2=0.85 which are used to set the values for D_{1t} and D_{2t} in equation (2).[6] The dummies for volatility regime are set such that

$$D_{1t} = 1 \text{ if } t < T$$
$$D_{1t} = 0 \text{ if } t \geq T$$
$$D_{2t} = 0 \text{ if } t < T$$
$$D_{2t} = 1 \text{ if } t \geq T$$

The model is estimated for each possible T value between T_1 and T_2, and Wald/Likelihood Ratio (LR) tests are performed for each of the estimated models. The test statistics used to determine whether or not a series contains a break are the *sup*, *ave*, and *exp* tests performed across all estimations.[7] The Wald/LR tests for the U.S. are all highly significant and estimate the most likely break date for the variance at 1983Q4.[8] A similar Lagrange Multiplier test for parameter stability of the U.S. model using p-values corrected using methods suggested by Hansen (1997) also estimates the break date at a nearby 1983Q1. Either of these dates could plausibly be used in the forthcoming Monte Carlo estimation, so the Wald tests were chosen for the ease of calculation.

It is also plausible that given the simple system of equations that have been laid out, the model could have experienced a break in the lag coefficients

[4]The choice for the variance estimation equation was due to the chosen methodology of McConnell and Perez-Quiros (2000) as the $\sqrt{\frac{\pi}{2}}|\epsilon_t|$ is an unbiased estimator of the standard deviation as per Davidian and Carroll (1987).

[5]For each model, a Newey-West weighting matrix with 12 lags was used to calculate standard errors. Estimation was carried out using iterated GMM with instruments used to construct the initial weighting matrix and an optimal weighting matrix to calculate final parameters.

[6]T_1=0.15 and T_2=0.85 are set to avoid the problems of detecting breaks at the beginning and ends of samples

[7]The *sup*, *ave*, and *exp* tests were proposed by Andrews (1993) Andrews and Ploberger (1994) using the p-value correction suggested by Hansen (1997)

[8]See Tables 1A-1C, column 3 for regressions involving break tests.

or the constant. In order to plausibly rule out this possibility, regressions were run on the following system as well.

$$\Delta y_t = D_{1t}\phi_o + D_{2t}\phi_1 + D_{1t}\phi_3\Delta y_{t-1} + D_{2t}\phi_4\Delta y_{t-1} + \epsilon_t \tag{3}$$

$$\sqrt{\frac{\pi}{2}}\,|\epsilon_t| = D_{1t}\alpha_1 + D_{2t}\alpha_2 + \mu_t \tag{4}$$

While the model in (3) and (4) has the shortcoming of not testing individual break dates, if we are searching for a plausible reason for the obvious break in the mid-1980s in volatility it is reasonable to expect that the coefficients might experience a concurrent break.[9]

2.2 Regression and Break Test Results

The estimation of the system of equations (1) and (2) for the U.S., the U.K., and Australia all appear to indicate that there was a break in the volatility.[10] Rates of growth are listed in Table 1 for each of the countries studied here. It can be seen that the U.S. volatility (or estimated standard deviation) of growth moderated from approximately 0.8888 to 0.5074. The U.K. experienced an even greater decline in volatility as estimated in this model, falling from 0.9389 to 0.2618 during approximately 1992.[11] Australia also experienced a significant decline in volatility around the fourth quarter of 1990, as the level of volatility fell from 0.4276 to 0.2462. As a note of importance, the values for each of the country's growth constants and lag coefficients all vary a great deal with this simple model. The estimated values for these parameters will play a substantial role during simulations. Estimates for the models estimating equations (3) and (4) with simultaneous breaks in all coefficients show that the restrictions of a constant rate of growth and a constant lag coefficient for the U.S. fail to reject the null hypothesis of no break.

Further tests were carried out conditional on the first break to determine if there were any additional variance breaks in the system. In the U.S. the Wald *sup* test has a high p-value for the post-1984 period conditioned on a break in 1983Q4, the *ave* and *exp* tests display evidence that there was not likely to be a break in the post 1983 period. Plots of the Wald test

[9] It is possible to test the model for independent breaks of the constant and lag coefficients along with the variance break, but those tests were not performed here.

[10] See the test statistics from column 3 in Tables 1A-1C.

[11] It should be noted that the break date of 1992Q1 was used for the simulations, but a LM test estimated the break date at 1985Q1

values for each of the possible break dates (excluding the beginning 15% and ending 15%) suggest that the system had not finished "breaking" by the late 1980's.[12] In fact *sup* tests for the detection of a break in the U.K. and Australia after the detected break show some evidence of another break in volatility in the period since output growth moderated if only using the *sup* test (See 12 and 9). However, given that the *ave* and *exp* tests do not agree that a break has occured the evidence does not yet fully support a break in the variance of the second period for these countries.

[12]This issue is left to future research where regressions with trend could be studied.

Table 1A: Estimated Rates of Growth and Volatility for the U.S.

		GMM Models						
				Test for Additional Breaks		Allow All Parameters to Break		
	Random Walk	No Break	Volatility Break	Period 1	Period 2	Test Constant Mean	Test Constant Persistence	Test Mean and Persistence
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean	0.8260	0.5552	0.5278	0.5709	0.5224	0.5616	0.3719	0.6722
(P-values in Parentheses)		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)
Mean (2)†						0.7612	0.5767	0.5278
						(0.000)	(0.000)	(0.000)
Persistence		0.3181	0.3214	0.3313	0.2702	0.3171	0.5309	0.3460
		(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)
Persistence (2)†						0.0463	0.2823	0.2724
						(0.632)	(0.000)	(0.000)
Standard Deviation	0.9570	0.8249	0.8888	1.0141	0.4930	0.8943	1.0005	0.9741
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Standard Deviation (2)†			0.5074	1.2939	0.3667	0.4663	0.7863	0.7135
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Wald Test Values								
Sup			63.4979	4.2640	10.2770	3.3963	11.3553	3.8270
			(0.000)	(0.648)	(0.068)	(0.837)	(0.056)	(0.990)
Ave			28.0274	0.5900	1.6700	0.4341	2.4927	0.7244
			(0.000)	(0.777)	(0.245)	(0.914)	(0.914)	(1.000)
Exp			19.7889	0.8960	1.0950	0.7166	2.1803	1.2244
			(0.000)	(0.816)	(0.706)	(0.917)	(0.917)	(1.000)
Break Dates								
Wald			1983Q4	1977Q4	1987Q2	1996Q4	1956Q2	1968Q4
LR			1983Q4	1977Q4	1987Q2	-	-	-
LM			1983Q1	1977Q4	2001Q2	-	-	-
Test for Change in Constant						Yes	No	Yes
Test for Change in Persistence						No	Yes	Yes

†Values are calculated for these parameters when allowing for breaks in the specific parameter. Parameter values are those from the most likely occuring break date. The first reported value for each parameter when allowing for breaks, such as the mean, represents the mean before the break. The second reported value for each parameter when allowing for breaks estimates the value after the break.

*Wald tests here signify a test of the restriction rather than a test for the break date. The parameter values are those from the estimated break date supported by the model.

Table 1B: Estimated Rates of Growth and Volatility for the U.K.

		GMM Models						
				Test for Additional Breaks		Allow All Parameters to Break		
	Random Walk	No Break	Volatility Break	Period 1	Period 2	Test Constant Mean	Test Constant Persistence	Test Mean and Persistence
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean	0.6125	0.6863	0.7364	0.6338	0.4991	0.6846	0.6931	0.5948
(P-values in Parentheses)	(0.068)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Mean (2)†						0.2652	0.2371	0.4096
						(0.000)	(0.008)	(0.000)
Persistence		-0.1118	-0.1126	-0.1199	0.3155	-0.1423	-0.1387	-0.1039
		(0.140)	(0.137)	(0.121)	(0.000)	(0.081)	(0.087)	(0.188)
Persistence (2)†						0.6033	0.6032	0.4290
						(0.000)	(0.000)	(0.000)
Standard Deviation	0.9661	0.8493	0.9389	1.1415	0.2579	1.0838	1.0821	1.0895
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Standard Deviation (2)†			0.2618	0.7811	0.1423	0.2719	0.3061	0.2558
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Wald Test Values								
Sup			62.9974	10.5013	17.5816	10.5825	42.8744	10.9551
			(0.000)	(0.071)	(0.003)	(0.074)	(0.000)	(0.288)
Ave			27.7606	2.9493	5.0157	2.4638	17.7389	3.0271
			(0.000)	(0.064)	(0.007)	(0.109)	(0.109)	(0.452)
Exp			23.9097	2.5908	2.1480	2.6294	10.7352	3.6382
			(0.000)	(0.246)	(0.343)	(0.237)	(0.237)	(0.754)
Break Dates								
Wald			1992Q1	1981Q2	2004Q4	1990Q2	1989Q4	1991Q4
LR			1992Q1	1981Q1	2004Q4	-	-	-
LM			1985Q1	1980Q4	2001Q1	-	-	-
Test for Change in Constant						Yes	No	Yes
Test for Change in Persistence						No	Yes	Yes

†Values are calculated for these parameters when allowing for breaks in the specific parameter. Parameter values are those from the most likely occuring break date. The first reported value for each parameter when allowing for breaks, such as the mean, represents the mean before the break. The second reported value for each parameter when allowing for breaks estimates the value after the break.

*Wald tests here signify a test of the restriction rather than a test for the break date. The parameter values are those from the estimated break date supported by the model.

Table 1C: Estimated Rates of Growth and Volatility for Australia

		GMM Models						
				Test for Additional Breaks		Allow All Parameters to Break		
	Random Walk	No Break	Volatility Break	Period 1	Period 2	Test Constant Mean	Test Constant Persistence	Test Mean and Persistence
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Mean	0.8940	0.2040	0.1840	0.1684	0.2527	0.1589	0.1940	0.4534
(P-values in Parentheses)	(0.048)	(0.000)	(0.000)	(0.003)	(0.000)	(0.003)	(0.000)	(0.000)
Mean (2)†						0.3070	0.2968	0.2027
						(0.000)	(0.000)	(0.000)
Persistence		0.7659	0.7777	0.7820	0.7205	0.7925	0.7794	0.6987
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Persistence (2)†						0.6682	0.6143	0.7446
						(0.000)	(0.000)	(0.000)
Standard Deviation	0.4514	0.3972	0.4276	0.5146	0.2630	0.4842	0.4301	0.5469
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Standard Deviation (2)†			0.2462	0.3702	0.1336	0.2359	0.2187	0.3739
			(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Wald Test Values								
Sup			30.0317	10.5327	15.1226	5.4521	11.6632	8.3763
			(0.000)	(0.066)	(0.008)	(0.479)	(0.046)	(0.548)
Ave			11.9695	1.8536	4.4552	1.0161	2.1366	1.2274
			(0.000)	(0.205)	(0.013)	(0.513)	(0.513)	(0.952)
Exp			16.8089	2.0028	1.6883	1.6136	2.0802	1.0414
			(0.000)	(0.382)	(0.470)	(0.513)	(0.513)	(0.971)
Break Dates								
Wald			1990Q4	1985Q4	2004Q3	1991Q2	1999Q4	1968Q3
LR			1991Q1	1985Q4	2004Q3	-	-	-
LM			1985Q4	1976Q3	2004Q3	-	-	-
Test for Change in Constant						Yes	No	Yes
Test for Change in Persistence						No	Yes	Yes

†Values are calculated for these parameters when allowing for breaks in the specific parameter. Parameter values are those from the most likely occuring break date. The first reported value for each parameter when allowing for breaks, such as the mean, represents the mean before the break. The second reported value for each parameter when allowing for breaks estimates the value after the break.

*Wald tests here signify a test of the restriction rather than a test for the break date. The parameter values are those from the estimated break date supported by the model.

2.3 Classical Cycles

The classical cycle studied here is a non-parametric method of determining the stylized facts or *reference cycle* displayed by an economic time series. Harding and Pagan (2002) show that the classical business cycle is a valid way to measure whether or not an economic model properly fits the data in question. Models are often selected based on parametric measures of fit for model specification or to give the lowest forecasting error. However, if one ignores the fundamental features of the underlying series with regard to the length, depth, and shape of recessions (expansions) a model is in danger of being assumed to fit the data rather well when it may in fact not be able to reproduce time series that remotely resemble the actual levels or features of output.

In the spirit of Harding and Pagan (2002) the stylized facts for the three countries are assembled using the methods summarized in their paper and altered to examine different reference cycles for the same economies. The economic time series are assumed to exhibit phases that last at least two quarters (for both recessions and expansions), and full cycles that last at least five quarters.[13] Also, the turning points are censored such that peaks and troughs must alternate so that a continuous cycle can be measured. Lastly, rather than use the typical rule of thumb "two consecutive quarters of negative growth signals a recession" a peak is found when $\Delta_2 y_t > 0, \Delta y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0$.[14]

All series measured using these methods are denoted as (2,2,5) for a two quarter minimum phase length, two quarters looking both forwards and backwards, and a five quarter minimum complete cycle length. Note that this characterization of turning points in output is dubbed the *classical cycle* versus the *growth cycle* since it detects peaks and troughs in output and not growth. While this algorithm is useful at timing business cycles, it would not capture a turning point where there is a single quarter of negative growth at $t+1$ if the economy returns to a higher level of output in $t+2$ relative to time t. A series would need to experience two consecutive quarters of negative growth or not return to a higher level of output after one quarter in order to be characterized as a recession.[15]

Growth that is below trend would not appear as a recession in output,

[13] These methods are based off of models proposed by Burns and Mitchell (1946), and Bry and Boschan (1971).

[14] A trough is found when $\Delta_2 y_t < 0, \Delta y_t < 0, \Delta y_{t+1} > 0, \Delta_2 y_{t+2} > 0$. As noted by Harding and Pagan (2002) the value for $\Delta_2 y_{t-2} = y_t - y_{t-2}$.

[15] This assumes that the two quarters previous to the peak had levels of output lower than at the peak.

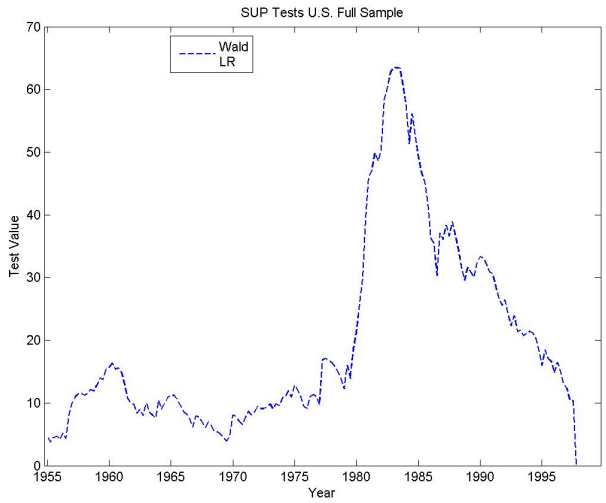

Figure 4: US Wald Sup Tests for a Single Break

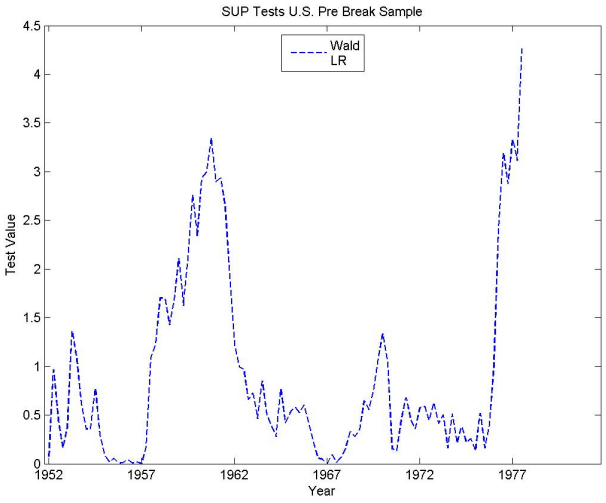

Figure 5: US Wald Sup Tests for Period Before Break

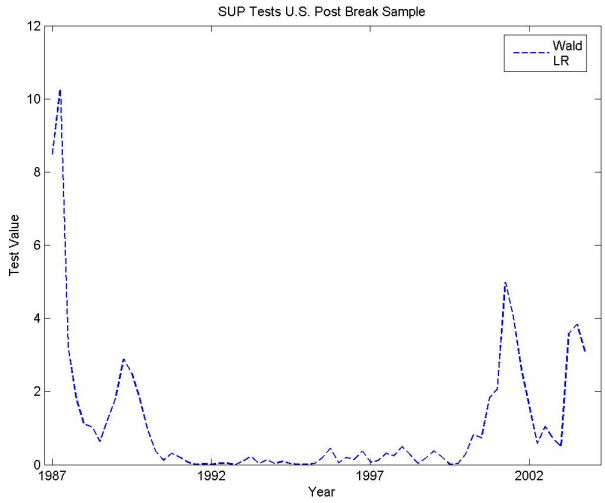

Figure 6: US Wald Sup Tests for Period After Break

and would fall under the umbrella of growth cycles. It should also be noted that a censoring algorithm is used in order to assure that complete cycles are being measured. The censoring mechanism used by Harding and Pagan (2002) and described by Cashin and Ouliaris (2004) behaves as follows...

1. Enforce alternating peaks and troughs.
2. Enforce peak-to-peak and trough-to-trough restriction on complete cycle length.
3. Eliminate dates at the beginning and final two quarters of the series.
4. Eliminate phases with a duration shorter than the restriction.
5. Again ensure that peaks and troughs alternate.

After peaks and troughs have been determined, the length of contractions (P→T) and the length of expansions (T→P) can be calculated. The amplitude between a peak and trough is calculated as the simple difference between the two levels of output in terms of percentage change. Cumulated amounts of output are calculated over the entire length of a recession or expansion as a percent change from the previous peak or trough. Lastly, percentage excess is calculated by measuring the difference between the *triangle approximation* of cumulated output and the actual path of the economy. A positive excess in a contraction implies that the actual data series

decreased slower when compared to a straight line. A model that can match the excess of a time series has the ability to replicate the asymmetries that appear in actual data. Work by Hess and Iwata (1997) shows that many of the models designed to correct for business cycle asymmetries actually created more problems with regard to being able to match the duration, amplitude, and cumulation measures for the same time series. In fact, Hess and Iwata (1997) found that a simple ARIMA(1,1,0) model fit most features of the economy better than those specifically designed to match asymmetries in the data including Markov switching models in mean. Likewise Harding and Pagan (2002) show that an ARIMA(1,1,0) model was able to nearly outperform a non-linear Markov switching model proposed by Hamilton (1989) with regard to fitting the estimated features of the economy.

Table 2A: U.S. Business Cycle Peaks and Characteristics by Algorithm (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter
Cycle Peak 1	1948	4	1948	4	1948	4	1948	4	1948	4	1948	4
Cycle Peak 2	1953	2	1953	2	1953	2	1953	2	1953	2	1953	2
Cycle Peak 3	1957	3	1957	3	1956	2	1956	2	1956	2	1956	2
Cycle Peak 4	1960	1	1960	1	1957	3	1957	3	1957	3	1957	3
Cycle Peak 5	1969	3	1969	3	1960	1	1960	1	1960	1	1960	1
Cycle Peak 6	1973	4	1973	4	1970	3	1970	3	1970	3	1970	3
Cycle Peak 7	1980	1	1980	1	1973	4	1973	4	1973	4	1973	4
Cycle Peak 8	1981	3	1981	3	1977	3	1977	3	1977	3	1977	3
Cycle Peak 9	1990	3	1990	3	1981	3	1980	1	1981	3	1980	1
Cycle Peak10					1990	3	1981	3	1990	3	1981	3
Cycle Peak11					2001	2	1990	3	2001	2	1990	3
Cycle Peak12							2001	2			2001	2

	Cycle Characteristics											
	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P
Durations	2.8889	16.4444	2.8889	16.4444	2.0909	17.5455	2.0833	15.9167	2.0909	17.5455	2.0833	15.9167
Amplitudes(%)	-2.2380	18.9260	-2.2380	18.9260	-1.7135	18.4390	-1.7545	17.0863	-1.7135	18.4390	-1.7545	17.0863
Cumulative(%)	-2.9379	238.2669	-2.9379	238.2669	-2.0560	260.9763	-2.1465	236.1779	-2.0560	260.9763	-2.1465	236.1779
Excess(%)	0.0244	1.0412	0.0244	1.0412	0.0597	0.8959	0.0157	0.8732	0.0597	0.8959	0.0157	0.8732

Table 2B: U.K. Business Cycle Peaks and Characteristics by Algorithm (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter
Cycle Peak 1	1955	3	1955	3	1955	3	1955	3	1955	3	1955	3
Cycle Peak 2	1958	1	1958	1	1958	1	1958	1	1958	1	1958	1
Cycle Peak 3	1961	2	1961	2	1960	1	1960	1	1960	1	1960	1
Cycle Peak 4	1973	2	1973	2	1961	2	1961	2	1961	2	1961	2
Cycle Peak 5	1974	3	1974	3	1964	4	1962	3	1964	4	1962	3
Cycle Peak 6	1979	2	1979	2	1966	3	1964	4	1966	3	1964	4
Cycle Peak 7	1990	2	1990	2	1968	1	1966	3	1968	1	1966	3
Cycle Peak 8					1970	4	1968	1	1970	4	1968	1
Cycle Peak 9					1973	2	1969	4	1973	2	1969	4
Cycle Peak10					1974	3	1970	4	1974	3	1970	4
Cycle Peak11					1977	1	1971	4	1977	1	1971	4
Cycle Peak12					1979	2	1973	2	1979	2	1973	2
Cycle Peak13					1982	2	1974	3	1982	2	1974	3
Cycle Peak14					1984	1	1977	1	1984	1	1977	1
Cycle Peak15					1990	2	1979	2	1990	2	1979	2
Cycle Peak16							1982	2			1982	2
Cycle Peak17							1984	1			1984	1
Cycle Peak18							1990	2			1990	2

	Cycle Characteristics											
	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P
Durations	3.7143	19.6667	3.7143	19.6667	2.0667	8.0714	1.8889	6.4706	2.0667	8.0714	1.8889	6.4706
Amplitudes(%)	-2.7174	16.9030	-2.7174	16.9030	-1.6087	7.6091	-1.4152	6.3452	-1.6087	7.6091	-1.4152	6.3452
Cumulative(%)	-6.3894	258.2457	-6.3894	258.2457	-3.0887	41.1670	-2.6112	31.2483	-3.0887	41.1670	-2.6112	31.2483
Excess(%)	-0.0406	-0.2523	-0.0406	-0.2523	-0.0163	-0.2132	-0.0136	0.0377	-0.0163	-0.2132	-0.0136	0.0377

Table 2C: Australia Business Cycle Peaks and Characteristics by Algorithm (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter	Year	Quarter
Cycle Peak 1	1960	3	1960	3	1960	3	1960	3	1960	3	1960	3
Cycle Peak 2	1974	1	1974	1	1965	3	1965	3	1965	3	1965	3
Cycle Peak 3	1981	4	1981	4	1974	1	1974	1	1974	1	1974	1
Cycle Peak 4	1990	2	1990	2	1981	4	1981	4	1981	4	1981	4
Cycle Peak 5					1985	4	1985	4	1985	4	1985	4
Cycle Peak 6					1990	2	1990	2	1990	2	1990	2
Cycle Peak 7					2000	3	2000	3	2000	3	2000	3

Cycle Characteristics

	(2,2,5)		(2,2,4)		(1,1,5)		(1,2,4)		(1,2,5)		(1,1,4)	
	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P	P→T	T→P
Durations	4.0000	36.0000	4.0000	36.0000	2.7143	23.6667	2.7143	23.6667	2.7143	23.6667	2.7143	23.6667
Amplitudes(%)	-1.9868	38.6480	-1.9868	38.6480	-1.1728	25.2807	-1.1728	25.2807	-1.1728	25.2807	-1.1728	25.2807
Cumulative(%)	-3.9013	842.6579	-3.9013	842.6579	-2.2481	348.9563	-2.2481	348.9563	-2.2481	348.9563	-2.2481	348.9563
Excess(%)	0.0782	1.3878	0.0782	1.3878	0.0447	0.4702	0.0447	0.4702	0.0447	0.4702	0.0447	0.4702

As can be seen in Table 2A, in the U.S. if turning points are determined by the methods suggested in Harding and Pagan (2002) then there are nine recorded significant downturns in economic activity. Most notably omitted from the business cycle measurements a (2,2,5) search algorithm fails to detect the 2001 recession. The failure to detect the mild recessions that are experienced under the "Great Moderation" might be a fatal flaw of using a dating algorithm to determine turning points in the level of GDP. The failure of a (2,2,5) algorithm to capture business cycles that may be large declines in economic activity even if only lasting a single quarter becomes a major problem for this algorithm and the models employed here. A major drawback to using the (2,2,5) algorithm (or any other algorithm used here for that matter) is that it only calculates measurements for complete cycles. Thus, for the U.S. the period between the last detected trough in 1991 spans 68 quarters to date and is not included in the averages for cycle duration length, cycle amplitude, cumulative growth, or excess. The cycle features produced by a model (if correctly specified) will have increasing difficulty matching an increasingly irrelevant set of statistics since the reference cycle will be calculating averages over two fundamentally different patterns. One could update the algorithm to include currently occurring business cycles into the model, but it might make more sense to get a model to fit the data using an algorithm with statistics that update more frequently, such as at each downturn.

Using the (2,2,5) algorithm to date the U.S. business cycle the typical "recession" length is 2.89 quarters, and results in a loss of 2.93% of accumulated output.[16] Recessions also appear to be very close to linear with only 0.024% of excess. A typical U.S. "expansion" is around 16.44 quarters in length and gains 238% of accumulated output on average, which is in excess of the triangle approximation.

In order to capture a more complete picture of the economy, it is worthwhile to examine the alternatives to a (2,2,5) dating algorithm and examine their performance side-by-side. All reasonable variations with regard to classical business cycle dating on the algorithm suggested by Harding and Pagan (2002) are presented in Tables 2A-2C to show which possible changes have an influence on the detection of business cycles. The most influential changes for each of these countries is adjustment of the phase length and the minimum cycle length.

If a model of the economy should capture the single large shocks as well as the long protracted downturns/expansions it may help to use an algorithm

[16] See Table 2A, column 1

with single quarter phases such as a (1,1,5)[17] or a (1,1,2)[18] algorithm to detect turning points.[19]

As can be seen in Table 2A-C, the detection of cycle peaks differs somewhat depending on the type of algorithm used.[20] For the U.S., it is notable that the duration length of expansions (Trough to Peak) for a (1,1,5) algorithm is longer than for a (2,2,5) reflecting the fact that the (1,1,5) algorithm picks up one of the longer cycles recorded between 1990 and 2001 which increases the average length of the reference cycle. This plays a major role when determining how effective a model is when comparing simulation data to actual output measures. If one uses a (1,1,5) dating algorithm in the U.S., eleven peaks are detected over the entire series compared to the nine detected using a (2,2,5) algorithm. In the U.K. and Australia series, the (1,1,5) algorithm detects 15 and 7 peaks over the series, versus the 7 and 4 respectively detected using the (2,2,5) algorithm.

Another alternative to the (2,2,5) reference cycle would be to use a shorter total cycle length such as two quarters. The desire of Harding and Pagan (2002) to remain true to Burns and Mitchell (1946) is reasonable when considering the "typical" shape of recessions prior to the shift in volatility. However, given the large change in volatility, it is possible that another method of fitting models to the data may be necessary. An example of using a non-traditional measure of business cycles is used in Hess and Iwata (1997) where each local peak and trough is counted as a phase. In their work, Hess and Iwata (1997) use an algorithm that signals a downturn whenever output falls below the "running peak" of output. All periods which are not considered contractions are then declared expansions (or peaks). In the spirit of Hess and Iwata (1997) and Harding and Pagan (2002) an alternative business cycle algorithm of (1,1,2) with a minimum cycle length of two quarters. Tables 3A-3C display the reference cycles for each the U.S., the

[17] This algorithm would detect a peak if all of the following hold true: $\Delta y_{t-1} > 0, \Delta y_{t+1} < 0$, the phase lasts a single quarter, and the entire cycle lasts five quarters.

[18] A (1,1,2) algorithm is the same as a (1,1,5) except the entire cycle only needs to last two quarters. This limitation allows for the same peaks and troughs as before, but is more subject to the censoring procedures.

[19] Other choices for the algorithm include a (2,2,4) which was used by Harding and Pagan (2002) for the U.K., a (1,2,4), or any other combination for that matter. The intuitive reasoning behind using minimum phase lengths of five quarters is attributed to Burns and Mitchell (1946) and others as it typically coincides with what are typically thought of as business cycles.

[20] The choice to focus on peaks here is that they are typically more remarkable and memorable as they signal the beginning of a recession or downturn. One could have just as easily reported the dates for troughs which typically coincide with the announcement of the end of a recession.

U.K., and Australia using a dating algorithm of (1,1,2). For comparison, Hess and Iwata (1997) calculate 18 complete cycles between 1949 and 1992, which compares to 20 using the (1,1,2) algorithm.

Table 3A: U.S. Business Cycle Peaks and Characteristics for Algorithm (1,1,2), (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(1,1,2)	
	Year	Quarter
Cycle Peak 1	1948	4
Cycle Peak 2	1949	3
Cycle Peak 3	1953	2
Cycle Peak 4	1955	4
Cycle Peak 5	1956	2
Cycle Peak 6	1957	1
Cycle Peak 7	1957	3
Cycle Peak 8	1959	2
Cycle Peak 9	1960	1
Cycle Peak10	1960	3
Cycle Peak11	1969	3
Cycle Peak12	1970	3
Cycle Peak13	1973	2
Cycle Peak14	1973	4
Cycle Peak15	1974	2
Cycle Peak16	1977	3
Cycle Peak 17	1980	1
Cycle Peak 18	1981	1
Cycle Peak 19	1981	3
Cycle Peak 20	1982	2
Cycle Peak 21	1990	3
Cycle Peak 22	2000	2
Cycle Peak 23	2000	4
Cycle Peak 24	2001	2

	Cycle Characteristics	
	(1,1,2)	
Durations	1.4167	7.5833
Amplitudes(%)	-1.0789	8.7447
Cumulative(%)	-1.0494	101.6148
Excess(%)	-0.0145	0.2383

Table 3B: U.K. Business Cycle Peaks and Characteristics for Algorithm (1,1,2), (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(1,1,2)	
	Year	Quarter
Cycle Peak 1	1955	3
Cycle Peak 2	1956	1
Cycle Peak 3	1957	1
Cycle Peak 4	1958	1
Cycle Peak 5	1960	1
Cycle Peak 6	1961	2
Cycle Peak 7	1962	3
Cycle Peak 8	1964	4
Cycle Peak 9	1966	3
Cycle Peak10	1968	1
Cycle Peak11	1969	4
Cycle Peak12	1970	4
Cycle Peak13	1971	4
Cycle Peak14	1973	2
Cycle Peak15	1974	3
Cycle Peak16	1975	1
Cycle Peak 17	1976	1
Cycle Peak 18	1977	1
Cycle Peak 19	1978	4
Cycle Peak 20	1979	2
Cycle Peak 21	1979	4
Cycle Peak 22	1981	3
Cycle Peak 23	1982	2
Cycle Peak 24	1984	1
Cycle Peak 25	1990	2
Cycle Peak 26	1992	1

	Cycle Characteristics	
	(1,1,2)	
Durations	1.5385	4.2800
Amplitudes(%)	-1.1539	4.4987
Cumulative(%)	-1.4308	20.0276
Excess(%)	-0.0123	0.0326

Table 3C: Australia Business Cycle Peaks and Characteristics for Algorithm (1,1,2), (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

	(1,1,2)	
	Year	Quarter
Cycle Peak 1	1960	3
Cycle Peak 2	1965	3
Cycle Peak 3	1974	1
Cycle Peak 4	1981	4
Cycle Peak 5	1985	4
Cycle Peak 6	1990	2
Cycle Peak 7	2000	3
	Cycle Characteristics	
	(1,1,2)	
Durations	2.7143	23.6667
Amplitudes(%)	-1.1728	25.2807
Cumulative(%)	-2.2481	348.9563
Excess(%)	0.0447	0.4702

3 Simulation Methods

In order to compare the effectiveness of the dating algorithms across models, a simple random walk with drift model for growth is estimated $\Delta y_t = \phi_o + \epsilon_t$ is estimated first. Additionally a model without a volatility break is estimated using GMM with two equations as in (1) and (2) for comparison. Lastly, models with variance breaks were estimated for each country. For each of the models, 5,000 replications were conducted using random draws from a normal distribution corresponding to the standard deviation of the models that were estimated. For each of these models it was assumed that there was no parameter uncertainty. Also, since the assumption of normally distributed errors and parameter uncertainty may come into question, the models were also simulated using residual resampling from the estimated models. In one case using only the residuals from the first equation to simulate the model, and in another case using the residuals for each equation to simulate the model.[21] The latter method of residual resampling approximates the possibility that there are random shocks to the volatility of shocks over time.

In order to compare the results from each of the estimated models with switching volatility, counterfactual estimation is conducted for each model.[22]

1. Examine simulations without breaks in volatility.
2. Apply pre-break standard deviation to the entire simulation.
3. Apply post-break standard deviation to the entire simulation.
4. Apply pre- and post-break standard deviations to their respective sample lengths and simulate ("Switching").
5. Reverse pre- and post-break standard deviations and apply to respectively opposite sample lengths and simulate.

After *each* of the simulations is conducted, summary statistics are calculated using the dating algorithms and stored for each run. The simulation statistics are then collected and their distributions are compared to the values estimated as the reference cycles calculated by using the same dating algorithm on the actual data sample.

[21] Standardized residuals were used for the resampling simulations.

[22] The counterfactual estimation in the context presented here displays the features of the simulation if given a long enough period to act upon. For example, in the U.S. using a (2,2,5) algorithm, cycles are detected frequently when average volatility was high. However, as results from the simulation show if given a longer period of time the algorithm may not detect any peaks or troughs at all.

3.1 Basic Simulation Results: Fitting the Data

For each model and country there are a number of possible statistics to report, but the most important would be to find the models that simulate data that do not resemble the actual reference cycles for duration, amplitudes, cumulation, and excess. The dating algorithms used in the Monte Carlo simulation are the (2,2,5), (1,1,5), and (1,1,2) algorithms. Each set of simulations is compared to the stylized facts and if the actual value from the data falls outside of a 90% or 95% confidence interval it is shown that the tested data generating process rejects a "fact" of the reference cycle. In the case of this study with the simple data generating processes examined, it is not likely the models truly fit all of the stylized facts of a referency cycle, but rather the reasoning is to build a case for using one algorithm relative to another.

3.1.1 U.S.

The results for the U.S. reveal a definite need to try either another dating algorithm to fit the business cycle in the face of the reduced volatility. Using a (2,2,5) reference cycle, a simple random walk model is almost as successful as the GMM model as estimated with no breaks in variance. The model with no breaks only fails to replicate the amplitudes of contractions and the asymmetry of expansions. Once switching to a model of the economy that simultaneously estimates the standard deviation according to equation (2) without a break in variance, the model appears to outperform the simple random walk but still fails at matching the amplitudes during contractions by underestimating them, as well as overestimating the excess exhibited during contraction. In the simulations that resample residuals by equation rather than using the estimated volatility, none of these models performs particularly well, with all models being far off the mark for a number of characteristics.

The most important results from Appendix Tables 1A-1C are in the fifth and sixth rows which use values from the models with estimated breaks in volatility to simulate the equations. If using only the first period variance, the (1,1,5) model performs quite well when tracking peak to trough movements, but matches none of the characteristics of the expansionary movements. The (1,1,2) model faces similar difficulties, being much more effective when used to locate turning points during recessionary periods. The worst performance of all basic simulations for the U.S. is exhibited by the (2,2,5) algorithm which performs fantastically when examining the movements un-

der the pre-moderation volatility level. However, once the post-moderation level of volatility is used to model the economy, the (2,2,5) algorithm goes through 1,232 (24.6%) simulations without detecting a full trough to peak measurement, and 7.6% of movements do not have either a trough to peak or vice versa.[23]

3.1.2 U.K.

The results for the U.K. highlight the shortcomings of using the models and algorithms here to estimate the business cycle. The (2,2,5) algorithm receives very mixed results when attempting to fit simulations using the GMM model without accounting for a volatility break. Once the break has been accounted for, and the simulations are run using the second period (post-moderation) volatility, the algorithm fails to detect a single complete cycle in 5,000 simulations. The (1,1,5) and (1,1,2) algorithms also attain only mixed results when attempting to fit the data to models that do not include a break in volatility with the model not fitting any full set of statistics for both the peak-to-trough or trough-to-peak cycles. While this may be good evidence that the model in use is a poor choice, it does not account for the failure of different models to create simulations without cycles. When examining the GMM model with variance break and simulations set at the lower post-moderation level of volatility, the (1,1,5) and (1,1,2) algorithms detect cycles in just over 53% of simulations.

3.1.3 Australia

The result of the simulation data returning no valid peaks or troughs is a running theme throughout the simulations, but is an especially apparent problem for Australia. The problem can be seen in comparing Table 2C to Table 3C which uses the (1,1,2) algorithm in order to date the business cycles of the actual data series. The (1,1,2) algorithm detects the same number of peaks and troughs as a (1,1,5) model meaning the actual series has typically very long cycles. The only factor changing the number of cycles between a (2,2,5) and a different model is a minimum phase length of two censors three peaks from the analysis which would otherwise exist. Very short downturns in 1965, 1985, and 2000 are detected using any model with a phase length

[23] In almost all cases, when an algorithm picks up zero peak to trough movements in a series, there are also no trough to peak movements. Thus in the example here, 1,232 of 5,000 simulations had zero trough to peak movements, and 379 missing peak to trough movements, but 853 (the difference) had measurable peak to trough movements.

shorter than two. Using a minimum phase length of two or a total cycle length of five leads to contractions averaging 2.7 quarters, and expansions averaging 23.7 quarters (or almost 6 years).

The problems inherent in the Australian data and models selected to study here show that the (2,2,5) algorithm has severe problems. Using a random walk model for growth with a mean value of 0.8940 and a standard deviation of 0.4514, the probability of observing a negative shock larger than $(\frac{\phi_0}{\sigma})$ would occur in only 2.3% of periods. For the length of the Australian data sample, a downturn would be expected to occur during approximately 4-5 periods per simulation. However, if the period following the downturn experienced a shock of zero (occurring with 50% probability) the downturn would not register as an official peak using a (2,2,5) algorithm. Additionally, in order to have a valid peak-to-peak measurement, a second valid peak would need to occur before the end of the simulation. Thus, it should be no surprise that there are simulations missing either a trough-to-peak, a peak-to-trough, or both.

The (2,2,5) algorithm fails to detect any complete cycles within over 99% of all random walk models. Using residual resampling creates simulations with nearly 100% failing to register a full cycle. The (2,2,5) algorithm fares better against the GMM model estimated with no breaks, but still fails to register a cycle in over 13% of simulations. The GMM model estimated to detect a break in volatility results in almost 95% success when using the pre-moderation volatility level. It might also be noted that those models that successfully register a complete cycle are fit rather well by the (2,2,5) algorithm hitting the mark on all cycle features. However, using the post-moderation level of volatility results in almost all simulations (96%) failing to register a complete cycle.

The (1,1,5) algorithm performs much better for the random walk and GMM estimated model but also performs terribly when using the resampled residuals. If the standardized residuals are resampled from the model without a break, all 5000 simulations fail to register a valid full cycle. The GMM model with a volatility break using the pre-moderation level of volatility fares rather well, picking up nearly 100% of simulations with valid cycles. However, when using the post-moderation period volatility for the full simulation, over 80% of simulations fail to register a valid full cycle.[24]

Even using the (1,1,2) algorithm with very short cycle lengths fails to

[24]It should be noted again, that if a peak or trough occurred in the middle of a simulation without another corresponding peak or trough within the same simulation that a full cycle would not be measured using the algorithm.

detect a full cycle in over 77% of GMM estimated models when setting the volatility equal to the post-moderation volatility level. In fact, it appears that in the case of Australia and the U.K., the "Great Moderation" matters quite a bit when trying to use classical cycle dating algorithms to fit a simulation of the model. The moderation itself may not be to blame for the failure of the algorithms, but it might rather be the failure of the model itself to replicate the facts that are detected using the different dating algorithms.

3.2 Switching and Counterfactual Results

In the previous estimations, no effort was made to model the entire history of the time series with a switching volatility, so additional simulations were run where the volatility was altered.[25] Additionally, the volatility levels were reversed allowing for a longer period of low volatility in the estimation to see if the reference cycle itself still applies to data modeled with different shocks.

3.2.1 U.S.

For the U.S. in the switching models, the (2,2,5) and (1,1,2) algorithms perform rather well, fitting almost all characteristics of the reference cycle, missing only excess characteristics. The (1,1,5) algorithm fails at detecting expansions in the U.S. but performs well relative to contractions. All dating schemes perform quite well when switching and reversing the volatility measures or shocks. The (2,2,5) algorithm fails at producing excesses when the low-volatility period is extended in the sample, while the (1,1,2) algorithm fares just as well. The concern of these results is that without consideration of the pre-moderation period volatility, the algorithms fail at detecting reference cycles.

It might also be considered that the mean growth rates or persistence may have changed simultaneously with a volatility shift likely leading to different results. However, as shown in Table 1A, the Wald tests for a parameter shift in the constant and lagged growth coefficient show only marginal evidence of a shift, and occurred at a time nowhere near the estimated moderation in volatility. Further work testing independent multiple breaks on the mean and persistence parameters may signal differing break dates and

[25]The Appendix Tables 2A-2C display the results from the switching and counterfactual simulations. The first two rows show the results from switching and counterfactual estimations for each reference cycle.

thus allow the ARIMA model to be a better fit to the currently modeled reference cycle.

3.2.2 U.K.

In the U.K. when the ARIMA model with switching variance is simulated, the simulations have trouble replicating the (2,2,5) and (1,1,2) contractionary reference cycles, predicting recessions that are too short, and too small. The (1,1,5) reference cycle fits the simulated data somewhat better but still fails to produce features that fall within even a 95% confidence interval. Similar results are obtained when comparing the simulated data from the model where volatilities from each period were reversed. Each reference cycle fails to match up unanimously with the produced data. Also, the simulated data from reversing volatilities results in over 400 simulations where there are no measurable peaks or troughs produced.

The best fit occurs for the (1,1,5) reference cycle to simulations where the residuals are resampled rather than using the estimated standard deviations. The (2,2,5) and (1,1,2) reference cycles fit these simulations rather nicely as well, missing only a few components. The ARIMA model for the U.K. actually shows marginal evidence of a break in persistence along with a volatility break. This model is left for future testing.

3.2.3 Australia

For the volatility switching models in Australia, all reference cycles fit the simulated data in all features. Also, there were a number of simulations that produced no full cycles. However, if the models with no cycles are not excluded, all simulations still fit the reference cycles.[26] All reference cycles are matched when reversing the volatilities between periods one and two. The biggest drawback to any of the reference cycles is that the (2,2,5) algorithm passes over 2,200 simulations without finding full trough-to-peak movements. The other reference cycle results here are substantially better at finding complete cycles.

As noted for the U.K., the results for Australia show that simple methods of dating cycles and simple models are problematic and do not always yield good results. However, the simple models do show some effectiveness at replicating reference cycles when allowing for switches in volatility.

[26] Results not reported.

4 Conclusion

The results from the simulations reported here are not an implication that the "business cycle is dead," but rather that either the models chosen here are poor fits to the reference cycles determined by the data, or that the dating algorithms and reference cycles need to be updated or abandoned. Also, the literature on classical cycles is unclear about how to correct dating algorithms and reference cycles in the face of parameter shifts such as the variance of shocks. What is clear, is that some sort of volatility moderation has occurred and that models that fit the reference cycles quite well at one time have extreme difficulty fitting the reference cycles following the moderation in volatility.

What is apparent from the research on business cycles is that their features change quite a bit. While all economies are different, and might be modeled differently, the methods for dating business cycles should apply broadly. If the Great Moderation is only temporary, then it might be expected that a simple (2,2,5) algorithm would continue to be useful. However, there is evidence using the ARIMA(1,1,0) model here that each of the countries studied experienced a downward break in volatility without a change in either the mean or persistence parameters. After accounting for the volatility break, the (2,2,5) reference cycle becomes very hard to replicate, resulting in many simulations without any full cycle measurements. In these instances, other dating algorithms such as the (1,1,5) and (1,1,2) are more effective. While these models are possibly more effective at replicating the different reference cycles noted here, these reference cycles do not have the common interpretation of a business cycle.

Evidence here shows that these ARIMA models perform quite poorly when trying to fit reference cycles measured by a variety of dating algorithms. There is intuitive appeal to a simple method for measuring business cycles and a simple model for the economy. However, in a world where volatility is more moderate, these simple models and dating methods encounter substantial problems when used together. If the volatility moderation continues as one might expect, new methods or models must be used to fit the economy. Non-linear models such as Hamilton's Markov-switching models in mean and volatility might be a solution, but these models must also be updated to add features when the stylized facts change, as shown by McConnell and Perez-Quiros (2000). The Hamilton models may result in estimating low-growth mean states (the recessionary state in the model) with positive rates of growth, which does not match the common interpretation of a recession.

With the results here, there is not broad evidence that tweaks to the dating algorithms helps to create reference cycles for the U.S., U.K., and Australia that fit a simple economic model accounting for breaks in volatility. However, there is some improvement when comparing the simulations of these models to more forgiving reference cycles.

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A Appendix Figures

B Appendix Tables

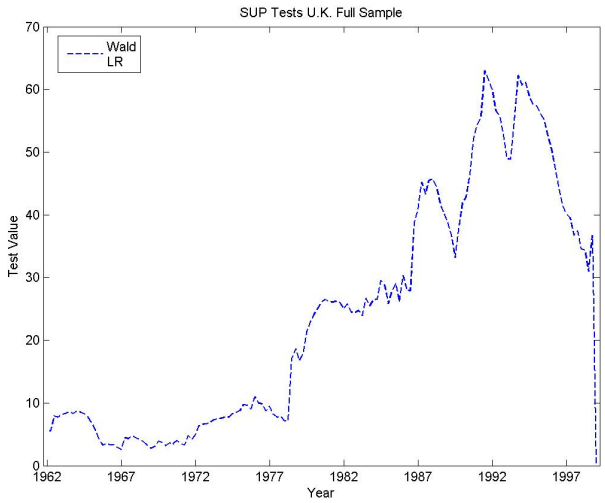

Figure 7: UK Wald Sup Tests for a Single Break

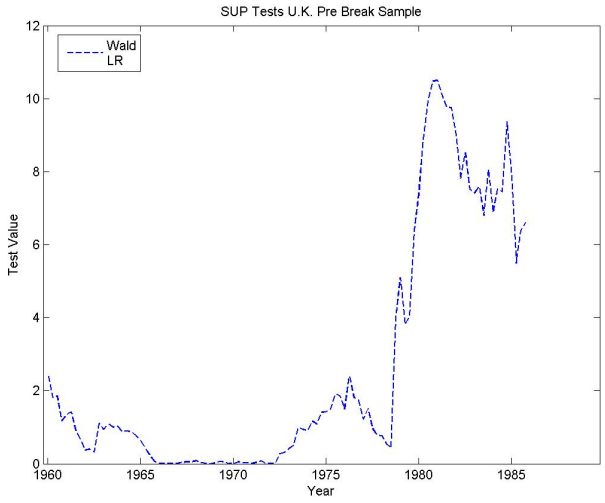

Figure 8: UK Wald Sup Tests for Period Before Break

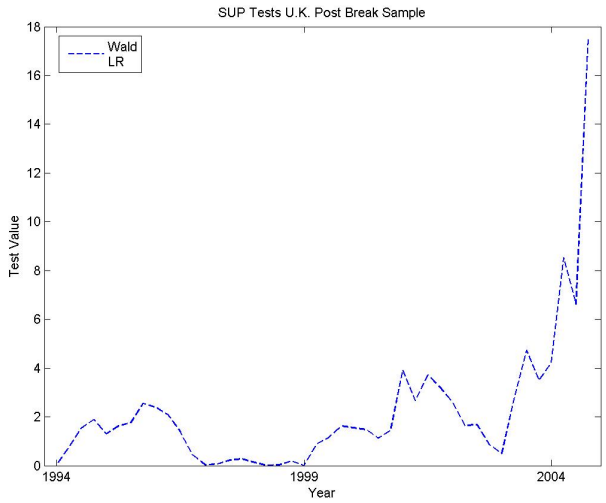

Figure 9: UK Wald Sup Tests for Period After Break

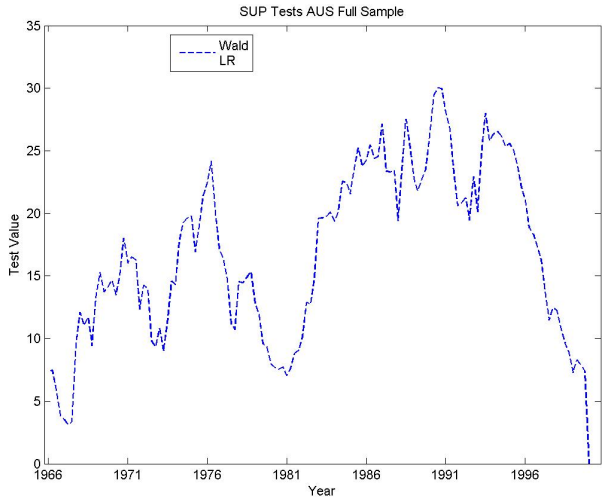

Figure 10: AUS Wald Sup Tests for a Single Break

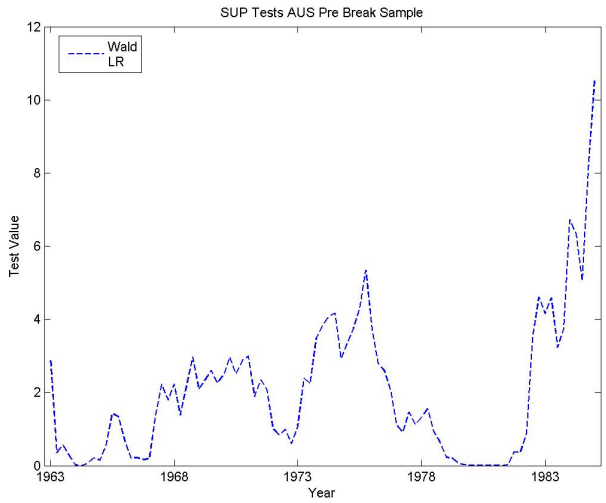

Figure 11: AUS Wald Sup Tests for Period Before Break

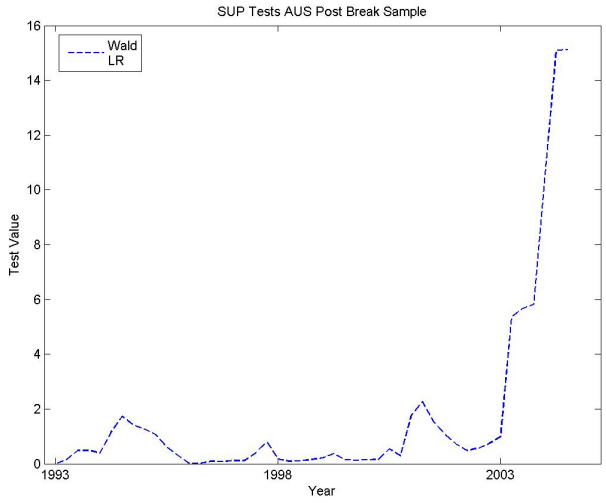

Figure 12: AUS Wald Sup Tests for Period After Break

Appendix Table 1A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	Random Walk (1)	P→T	Durations	1.4759	99%	1%	1.1304	1.1752	1.4545	1.8500	1.9545	2.0909	0	0
			Amplitudes(%)	-0.7726	0%	100%	-1.0864	-1.0203	-0.7627	-0.5460	-0.5124	-1.7135	0	0
			Cumulative(%)	-0.7431	0%	100%	-1.4675	-1.2857	-0.6918	-0.3786	-0.3367	-2.0560	0	0
			Excess(%)	0.0008	99%	1%	-0.0439	-0.0345	0.0008	0.0374	0.0448	0.0597	0	0
		T→P	Durations	9.3643	100%	0%	7.5486	7.7600	9.2381	11.4118	11.8824	17.5455	0	0
			Amplitudes(%)	9.7376	100%	0%	7.5555	7.8328	9.6025	12.1193	12.6737	18.4390	0	0
			Cumulative(%)	60.2443	100%	0%	34.5840	37.1976	57.0078	95.2352	106.0486	260.9763	0	0
			Excess(%)	0.0793	100%	0%	-0.2203	-0.1699	0.0750	0.3303	0.3841	0.8959	0	0
	GMM No Breaks (2)	P→T	Durations	1.8225	81%	19%	1.3000	1.3684	1.7895	2.3913	2.5000	2.0909	1	1
			Amplitudes(%)	-0.8466	0%	100%	-1.2732	-1.1917	-0.8298	-0.5485	-0.5044	-1.7135	0	0
			Cumulative(%)	-1.1666	7%	93%	-2.4957	-2.1950	-1.0642	-0.4910	-0.4187	-2.0560	1	1
			Excess(%)	0.0010	98%	2%	-0.0583	-0.0474	0.0013	0.0475	0.0597	0.0597	0	0
		T→P	Durations	10.7650	100%	0%	8.1779	8.5000	10.5789	13.6667	14.5833	17.5455	0	0
			Amplitudes(%)	11.0716	100%	0%	7.8841	8.2824	10.8158	14.7487	15.7687	18.4390	0	0
			Cumulative(%)	86.7430	100%	0%	41.2974	45.6881	79.7772	153.4816	174.3451	260.9763	0	0
			Excess(%)	0.0752	100%	0%	-0.2778	-0.2142	0.0740	0.3673	0.4337	0.8959	0	0
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.7558	85%	15%	1.2143	1.2857	1.7222	2.3529	2.5333	2.0909	1	1
			Amplitudes(%)	-1.7081	45%	55%	-3.0247	-2.7516	-1.6348	-0.9130	-0.8166	-1.7135	1	1
			Cumulative(%)	-2.7535	62%	38%	-6.8422	-5.7574	-2.3930	-0.8864	-0.7322	-2.0560	1	1
			Excess(%)	-0.1416	99%	1%	-0.3947	-0.3380	-0.1263	0.0023	0.0226	0.0597	0	0
		T→P	Durations	13.7532	91%	9%	9.6000	10.0556	13.3333	18.8182	20.5000	17.5455	1	1
			Amplitudes(%)	14.3305	89%	11%	9.2646	9.8895	13.8569	20.5230	22.2878	18.4390	1	1
			Cumulative(%)	154.5226	91%	9%	59.7630	66.7461	135.3638	309.2884	369.9246	260.9763	1	1
			Excess(%)	-0.0695	100%	0%	-0.6622	-0.5543	-0.0733	0.4324	0.5688	0.8959	0	0
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.6409	92%	8%	1.1765	1.2308	1.6000	2.1765	2.2857	2.0909	1	1
			Amplitudes(%)	-0.8683	0%	100%	-1.3184	-1.2328	-0.8465	-0.5758	-0.5310	-1.7135	0	0
			Cumulative(%)	-1.0645	5%	95%	-2.3435	-2.0378	-0.9554	-0.4512	-0.3952	-2.0560	0	1
			Excess(%)	-0.0164	99%	1%	-0.0814	-0.0673	-0.0155	0.0322	0.0416	0.0597	0	0
		T→P	Durations	12.9407	96%	4%	9.3333	9.7500	12.6000	17.2500	18.5455	17.5455	0	1
			Amplitudes(%)	12.7200	97%	3%	8.6771	9.1183	12.3211	17.6612	18.8899	18.4390	0	1
			Cumulative(%)	125.6632	97%	3%	53.4799	59.4001	111.6803	237.9308	278.6938	260.9763	0	1
			Excess(%)	-0.0155	100%	0%	-0.4357	-0.3600	-0.0173	0.3403	0.4119	0.8959	0	0
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	2.3651	26%	75%	1.6400	1.7391	2.3333	3.0607	3.2500	2.0909	1	1
			Amplitudes(%)	-1.5745	31%	69%	-2.2845	-2.1619	-1.5545	-1.0743	-0.9839	-1.7135	1	1
			Cumulative(%)	-2.8733	75%	25%	-5.7351	-5.1531	-2.6896	-1.3107	-1.1065	-2.0560	1	1
			Excess(%)	0.0008	89%	11%	-0.0956	-0.0793	0.0010	0.0807	0.0951	0.0597	1	1
		T→P	Durations	8.7542	100%	0%	6.8696	7.1154	8.6432	10.7222	11.2941	17.5455	0	0
			Amplitudes(%)	10.2067	100%	0%	7.4664	7.8427	10.0234	13.1654	14.0136	18.4390	0	0
			Cumulative(%)	61.9786	100%	0%	32.8458	35.5566	58.0194	101.1399	114.4019	260.9763	0	0
			Excess(%)	0.0758	100%	0%	-0.2892	-0.2192	0.0745	0.3792	0.4334	0.8959	0	0
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.2446	100%	0%	1.0000	1.0000	1.2000	1.6250	1.7500	2.0909	0	0
			Amplitudes(%)	-0.2633	0%	100%	-0.4627	-0.4198	-0.2518	-0.1392	-0.1179	-1.7135	0	0
			Cumulative(%)	-0.2096	0%	100%	-0.5530	-0.4493	-0.1751	-0.0753	-0.0636	-2.0560	0	0
			Excess(%)	-0.0001	100%	0%	-0.0248	-0.0190	0.0000	0.0181	0.0233	0.0597	0	0
		T→P	Durations	23.4910	19%	81%	13.7692	14.8091	21.8889	37.4000	42.6000	17.5455	1	1
			Amplitudes(%)	19.4439	53%	47%	10.9202	11.7732	18.1040	31.8023	36.4908	18.4390	1	1
			Cumulative(%)	394.7882	40%	60%	106.6265	124.8250	305.9526	967.5108	1258.2936	260.9763	1	1
			Excess(%)	0.0524	99%	1%	-0.5458	-0.4117	0.0521	0.5051	0.6501	0.8959	0	0

Appendix Table 1A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	Random Walk (1)	P→T	Durations	2.2416	95%	6%	1.5833	1.6667	2.2000	2.9091	3.1429	2.8889	1	1
			Amplitudes(%)	-1.2468	0%	100%	-1.7678	-1.6556	-1.2337	-0.8821	-0.8113	-2.2380	0	0
			Cumulative(%)	-1.5127	3%	97%	-2.9476	-2.6159	-1.4016	-0.7913	-0.7042	-2.9379	0	1
			Excess(%)	-0.0003	69%	31%	-0.1103	-0.0898	-0.0004	0.0872	0.1039	0.0244	1	1
		T→P	Durations	23.4116	15%	85%	13.0000	14.1429	21.5000	39.0000	44.3333	16.4444	1	1
		Missing	Amplitudes(%)	22.3889	38%	62%	12.3971	13.3818	20.6117	37.3985	42.5018	18.9260	1	1
		1	Cumulative(%)	461.8365	26%	74%	114.5813	135.5665	342.6013	1159.5202	1478.7186	238.2669	1	1
			Excess(%)	0.0264	99%	1%	-0.8693	-0.6656	0.0219	0.6964	0.8595	1.0412	0	0
	GMM No Breaks (2)	P→T	Durations	2.6170	75%	26%	1.8182	1.9091	2.5556	3.4545	3.7143	2.8889	1	1
			Amplitudes(%)	-1.3180	1%	99%	-1.9888	-1.8642	-1.2900	-0.8848	-0.8206	-2.2380	0	0
			Cumulative(%)	-2.0982	16%	84%	-4.7167	-3.9597	-1.9009	-0.9359	-0.8150	-2.9379	1	1
			Excess(%)	0.0009	68%	32%	-0.1103	-0.0887	0.0002	0.0908	0.1113	0.0244	1	1
		T→P	Durations	21.0098	25%	75%	12.3750	13.1909	19.6000	33.3333	37.2917	16.4444	1	1
			Amplitudes(%)	20.5402	49%	51%	11.5959	12.3316	19.0673	33.3885	38.0243	18.9260	1	1
			Cumulative(%)	383.1763	36%	64%	102.2957	118.4617	295.8483	905.8852	1156.1105	238.2669	1	1
			Excess(%)	0.0102	99%	1%	-0.7969	-0.6347	0.0076	0.6666	0.8339	1.0412	0	0
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	2.3292	87%	13%	1.5000	1.6250	2.2727	3.2222	3.4773	2.8889	1	1
			Amplitudes(%)	-2.6855	68%	32%	-4.5932	-4.1944	-2.5754	-1.5242	-1.3628	-2.2380	1	1
			Cumulative(%)	-4.6702	74%	26%	-11.3487	-9.6127	-4.0947	-1.6820	-1.3755	-2.9379	1	1
			Excess(%)	-0.2469	96%	4%	-0.6669	-0.5790	-0.2266	0.0116	0.0548	0.0244	0	1
		T→P	Durations	24.6198	12%	88%	13.7033	14.6795	22.6667	41.2000	47.3750	16.4444	1	1
		Missing	Amplitudes(%)	24.6280	29%	71%	12.7744	13.9138	22.5178	42.1069	48.8827	18.9260	1	1
		3	Cumulative(%)	538.4562	21%	79%	127.8744	151.0469	394.1678	1422.1572	1846.6882	238.2669	1	1
			Excess(%)	-0.0996	97%	3%	-1.3376	-1.0471	-0.1096	0.8719	1.1485	1.0412	0	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	2.3094	89%	12%	1.5000	1.6000	2.2500	3.1667	3.4000	2.8889	1	1
			Amplitudes(%)	-1.3517	1%	99%	-2.0547	-1.8996	-1.3205	-0.9063	-0.8321	-2.2380	0	0
			Cumulative(%)	-1.9452	13%	87%	-4.3716	-3.6840	-1.7413	-0.8316	-0.7274	-2.9379	1	1
			Excess(%)	-0.0316	84%	16%	-0.1509	-0.1306	-0.0315	0.0668	0.0941	0.0244	1	1
		T→P	Durations	26.3680	9%	91%	14.2750	15.3333	24.0000	44.3667	52.0000	16.4444	1	1
		Missing	Amplitudes(%)	24.7216	30%	70%	12.6569	13.7343	22.3195	42.7829	49.7604	18.9260	1	1
		3	Cumulative(%)	589.8056	19%	81%	127.0222	153.6183	422.5947	1536.8998	2033.6507	238.2669	1	1
			Excess(%)	-0.0414	97%	3%	-1.0949	-0.8778	-0.0402	0.8061	1.0418	1.0412	0	1
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	3.0933	38%	63%	2.2000	2.3125	3.0625	4.0000	4.2000	2.8889	1	1
			Amplitudes(%)	-2.1361	38%	62%	-2.9975	-2.8501	-2.1097	-1.5211	-1.4234	-2.2380	1	1
			Cumulative(%)	-4.2351	78%	22%	-8.5149	-7.4772	-3.9203	-2.0628	-1.8027	-2.9379	1	1
			Excess(%)	0.0006	64%	36%	-0.1464	-0.1191	0.0012	0.1151	0.1388	0.0244	1	1
		T→P	Durations	13.4490	85%	15%	8.9000	9.4000	12.9231	19.5000	21.2222	16.4444	1	1
			Amplitudes(%)	15.0074	86%	14%	9.3962	10.0302	14.2896	22.2575	24.4667	18.9260	1	1
			Cumulative(%)	169.3106	83%	17%	61.3161	68.0419	142.0419	360.6416	432.2494	238.2669	1	1
			Excess(%)	0.0160	100%	0%	-0.5962	-0.4813	0.0153	0.5094	0.6182	1.0412	0	0
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	2.0462	91%	9%	1.0000	1.0000	2.0000	3.0000	3.5000	2.8889	1	1
		Missing	Amplitudes(%)	-0.5387	0%	100%	-1.0833	-0.9561	-0.5135	-0.2178	-0.1599	-2.2380	0	0
		379	Cumulative(%)	-0.5844	0%	100%	-1.7170	-1.3761	-0.4801	-0.1642	-0.1261	-2.9379	0	0
			Excess(%)	0.0011	74%	26%	-0.0978	-0.0767	0.0000	0.0794	0.1028	0.0244	1	1
		T→P	Durations	66.7568	5%	95%	10.2500	16.0000	56.5000	158.0000	184.0000	16.4444	1	1
		Missing	Amplitudes(%)	52.9623	10%	90%	7.1952	12.1793	44.4464	128.1700	149.1609	18.9260	1	1
		1,232	Cumulative(%)	2829.5788	9%	91%	37.1323	114.7656	1651.0438	10142.1454	13711.6028	238.2669	1	1
			Excess(%)	0.0286	85%	15%	-2.4620	-1.8566	0.0222	1.9575	2.4923	1.0412	1	1

Appendix Table 1A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	Random Walk (1)	P→T	Durations	1.2423	96%	4%	1.0833	1.1053	1.2353	1.4000	1.4390	1.4167	0	1
			Amplitudes(%)	-0.6566	0%	100%	-0.8580	-0.8239	-0.6517	-0.5098	-0.4883	-1.0789	0	0
			Cumulative(%)	-0.4860	0%	100%	-0.7641	-0.7130	-0.4707	-0.3186	-0.2964	-1.0494	0	0
			Excess(%)	0.0001	11%	89%	-0.0247	-0.0200	0.0001	0.0211	0.0254	-0.0145	1	1
		T→P	Durations	5.1484	99%	1%	3.8864	4.0455	5.0541	6.5667	6.9286	7.5833	0	0
			Amplitudes(%)	5.9334	99%	1%	4.3748	4.5694	5.8273	7.6565	8.0401	8.7447	0	0
			Cumulative(%)	27.6461	100%	0%	13.8874	15.1541	25.6728	46.4623	53.9153	101.6148	0	0
			Excess(%)	-0.0015	100%	0%	-0.1585	-0.1317	-0.0004	0.1246	0.1512	0.2383	0	0
	GMM No Breaks (2)	P→T	Durations	1.4605	42%	58%	1.1923	1.2273	1.4483	1.7343	1.8032	1.4167	1	1
			Amplitudes(%)	-0.6863	1%	99%	-0.9816	-0.9252	-0.6744	-0.4864	-0.4548	-1.0789	0	0
			Cumulative(%)	-0.7128	11%	89%	-1.3880	-1.2223	-0.6619	-0.3687	-0.3325	-1.0494	1	1
			Excess(%)	0.0004	19%	81%	-0.0356	-0.0291	0.0003	0.0299	0.0357	-0.0145	1	1
		T→P	Durations	6.8683	75%	25%	4.8611	5.0882	6.6923	9.2727	9.9261	7.5833	1	1
			Amplitudes(%)	7.4656	81%	19%	4.9582	5.2745	7.2574	10.4317	11.2169	8.7447	1	1
			Cumulative(%)	50.9054	96%	4%	21.5102	24.1927	45.6828	94.8022	111.9874	101.6148	0	1
			Excess(%)	-0.0007	98%	2%	-0.2209	-0.1782	-0.0013	0.1749	0.2086	0.2383	0	0
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.4265	51%	50%	1.1364	1.1765	1.4118	1.7143	1.7895	1.4167	1	1
			Amplitudes(%)	-1.3844	77%	23%	-2.3312	-2.1112	-1.3423	-0.7956	-0.7124	-1.0789	1	1
			Cumulative(%)	-1.7576	80%	20%	-3.9688	-3.4191	-1.5949	-0.6481	-0.5536	-1.0494	1	1
			Excess(%)	-0.1146	94%	6%	-0.3029	-0.2601	-0.1033	-0.0103	0.0060	-0.0145	1	1
		T→P	Durations	10.1539	10%	90%	6.6786	7.0769	9.8333	14.4226	15.8258	7.5833	1	1
			Amplitudes(%)	10.7904	24%	76%	6.6027	7.0476	10.4028	15.8312	17.5249	8.7447	1	1
			Cumulative(%)	109.4002	56%	44%	38.4048	44.6044	93.4845	226.6039	272.6439	101.6148	1	1
			Excess(%)	-0.0788	94%	7%	-0.4864	-0.4141	-0.0837	0.2747	0.3545	0.2383	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.3613	68%	34%	1.1071	1.1429	1.3478	1.6296	1.6957	1.4167	1	1
			Amplitudes(%)	-0.7384	2%	98%	-1.0662	-1.0034	-0.7262	-0.5171	-0.4795	-1.0789	0	0
			Cumulative(%)	-0.6981	10%	90%	-1.3686	-1.2041	-0.6483	-0.3549	-0.3137	-1.0494	1	1
			Excess(%)	-0.0134	46%	54%	-0.0568	-0.0485	-0.0125	0.0193	0.0249	-0.0145	1	1
		T→P	Durations	9.2378	20%	80%	6.1667	6.5172	8.9069	13.0000	14.2487	7.5833	1	1
			Amplitudes(%)	9.3822	44%	56%	5.9354	6.3126	9.0122	13.5289	14.9591	8.7447	1	1
			Cumulative(%)	86.3846	74%	26%	32.2463	36.6118	74.5291	175.1206	212.3796	101.6148	1	1
			Excess(%)	-0.0628	98%	2%	-0.3589	-0.2988	-0.0618	0.1764	0.2239	0.2383	0	0
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	1.6370	8%	92%	1.3333	1.3810	1.6216	1.9412	2.0254	1.4167	1	1
			Amplitudes(%)	-1.1286	58%	42%	-1.5481	-1.4684	-1.1151	-0.8372	-0.7892	-1.0789	1	1
			Cumulative(%)	-1.3975	75%	25%	-2.5923	-2.3108	-1.3107	-0.7533	-0.6852	-1.0494	1	1
			Excess(%)	0.0003	27%	73%	-0.0516	-0.0431	0.0000	0.0432	0.0520	-0.0145	1	1
		T→P	Durations	4.9049	100%	0%	3.6304	3.8046	4.8108	6.3548	6.6786	7.5833	0	0
			Amplitudes(%)	6.2175	97%	3%	4.3209	4.5961	6.1023	8.3190	8.8821	8.7447	0	1
			Cumulative(%)	29.6983	100%	0%	13.7282	15.3445	27.3567	52.6987	60.3745	101.6148	0	0
			Excess(%)	0.0010	99%	1%	-0.1838	-0.1525	0.0020	0.1538	0.1840	0.2383	0	0
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.1916	92%	8%	1.0000	1.0000	1.1667	1.5000	1.5455	1.4167	1	1
			Amplitudes(%)	-0.2504	0%	100%	-0.4279	-0.3847	-0.2422	-0.1415	-0.1232	-1.0789	0	0
			Cumulative(%)	-0.1821	0%	100%	-0.4350	-0.3620	-0.1597	-0.0759	-0.0664	-1.0494	0	0
			Excess(%)	0.0001	6%	94%	-0.0200	-0.0154	0.0000	0.0159	0.0208	-0.0145	1	1
		T→P Missing 2	Durations	19.7635	0%	100%	10.7361	11.7059	18.2000	32.5357	37.5500	7.5833	0	0
			Amplitudes(%)	16.5278	3%	97%	8.5430	9.4416	15.1882	27.6802	32.1831	8.7447	0	1
			Cumulative(%)	331.4823	6%	94%	82.5612	98.3814	250.3365	803.2532	1055.4393	101.6148	1	1
			Excess(%)	-0.0072	87%	13%	-0.4838	-0.3907	-0.0062	0.3708	0.4695	0.2383	1	1

Appendix Table 1B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	Random Walk (1)	P→T	Durations	1.8976	73%	27%	1.3565	1.4286	1.8604	2.4737	2.6111	2.0667	1	1
			Amplitudes(%)	-1.0501	1%	99%	-1.4716	-1.4000	-1.0378	-0.7489	-0.6971	-1.6087	0	0
			Cumulative(%)	-1.3829	1%	99%	-2.6402	-2.3458	-1.3031	-0.6677	-0.5915	-3.0887	0	0
			Excess(%)	0.0006	29%	71%	-0.0657	-0.0546	0.0003	0.0550	0.0666	-0.0163	1	1
		T→P	Durations	8.1111	52%	48%	6.6087	6.8095	8.0000	9.7059	10.0667	8.0714	1	1
			Amplitudes(%)	7.1800	70%	30%	5.5744	5.7791	7.1001	8.8821	9.3154	7.6091	1	1
			Cumulative(%)	36.5635	74%	27%	21.8713	23.5126	34.7868	55.8156	61.0140	41.1670	1	1
			Excess(%)	0.0542	2%	98%	-0.2098	-0.1654	0.0539	0.2809	0.3252	-0.2132	0	0
	GMM No Breaks (2)	P→T	Durations	1.5498	97%	3%	1.1579	1.2105	1.5238	2.0000	2.1026	2.0667	0	1
			Amplitudes(%)	-0.7484	0%	100%	-1.0377	-0.9851	-0.7397	-0.5398	-0.5066	-1.6087	0	0
			Cumulative(%)	-0.7442	0%	100%	-1.4214	-1.2812	-0.6941	-0.3749	-0.3405	-3.0887	0	0
			Excess(%)	0.0020	19%	81%	-0.0417	-0.0340	0.0015	0.0405	0.0487	-0.0163	1	1
		T→P	Durations	8.4237	39%	61%	6.8333	7.0476	8.3333	10.1176	10.4853	8.0714	1	1
			Amplitudes(%)	6.9139	80%	20%	5.4178	5.6165	6.8229	8.4955	8.9194	7.6091	1	1
			Cumulative(%)	36.4992	74%	26%	21.7029	23.4256	34.5940	55.3494	61.2859	41.1670	1	1
			Excess(%)	0.0541	1%	99%	-0.1784	-0.1376	0.0524	0.2511	0.2874	-0.2132	0	0
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.3051	99%	1%	1.0000	1.0000	1.2667	1.7333	1.8333	2.0667	0	0
			Amplitudes(%)	-1.5840	43%	57%	-2.6754	-2.4499	-1.5230	-0.8919	-0.7953	-1.6087	1	1
			Cumulative(%)	-1.2629	3%	97%	-3.2055	-2.6418	-1.0772	-0.5063	-0.4486	-3.0887	0	1
			Excess(%)	0.0206	22%	78%	-0.1007	-0.0784	0.0098	0.1458	0.1903	-0.0163	1	1
		T→P	Durations	11.1292	1%	99%	8.3333	8.6500	10.8750	14.5000	15.2864	8.0714	0	0
			Amplitudes(%)	9.2539	14%	86%	6.6526	6.9639	9.0307	12.2361	13.0994	7.6091	1	1
			Cumulative(%)	70.5974	9%	91%	34.4656	37.6745	64.1264	124.4640	144.0186	41.1670	1	1
			Excess(%)	-0.0215	19%	81%	-0.5837	-0.4597	0.0050	0.3445	0.4170	-0.2132	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.3855	100%	1%	1.0588	1.1053	1.3604	1.7647	1.8377	2.0667	0	0
			Amplitudes(%)	-0.8613	0%	100%	-1.2346	-1.1650	-0.8473	-0.6039	-0.5564	-1.6087	0	0
			Cumulative(%)	-0.7413	0%	100%	-1.4757	-1.3027	-0.6795	-0.3754	-0.3363	-3.0887	0	0
			Excess(%)	0.0004	23%	77%	-0.0507	-0.0413	0.0000	0.0426	0.0518	-0.0163	1	1
		T→P	Durations	9.6196	9%	91%	7.5909	7.8182	9.4444	12.0000	12.5000	8.0714	1	1
			Amplitudes(%)	7.6488	54%	46%	5.7888	6.0388	7.5138	9.7220	10.2102	7.6091	1	1
			Cumulative(%)	48.7372	36%	64%	26.4609	28.6402	45.6047	79.3283	89.4636	41.1670	1	1
			Excess(%)	0.0705	2%	98%	-0.1900	-0.1478	0.0704	0.2911	0.3381	-0.2132	0	0
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	1.7132	89%	11%	1.2500	1.3000	1.6842	2.2105	2.3182	2.0667	1	1
			Amplitudes(%)	-1.0419	1%	99%	-1.4596	-1.3760	-1.0331	-0.7514	-0.7022	-1.6087	0	0
			Cumulative(%)	-1.1752	0%	100%	-2.2580	-2.0459	-1.0977	-0.5865	-0.5236	-3.0887	0	0
			Excess(%)	0.0029	26%	74%	-0.0584	-0.0483	0.0024	0.0552	0.0662	-0.0163	1	1
		T→P	Durations	8.0556	55%	45%	6.6400	6.8297	7.9524	9.5590	10.0278	8.0714	1	1
			Amplitudes(%)	7.6185	53%	47%	5.9963	6.2485	7.5419	9.2655	9.6221	7.6091	1	1
			Cumulative(%)	37.8630	69%	31%	23.2159	25.0590	36.1195	56.1078	61.2902	41.1670	1	1
			Excess(%)	0.0535	3%	97%	-0.2167	-0.1742	0.0534	0.2828	0.3249	-0.2132	0	1
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.0039	100%	0%	1.0000	1.0000	1.0000	1.0000	1.0000	2.0667	0	0
		Missing	Amplitudes(%)	-0.0937	0%	100%	-0.2644	-0.2226	-0.0799	-0.0123	-0.0066	-1.6087	0	0
		868	Cumulative(%)	-0.0472	0%	100%	-0.1339	-0.1119	-0.0400	-0.0061	-0.0033	-3.0887	0	0
			Excess(%)	0.0000	0%	100%	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0163	0	0
		T→P	Durations	59.2744	3%	97%	7.0000	11.2500	50.0000	143.0000	160.0000	8.0714	0	1
		Missing	Amplitudes(%)	40.4691	5%	95%	5.2666	7.6793	34.4221	97.2928	110.4509	7.6091	0	1
		2,365	Cumulative(%)	1865.0002	5%	95%	19.5922	47.1405	1025.3440	6888.1527	8848.7358	41.1670	0	1
			Excess(%)	0.0168	26%	74%	-1.0808	-0.8018	0.0047	0.8495	1.0960	-0.2132	1	1

Appendix Table 1B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	Random Walk (1)	P→T	Durations	2.6129	98%	2%	1.8462	1.9286	2.5714	3.4444	3.6364	3.7143	0	0
			Amplitudes(%)	-1.4908	0%	100%	-2.0579	-1.9508	-1.4727	-1.0945	-1.0292	-2.7174	0	0
			Cumulative(%)	-2.2599	0%	100%	-4.4563	-3.9090	-2.0968	-1.1785	-1.0562	-6.3894	0	0
			Excess(%)	-0.0004	23%	77%	-0.1152	-0.0939	-0.0011	0.0912	0.1116	-0.0406	1	1
		T→P	Durations	14.2513	93%	8%	9.1875	9.7500	13.5833	20.8750	23.3810	19.6667	1	1
			Amplitudes(%)	11.8032	94%	6%	7.4506	7.9470	11.2808	17.3960	19.6538	16.9030	1	1
			Cumulative(%)	136.2266	93%	7%	46.4533	53.4191	111.6421	297.3570	372.8291	258.2457	1	1
			Excess(%)	0.0129	13%	87%	-0.4815	-0.3868	0.0158	0.4016	0.5035	-0.2523	1	1
	GMM No Breaks (2)	P→T	Durations	2.3029	100%	0%	1.6000	1.7000	2.2500	3.0000	3.2222	3.7143	0	0
			Amplitudes(%)	-1.1459	0%	100%	-1.5990	-1.5185	-1.1346	-0.8124	-0.7535	-2.7174	0	0
			Cumulative(%)	-1.4347	0%	100%	-2.7822	-2.4866	-1.3361	-0.7566	-0.6834	-6.3894	0	0
			Excess(%)	-0.0016	21%	79%	-0.1002	-0.0821	-0.0014	0.0815	0.0956	-0.0406	1	1
		T→P	Durations	19.0512	65%	36%	11.0871	11.9231	17.7778	30.1667	34.2000	19.6667	1	1
			Amplitudes(%)	14.3134	79%	21%	8.2297	8.9860	13.3658	22.7560	25.8306	16.9030	1	1
			Cumulative(%)	230.9188	73%	27%	63.7084	75.3905	179.1711	543.1549	683.6047	258.2457	1	1
			Excess(%)	0.0265	15%	85%	-0.5532	-0.4289	0.0306	0.4897	0.5961	-0.2523	1	1
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.6080	100%	0%	1.0000	1.1000	1.5556	2.3333	2.5556	3.7143	0	0
			Amplitudes(%)	-2.6607	41%	59%	-4.5063	-4.1215	-2.5398	-1.6018	-1.4456	-2.7174	1	1
			Cumulative(%)	-2.2745	2%	98%	-5.6991	-4.6572	-1.9273	-0.9841	-0.8859	-6.3894	0	0
			Excess(%)	0.0354	22%	78%	-0.2106	-0.1498	0.0154	0.2712	0.3563	-0.0406	1	1
		T→P	Durations	22.2626	45%	56%	12.6667	13.4615	20.3750	37.1000	43.5000	19.6667	1	1
			Amplitudes(%)	17.4171	58%	42%	9.6064	10.3559	15.8989	29.1275	34.4040	16.9030	1	1
			Cumulative(%)	333.1383	55%	45%	82.9303	96.4923	242.1522	817.6667	1106.8043	258.2457	1	1
			Excess(%)	0.0858	18%	82%	-0.8815	-0.6208	0.0938	0.7971	1.0050	-0.2523	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.9456	100%	0%	1.2857	1.3750	1.8889	2.6667	2.8889	3.7143	0	0
			Amplitudes(%)	-1.4032	0%	100%	-1.9805	-1.8687	-1.3926	-0.9806	-0.8969	-2.7174	0	0
			Cumulative(%)	-1.4360	0%	100%	-2.9541	-2.5565	-1.3133	-0.7465	-0.6792	-6.3894	0	0
			Excess(%)	-0.0022	24%	76%	-0.1234	-0.1003	-0.0018	0.0947	0.1184	-0.0406	1	1
		T→P	Durations	22.9734	41%	60%	12.8182	13.7000	21.1250	38.2000	44.0000	19.6667	1	1
			Amplitudes(%)	16.8014	60%	40%	9.1356	9.8677	15.5263	27.9649	32.4102	16.9030	1	1
			Cumulative(%)	329.1865	53%	47%	80.7209	94.8581	248.6646	823.1807	1083.3108	258.2457	1	1
			Excess(%)	0.0417	18%	83%	-0.6449	-0.5151	0.0383	0.6271	0.7794	-0.2523	1	1
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	2.4540	99%	1%	1.7143	1.8284	2.4167	3.2222	3.3846	3.7143	0	0
			Amplitudes(%)	-1.5115	0%	100%	-2.0865	-1.9730	-1.4921	-1.1154	-1.0502	-2.7174	0	0
			Cumulative(%)	-2.0609	0%	100%	-3.9531	-3.5094	-1.9140	-1.0896	-0.9824	-6.3894	0	0
			Excess(%)	0.0004	24%	76%	-0.1174	-0.0964	0.0003	0.0975	0.1196	-0.0406	1	1
		T→P	Durations	15.7836	85%	15%	10.0000	10.5385	15.0000	23.5714	26.2857	19.6667	1	1
			Amplitudes(%)	13.8353	83%	17%	8.6655	9.1534	13.1235	20.7496	22.9339	16.9030	1	1
			Cumulative(%)	178.1805	84%	16%	59.1873	67.4232	146.7852	393.7149	482.9134	258.2457	1	1
			Excess(%)	0.0212	14%	86%	-0.5308	-0.4307	0.0212	0.4598	0.5739	-0.2523	1	1
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.9615	100%	0%	1.0000	1.5000	2.0000	2.0000	2.0000	3.7143	0	0
		Missing	Amplitudes(%)	-0.1595	0%	100%	-0.5613	-0.5095	-0.1296	-0.0321	-0.0312	-2.7174	0	0
		4,974	Cumulative(%)	-0.1537	0%	100%	-0.5572	-0.4189	-0.1448	-0.0421	-0.0416	-6.3894	0	0
			Excess(%)	-0.0025	4%	96%	-0.0498	-0.0369	-0.0050	0.0593	0.0736	-0.0406	0	1
		T→P	Durations				0.0000	0.0000	0.0000	0.0000	0.0000	19.6667		
		Missing	Amplitudes(%)				0.0000	0.0000	0.0000	0.0000	0.0000	16.9030		
		5,000	Cumulative(%)				0.0000	0.0000	0.0000	0.0000	0.0000	258.2457		
			Excess(%)				0.0000	0.0000	0.0000	0.0000	0.0000	-0.2523		

Appendix Table 1B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	Random Walk (1)	P→T	Durations	1.3590	94%	6%	1.1628	1.1905	1.3514	1.5526	1.6000	1.5385	1	1
			Amplitudes(%)	-0.7970	0%	100%	-1.0343	-0.9896	-0.7903	-0.6200	-0.5915	-1.1539	0	0
			Cumulative(%)	-0.6839	0%	100%	-1.1115	-1.0071	-0.6594	-0.4337	-0.4015	-1.4308	0	0
			Excess(%)	-0.0002	21%	79%	-0.0337	-0.0277	0.0001	0.0263	0.0316	-0.0123	1	1
		T→P	Durations	3.8114	82%	18%	2.8958	3.0227	3.7500	4.7879	5.0156	4.2800	1	1
			Amplitudes(%)	3.9652	82%	18%	2.9411	3.0860	3.9070	5.0532	5.3218	4.4987	1	1
			Cumulative(%)	13.1127	92%	8%	6.6964	7.4158	12.2591	21.5813	24.1067	20.0276	1	1
			Excess(%)	0.0001	71%	29%	-0.1202	-0.1009	-0.0002	0.0996	0.1191	0.0326	1	1
	GMM No Breaks (2)	P→T	Durations	1.2379	100%	0%	1.0857	1.1081	1.2308	1.3947	1.4286	1.5385	0	0
			Amplitudes(%)	-0.6191	0%	100%	-0.7933	-0.7629	-0.6163	-0.4907	-0.4663	-1.1539	0	0
			Cumulative(%)	-0.4492	0%	100%	-0.6970	-0.6448	-0.4359	-0.3014	-0.2826	-1.4308	0	0
			Excess(%)	-0.0002	12%	88%	-0.0217	-0.0182	-0.0003	0.0178	0.0215	-0.0123	1	1
		T→P	Durations	4.0150	71%	29%	3.0674	3.1957	3.9474	5.0625	5.3441	4.2800	1	1
			Amplitudes(%)	3.8607	87%	13%	2.9316	3.0542	3.7940	4.8821	5.1611	4.4987	1	1
			Cumulative(%)	13.3632	91%	9%	6.8612	7.5507	12.5155	22.0220	24.3827	20.0276	1	1
			Excess(%)	0.0004	73%	27%	-0.1067	-0.0887	-0.0002	0.0904	0.1064	0.0326	1	1
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.1153	100%	0%	1.0000	1.0000	1.1071	1.2500	1.2759	1.5385	0	0
			Amplitudes(%)	-1.2707	60%	40%	-2.0070	-1.8719	-1.2360	-0.8031	-0.7469	-1.1539	1	1
			Cumulative(%)	-0.7559	2%	98%	-1.3300	-1.1956	-0.7169	-0.4451	-0.4019	-1.4308	0	0
			Excess(%)	0.0092	18%	82%	-0.0495	-0.0365	0.0035	0.0688	0.0896	-0.0123	1	1
		T→P	Durations	7.2670	0%	100%	5.0303	5.2667	7.0445	9.9333	10.7059	4.2800	0	0
			Amplitudes(%)	6.4489	3%	97%	4.4305	4.6580	6.2583	8.7604	9.4440	4.4987	0	1
			Cumulative(%)	42.0212	5%	95%	17.8832	19.5588	37.1466	78.9697	93.4273	20.0276	1	1
			Excess(%)	-0.0427	66%	34%	-0.4102	-0.3290	-0.0255	0.1872	0.2279	0.0326	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.1867	100%	0%	1.0400	1.0645	1.1786	1.3425	1.3784	1.5385	0	0
			Amplitudes(%)	-0.7262	0%	100%	-0.9636	-0.9197	-0.7211	-0.5519	-0.5247	-1.1539	0	0
			Cumulative(%)	-0.4903	0%	100%	-0.7813	-0.7108	-0.4733	-0.3252	-0.3061	-1.4308	0	0
			Excess(%)	-0.0013	20%	80%	-0.0292	-0.0241	-0.0013	0.0220	0.0283	-0.0123	1	1
		T→P	Durations	5.4361	8%	92%	3.9730	4.1316	5.3000	7.1667	7.6087	4.2800	1	1
			Amplitudes(%)	4.8228	38%	62%	3.5232	3.6657	4.7133	6.3531	6.6774	4.4987	1	1
			Cumulative(%)	23.5079	42%	58%	11.2507	12.2925	21.6021	41.3259	47.2793	20.0276	1	1
			Excess(%)	0.0169	58%	42%	-0.1315	-0.1046	0.0167	0.1375	0.1650	0.0326	1	1
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	1.2780	99%	1%	1.1163	1.1389	1.2708	1.4390	1.4773	1.5385	0	0
			Amplitudes(%)	-0.8206	0%	100%	-1.0373	-1.0009	-0.8182	-0.6548	-0.6281	-1.1539	0	0
			Cumulative(%)	-0.6235	0%	100%	-0.9559	-0.8855	-0.6059	-0.4245	-0.3928	-1.4308	0	0
			Excess(%)	0.0003	18%	82%	-0.0284	-0.0234	0.0001	0.0242	0.0297	-0.0123	1	1
		T→P	Durations	3.6014	91%	9%	2.7843	2.8936	3.5581	4.4571	4.6875	4.2800	1	1
			Amplitudes(%)	4.1097	78%	22%	3.1291	3.2608	4.0634	5.1066	5.3880	4.4987	1	1
			Cumulative(%)	12.4970	95%	5%	6.5599	7.3306	11.8640	20.1602	22.4964	20.0276	1	1
			Excess(%)	-0.0002	71%	29%	-0.1179	-0.0989	-0.0001	0.0994	0.1203	0.0326	1	1
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.0032	100%	0%	1.0000	1.0000	1.0000	1.0000	1.0000	1.5385	0	0
		Missing	Amplitudes(%)	-0.0963	0%	100%	-0.2693	-0.2272	-0.0825	-0.0126	-0.0062	-1.1539	0	0
		863	Cumulative(%)	-0.0484	0%	100%	-0.1370	-0.1157	-0.0414	-0.0063	-0.0031	-1.4308	0	0
			Excess(%)	0.0000	0%	100%	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0123	0	0
		T→P	Durations	58.0321	3%	97%	4.0000	9.0000	49.5000	142.0000	159.0000	4.2800	0	1
		Missing	Amplitudes(%)	39.6656	4%	96%	3.1207	5.9120	34.0105	97.7070	108.3071	4.4987	0	1
		2,388	Cumulative(%)	1829.9798	4%	96%	7.2101	28.3851	1029.7388	6952.3525	8682.0556	20.0276	0	1
			Excess(%)	0.0010	54%	46%	-0.9433	-0.7390	0.0000	0.7550	0.9733	0.0326	1	1

Appendix Table 1C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	Random Walk (1)	P→T	Durations	1.0246	100%	0%	1.0000	1.0000	1.0000	1.2000	1.3333	2.7143	0	0
		Missing	Amplitudes(%)	-0.1747	0%	100%	-0.3782	-0.3328	-0.1629	-0.0540	-0.0404	-1.1728	0	0
		61	Cumulative(%)	-0.0912	0%	100%	-0.2101	-0.1801	-0.0826	-0.0272	-0.0202	-2.2481	0	0
			Excess(%)	0.0001	100%	0%	-0.0054	0.0000	0.0000	0.0000	0.0067	0.0447	0	0
		T→P	Durations	40.1241	21%	79%	12.1000	15.5000	33.7083	84.5000	114.0000	23.6667	1	1
		Missing	Amplitudes(%)	36.8239	33%	67%	11.2590	14.0204	30.7824	77.8072	104.9180	25.2807	1	1
		350	Cumulative(%)	1235.0027	21%	79%	79.2720	137.3742	715.6751	4041.7960	5996.1232	348.9563	1	1
			Excess(%)	0.0262	85%	15%	-1.1028	-0.8252	0.0233	0.8593	1.1482	0.4702	1	1
	GMM No Breaks (2)	P→T	Durations	2.4901	66%	35%	1.0000	1.1833	2.3333	4.3333	4.9444	2.7143	1	1
		Missing	Amplitudes(%)	-0.6874	12%	88%	-1.7880	-1.5082	-0.5925	-0.1628	-0.1181	-1.1728	1	1
		13	Cumulative(%)	-1.7049	24%	76%	-7.5771	-5.4225	-1.0581	-0.1081	-0.0671	-2.2481	1	1
			Excess(%)	0.0000	89%	11%	-0.0958	-0.0715	0.0000	0.0685	0.0928	0.0447	1	1
		T→P	Durations	29.5112	45%	56%	10.8571	13.0000	25.1667	60.5000	75.2500	23.6667	1	1
		Missing	Amplitudes(%)	28.3855	57%	43%	7.5323	9.7076	23.2302	64.1212	81.2055	25.2807	1	1
		89	Cumulative(%)	835.1924	37%	63%	60.1726	97.7958	467.4244	2772.7551	3875.3250	348.9563	1	1
			Excess(%)	0.0884	76%	24%	-1.6026	-1.1417	0.0575	1.4114	1.9516	0.4702	1	1
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	1.6486	88%	12%	1.0000	1.0000	1.2500	3.5000	4.0000	2.7143	1	1
		Missing	Amplitudes(%)	-0.3209	4%	96%	-1.3579	-1.0149	-0.1997	-0.0188	-0.0088	-1.1728	0	1
		1873	Cumulative(%)	-0.5247	5%	95%	-3.4152	-2.3108	-0.1341	-0.0094	-0.0044	-2.2481	1	1
			Excess(%)	-0.0229	99%	1%	-0.1754	-0.1169	0.0000	0.0134	0.0288	0.0447	0	0
		T→P	Durations	55.2527	23%	77%	5.0000	6.2500	47.5000	132.0000	145.0000	23.6667	1	1
		Missing	Amplitudes(%)	45.8923	34%	66%	1.5311	2.2602	37.6991	115.1829	128.0912	25.2807	1	1
		3633	Cumulative(%)	2003.1168	30%	70%	2.6221	6.3230	1020.1014	7644.6040	9057.1919	348.9563	1	1
			Excess(%)	-0.4652	83%	17%	-4.0101	-2.9409	-0.3485	1.8804	2.7185	0.4702	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	1.2429	95%	5%	1.0000	1.0000	1.0000	2.5000	3.0000	2.7143	0	1
		Missing	Amplitudes(%)	-0.0614	0%	100%	-0.2950	-0.2123	-0.0367	-0.0038	-0.0023	-1.1728	0	0
		4860	Cumulative(%)	-0.0526	0%	100%	-0.4044	-0.2242	-0.0193	-0.0019	-0.0012	-2.2481	0	0
			Excess(%)	-0.0001	100%	0%	-0.0212	-0.0123	0.0000	0.0109	0.0308	0.0447	0	0
		T→P	Durations				0.0000	0.0000	0.0000	0.0000	0.0000	23.6667		
		Missing	Amplitudes(%)				0.0000	0.0000	0.0000	0.0000	0.0000	25.2807		
		5000	Cumulative(%)				0.0000	0.0000	0.0000	0.0000	0.0000	348.9563		
			Excess(%)				0.0000	0.0000	0.0000	0.0000	0.0000	0.4702		
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	2.6856	57%	44%	1.2500	1.4286	2.5556	4.4000	5.0000	2.7143	1	1
		Missing	Amplitudes(%)	-0.9394	26%	74%	-2.2219	-1.9166	-0.8373	-0.3214	-0.2528	-1.1728	1	1
		1	Cumulative(%)	-2.5012	38%	62%	-9.5112	-7.2887	-1.7309	-0.3024	-0.1968	-2.2481	1	1
			Excess(%)	0.0007	85%	15%	-0.1041	-0.0796	0.0000	0.0813	0.1035	0.0447	1	1
		T→P	Durations	22.7104	66%	34%	11.0000	12.1623	20.2857	40.5000	49.1667	23.6667	1	1
		Missing	Amplitudes(%)	23.7717	67%	33%	8.9122	10.4181	20.7130	45.5993	57.4802	25.2807	1	1
		11	Cumulative(%)	531.0371	50%	50%	75.8217	99.6246	346.4487	1472.2952	2076.5349	348.9563	1	1
			Excess(%)	0.0619	79%	21%	-1.2083	-0.9225	0.0531	1.0860	1.4360	0.4702	1	1
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	1.5086	90%	10%	1.0000	1.0000	1.0000	3.0000	4.0000	2.7143	1	1
		Missing	Amplitudes(%)	-0.1824	1%	99%	-0.7021	-0.5593	-0.1234	-0.0110	-0.0066	-1.1728	0	0
		2268	Cumulative(%)	-0.2189	1%	99%	-1.3095	-0.9178	-0.0668	-0.0056	-0.0033	-2.2481	0	0
			Excess(%)	0.0008	96%	4%	-0.0493	-0.0324	0.0000	0.0380	0.0545	0.0447	0	1
		T→P	Durations	54.0200	24%	76%	4.0000	6.0000	47.5833	134.5000	148.0000	23.6667	1	1
		Missing	Amplitudes(%)	45.8695	33%	67%	1.2951	2.0511	39.0471	118.6546	134.2764	25.2807	1	1
		4050	Cumulative(%)	2000.1446	29%	71%	2.7499	6.1203	1028.1123	8272.2430	9426.5220	348.9563	1	1
			Excess(%)	-0.0519	71%	29%	-3.3970	-2.6886	-0.0069	2.2849	3.0243	0.4702	1	1

Appendix Table 1C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	Random Walk (1)	P→T	Durations	1.8470	100%	0%	1.0000	1.0000	2.0000	2.0000	3.0000	4.0000	0	0
		Missing	Amplitudes(%)	-0.3719	0%	100%	-0.9115	-0.8156	-0.3394	-0.0828	-0.0590	-1.9868	0	0
		4425	Cumulative(%)	-0.3317	0%	100%	-0.8802	-0.7305	-0.2808	-0.0681	-0.0469	-3.9013	0	0
			Excess(%)	-0.0015	95%	5%	-0.0978	-0.0762	0.0000	0.0759	0.1029	0.0782	0	1
		T→P	Durations	74.5455	24%	76%	6.0000	7.5000	71.0000	156.0000	160.0000	36.0000	1	1
		Missing	Amplitudes(%)	66.5464	24%	76%	5.3103	5.9777	59.3452	139.4155	143.4247	38.6480	1	1
		4967	Cumulative(%)	3391.5481	24%	76%	15.7449	19.3647	2066.1412	11093.0081	11178.2434	842.6579	1	1
			Excess(%)	0.4692	79%	21%	-3.0882	-2.6973	0.2934	2.7539	2.8118	1.3878	1	1
	GMM No Breaks (2)	P→T	Durations	3.5114	75%	34%	1.5000	1.7500	3.2500	6.0000	7.0000	4.0000	1	1
		Missing	Amplitudes(%)	-1.0613	8%	92%	-2.8784	-2.3570	-0.8958	-0.3155	-0.2457	-1.9868	1	1
		156	Cumulative(%)	-2.9013	21%	79%	-12.9515	-9.1222	-1.7399	-0.2644	-0.1929	-3.9013	1	1
			Excess(%)	0.0008	89%	11%	-0.1555	-0.1165	0.0000	0.1189	0.1697	0.0782	1	1
		T→P	Durations	41.1127	55%	46%	7.5000	11.0000	34.0000	102.0000	124.0000	36.0000	1	1
		Missing	Amplitudes(%)	38.3029	65%	35%	3.7072	6.8535	30.7122	102.7903	128.2101	38.6480	1	1
		663	Cumulative(%)	1448.9997	54%	46%	14.8485	44.1329	764.1588	5484.8961	7904.5532	842.6579	1	1
			Excess(%)	0.0135	89%	11%	-3.3785	-2.3625	0.0010	2.3716	3.5651	1.3878	1	1
	GMM No Breaks (2) Residual Resample Both Equations	P→T	Durations	2.6215	92%	16%	1.0000	1.0000	2.0000	5.0000	6.0000	4.0000	1	1
		Missing	Amplitudes(%)	-0.6431	3%	97%	-2.1271	-1.6867	-0.4768	-0.1248	-0.0895	-1.9868	0	1
		3279	Cumulative(%)	-1.3304	6%	94%	-6.8446	-4.7008	-0.6428	-0.0964	-0.0725	-3.9013	1	1
			Excess(%)	-0.0581	98%	2%	-0.2665	-0.2155	-0.0459	0.0415	0.0763	0.0782	0	0
		T→P	Durations	56.5970	40%	62%	3.0000	4.0000	51.0000	145.0000	158.0000	36.0000	1	1
		Missing	Amplitudes(%)	46.3055	48%	52%	0.7833	1.0968	39.2100	122.1835	131.4894	38.6480	1	1
		4641	Cumulative(%)	2132.7651	47%	53%	0.9069	1.9047	1053.3134	8705.8619	10085.7897	842.6579	1	1
			Excess(%)	-0.5839	92%	8%	-4.9925	-3.5047	-0.3280	1.8950	3.0861	1.3878	1	1
	GMM No Breaks (2) Residual Resample First Equation	P→T	Durations	2.2069	97%	3%	1.0000	1.0000	2.0000	4.5000	6.0000	4.0000	1	1
		Missing	Amplitudes(%)	-0.1424	0%	100%	-0.5620	-0.4607	-0.1253	-0.0248	-0.0216	-1.9868	0	0
		4971	Cumulative(%)	-0.1722	0%	100%	-1.3155	-0.8512	-0.1271	-0.0185	-0.0179	-3.9013	0	0
			Excess(%)	0.0022	100%	0%	-0.0235	-0.0226	-0.0024	0.0563	0.0617	0.0782	0	0
		T→P	Durations	72.0000	0%	100%	72.0000	72.0000	72.0000	72.0000	72.0000	36.0000	0	0
		Missing	Amplitudes(%)	49.3992	0%	100%	49.3992	49.3992	49.3992	49.3992	49.3992	38.6480	0	0
		4999	Cumulative(%)	1786.6402	0%	100%	1786.6402	1786.6402	1786.6402	1786.6402	1786.6402	842.6579	0	0
			Excess(%)	0.1149	100%	0%	0.1149	0.1149	0.1149	0.1149	0.1149	1.3878	0	0
	GMM Volatility Break (3) Set at First Period Volatility	P→T	Durations	3.7158	69%	37%	1.8000	2.0000	3.5000	6.0000	7.0000	4.0000	1	1
		Missing	Amplitudes(%)	-1.4125	18%	82%	-3.4092	-2.8316	-1.2452	-0.5110	-0.4230	-1.9868	1	1
		33	Cumulative(%)	-4.1198	35%	65%	-16.5370	-12.1274	-2.7319	-0.5550	-0.4001	-3.9013	1	1
			Excess(%)	-0.0001	86%	14%	-0.1904	-0.1358	0.0001	0.1362	0.1773	0.0782	1	1
		T→P	Durations	34.0981	67%	34%	10.0000	12.8819	29.0000	74.0000	90.0000	36.0000	1	1
		Missing	Amplitudes(%)	34.9191	69%	31%	6.9612	10.0635	28.7328	81.5426	103.8318	38.6480	1	1
		223	Cumulative(%)	1127.3495	60%	40%	46.8294	95.8395	641.6006	3756.0663	5317.5956	842.6579	1	1
			Excess(%)	0.0198	91%	9%	-2.6438	-1.8845	0.0107	1.9224	2.6460	1.3878	1	1
	GMM Volatility Break (3) Set at Second Period Volatility	P→T	Durations	2.4501	94%	13%	1.0000	1.0000	2.0000	5.0000	5.0000	4.0000	1	1
		Missing	Amplitudes(%)	-0.3764	0%	100%	-1.1586	-0.9224	-0.2988	-0.0765	-0.0571	-1.9868	0	0
		3753	Cumulative(%)	-0.5752	1%	99%	-2.8667	-1.9441	-0.2898	-0.0611	-0.0465	-3.9013	0	0
			Excess(%)	0.0031	96%	4%	-0.0880	-0.0679	0.0000	0.0714	0.0919	0.0782	0	1
		T→P	Durations	55.8889	42%	59%	3.0000	4.0000	48.0000	136.0000	148.0000	36.0000	1	1
		Missing	Amplitudes(%)	46.5189	51%	49%	0.2610	0.9126	36.2890	124.5633	132.1365	38.6480	1	1
		4820	Cumulative(%)	2208.9351	51%	49%	0.3633	1.9106	841.2167	8548.6616	9398.9803	842.6579	1	1
			Excess(%)	0.1271	84%	16%	-3.6254	-2.4781	0.0328	2.9605	4.7312	1.3878	1	1

Appendix Table 1C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	Random Walk	P→T	Durations	1.0257	100%	0%	1.0000	1.0000	1.0000	1.2000	1.3333	2.7143	0	0
	(1)	Missing	Amplitudes(%)	-0.1742	0%	100%	-0.3794	-0.3199	-0.1644	-0.0572	-0.0405	-1.1728	0	0
		61	Cumulative(%)	-0.0916	0%	100%	-0.2150	-0.1814	-0.0839	-0.0286	-0.0203	-2.2481	0	0
			Excess(%)	0.0000	100%	0%	-0.0063	0.0000	0.0000	0.0000	0.0056	0.0447	0	0
		T→P	Durations	37.0212	28%	72%	10.5000	13.6667	31.0000	79.7500	106.5000	23.6667	1	1
		Missing	Amplitudes(%)	34.0806	40%	60%	9.4823	12.4218	28.5752	73.2512	99.1131	25.2807	1	1
		317	Cumulative(%)	1133.3492	25%	75%	59.0854	113.3344	661.1931	3613.4382	5456.0990	348.9563	1	1
			Excess(%)	-0.0035	88%	12%	-1.0429	-0.7661	0.0007	0.7754	1.0569	0.4702	1	1
	GMM No Breaks	P→T	Durations	2.0212	87%	14%	1.0000	1.1429	1.9091	3.2792	3.6667	2.7143	1	1
	(2)	Missing	Amplitudes(%)	-0.5705	6%	94%	-1.4963	-1.2292	-0.4904	-0.1657	-0.1260	-1.1728	1	1
		11	Cumulative(%)	-1.2233	14%	86%	-5.2823	-3.8610	-0.7268	-0.1062	-0.0716	-2.2481	1	1
			Excess(%)	0.0001	94%	6%	-0.0667	-0.0490	0.0000	0.0479	0.0624	0.0447	1	1
		T→P	Durations	22.4403	69%	31%	7.5000	9.1167	18.5000	48.3333	60.0000	23.6667	1	1
		Missing	Amplitudes(%)	21.6458	74%	26%	5.6155	7.0282	17.3223	49.9768	62.9134	25.2807	1	1
		56	Cumulative(%)	608.9817	51%	49%	43.3130	69.5141	342.4847	1947.5064	2806.6867	348.9563	1	1
			Excess(%)	-0.0077	85%	15%	-1.3804	-0.9283	-0.0043	0.9787	1.2981	0.4702	1	1
	GMM No Breaks	P→T	Durations	1.5094	93%	7%	1.0000	1.0000	1.0000	3.0000	3.3333	2.7143	1	1
	(2) Residual	Missing	Amplitudes(%)	-0.2821	2%	98%	-1.1541	-0.8770	-0.1813	-0.0196	-0.0095	-1.1728	0	0
	Resample Both	1795	Cumulative(%)	-0.4125	3%	97%	-2.5071	-1.6657	-0.1113	-0.0098	-0.0048	-2.2481	0	1
	Equations		Excess(%)	-0.0193	99%	1%	-0.1372	-0.0980	0.0000	0.0106	0.0239	0.0447	0	0
		T→P	Durations	44.0220	37%	63%	1.0000	1.0000	35.0000	125.5000	146.0000	23.6667	1	1
		Missing	Amplitudes(%)	36.5703	47%	53%	0.0759	0.1531	27.4744	110.7431	129.6366	25.2807	1	1
		3438	Cumulative(%)	1622.4751	40%	60%	0.0382	0.0786	663.8866	6729.6966	9633.8931	348.9563	1	1
			Excess(%)	-0.3951	86%	14%	-3.2910	-2.5672	-0.2080	1.3912	2.1281	0.4702	1	1
	GMM No Breaks	P→T	Durations	1.2103	95%	5%	1.0000	1.0000	1.0000	2.5000	3.0000	2.7143	0	1
	(2) Residual	Missing	Amplitudes(%)	-0.0639	0%	100%	-0.2558	-0.2076	-0.0441	-0.0025	-0.0018	-1.1728	0	0
	Resample First	4855	Cumulative(%)	-0.0500	0%	100%	-0.4407	-0.2250	-0.0220	-0.0012	-0.0009	-2.2481	0	0
	Equation		Excess(%)	0.0012	99%	1%	-0.0176	-0.0034	0.0000	0.0151	0.0407	0.0447	0	0
		T→P	Durations	11.6000	80%	20%	1.0000	1.0000	1.0000	51.0000	51.0000	23.6667	1	1
		Missing	Amplitudes(%)	7.3153	80%	20%	0.0051	0.0051	0.0392	36.1372	36.1372	25.2807	1	1
		4995	Cumulative(%)	182.1244	80%	20%	0.0026	0.0026	0.0196	910.0738	910.0738	348.9563	1	1
			Excess(%)	-0.0560	100%	0%	-0.2240	-0.2240	0.0000	0.0000	0.0000	0.4702	0	0
	GMM Volatility	P→T	Durations	2.0949	86%	14%	1.1818	1.2857	2.0000	3.1818	3.5000	2.7143	1	1
	Break (3) Set at	Missing	Amplitudes(%)	-0.7441	12%	88%	-1.7143	-1.4565	-0.6664	-0.2735	-0.2233	-1.1728	1	1
	First Period	1	Cumulative(%)	-1.6508	22%	78%	-6.2492	-4.7266	-1.1160	-0.2218	-0.1541	-2.2481	1	1
	Volatility		Excess(%)	0.0001	92%	8%	-0.0707	-0.0562	0.0000	0.0566	0.0700	0.0447	1	1
		T→P	Durations	16.8533	86%	14%	7.3798	8.1667	14.7778	32.2000	38.3333	23.6667	1	1
		Missing	Amplitudes(%)	17.8475	84%	16%	6.2576	7.2368	15.3029	36.7965	44.7150	25.2807	1	1
		4	Cumulative(%)	388.3857	66%	34%	52.0328	67.4812	249.2524	1123.3569	1617.6503	348.9563	1	1
			Excess(%)	-0.0062	90%	10%	-0.9620	-0.7114	-0.0067	0.7404	0.9681	0.4702	1	1
	GMM Volatility	P→T	Durations	1.4324	93%	7%	1.0000	1.0000	1.0000	3.0000	3.0000	2.7143	1	1
	Break (3) Set at	Missing	Amplitudes(%)	-0.1701	1%	99%	-0.6784	-0.5102	-0.1124	-0.0129	-0.0067	-1.1728	0	0
	Second Period	2266	Cumulative(%)	-0.2002	1%	99%	-1.2991	-0.8241	-0.0628	-0.0066	-0.0034	-2.2481	0	0
	Volatility		Excess(%)	-0.0002	97%	3%	-0.0495	-0.0320	0.0000	0.0308	0.0511	0.0447	0	1
		T→P	Durations	43.5030	39%	61%	1.0000	1.0000	33.0000	126.0000	145.5000	23.6667	1	1
		Missing	Amplitudes(%)	37.1502	49%	51%	0.0551	0.1043	26.3536	114.8842	132.2006	25.2807	1	1
		3855	Cumulative(%)	1681.3096	42%	58%	0.0283	0.0572	584.0901	7157.0267	9704.3141	348.9563	1	1
			Excess(%)	0.0710	75%	25%	-3.3112	-2.1413	0.0000	2.2935	3.2706	0.4702	1	1

Appendix Table 2A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	GMM Volatility Break (3) Switching (Vol 1/Vol2)	P→T	Durations	2.1245	51%	49%	1.4118	1.5000	2.0714	2.8750	3.0714	2.0909	1	1
			Amplitudes(%)	-1.2832	11%	89%	-2.0882	-1.9229	-1.2475	-0.7765	-0.7018	-1.7135	1	1
			Cumulative(%)	-2.2830	50%	50%	-5.1826	-4.4893	-2.0443	-0.8213	-0.6756	-2.0560	1	1
			Excess(%)	0.0011	89%	11%	-0.0967	-0.0772	0.0001	0.0804	0.0994	0.0597	1	1
		T→P	Durations	11.3854	99%	1%	8.1765	8.5882	11.1765	14.7692	15.6923	17.5455	0	0
			Amplitudes(%)	11.7818	99%	1%	8.1125	8.5711	11.4989	15.8679	16.9211	18.4390	0	0
			Cumulative(%)	113.4568	98%	3%	45.2361	51.3495	99.7952	221.0336	261.0167	260.9763	0	1
			Excess(%)	0.1021	100%	0%	-0.2995	-0.2314	0.0993	0.4555	0.5352	0.8959	0	0
	GMM Volatility Break (3) Reversing (Vol 2/Vol1)	P→T	Durations	1.9156	72%	29%	1.2500	1.3333	1.8667	2.6667	2.8516	2.0909	1	1
			Amplitudes(%)	-1.0512	4%	96%	-1.8202	-1.6852	-1.0029	-0.5894	-0.5225	-1.7135	0	1
			Cumulative(%)	-1.7959	31%	69%	-4.6513	-3.9401	-1.5378	-0.5374	-0.4390	-2.0560	1	1
			Excess(%)	0.0001	91%	9%	-0.0936	-0.0754	0.0001	0.0758	0.0930	0.0597	1	1
		T→P	Durations	14.3890	87%	13%	9.9258	10.5294	13.9333	19.7000	21.1000	17.5455	1	1
			Amplitudes(%)	13.7230	93%	7%	9.0769	9.6196	13.2413	19.2394	21.1705	18.4390	1	1
			Cumulative(%)	180.8471	84%	16%	63.4928	72.5776	151.8428	385.2344	468.1072	260.9763	1	1
			Excess(%)	0.0289	100%	0%	-0.4308	-0.3496	0.0336	0.4055	0.4922	0.8959	0	0
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) Both Equations	P→T	Durations	2.0913	55%	45%	1.4000	1.4706	2.0588	2.8571	3.0588	2.0909	1	1
			Amplitudes(%)	-2.3525	81%	19%	-4.0160	-3.6555	-2.2737	-1.3315	-1.1781	-1.7135	1	1
			Cumulative(%)	-4.5809	88%	12%	-10.9603	-9.3697	-4.0269	-1.5634	-1.2610	-2.0560	1	1
			Excess(%)	-0.1512	97%	3%	-0.4423	-0.3848	-0.1372	0.0361	0.0717	0.0597	0	1
		T→P	Durations	12.2681	98%	2%	8.7000	9.1176	12.0000	16.3205	17.4545	17.5455	0	0
			Amplitudes(%)	13.9453	92%	8%	9.2341	9.9065	13.4821	19.5369	21.0468	18.4390	1	1
			Cumulative(%)	137.0559	94%	6%	54.2210	60.8926	120.8535	268.8093	318.1775	260.9763	1	1
			Excess(%)	0.0405	99%	1%	-0.5782	-0.4649	0.0308	0.6016	0.7304	0.8959	0	0
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) First Equation	P→T	Durations	2.1050	54%	46%	1.4118	1.5000	2.0625	2.8571	3.0625	2.0909	1	1
			Amplitudes(%)	-2.3535	82%	18%	-3.8791	-3.6098	-2.2869	-1.3364	-1.1783	-1.7135	1	1
			Cumulative(%)	-4.6143	89%	11%	-10.7711	-9.2816	-4.1142	-1.5956	-1.3276	-2.0560	1	1
			Excess(%)	-0.1505	97%	3%	-0.4480	-0.3773	-0.1366	0.0351	0.0720	0.0597	0	1
		T→P	Durations	12.2918	97%	3%	8.7303	9.1333	12.0000	16.5000	17.7386	17.5455	0	1
			Amplitudes(%)	13.9658	92%	8%	9.2606	9.8945	13.5675	19.5140	21.2320	18.4390	1	1
			Cumulative(%)	138.8911	94%	6%	54.1487	61.2635	121.7929	277.5877	320.2052	260.9763	1	1
			Excess(%)	0.0385	99%	1%	-0.6002	-0.4772	0.0242	0.5914	0.7312	0.8959	0	0
	GMM Volatility Break (3) Reversing Residual Resample (Per 2/Per 1) Both Equations	P→T	Durations	1.9088	72%	28%	1.2500	1.3333	1.8571	2.6667	2.8462	2.0909	1	1
			Amplitudes(%)	-1.9832	61%	39%	-3.5091	-3.2105	-1.9037	-1.0240	-0.8936	-1.7135	1	1
			Cumulative(%)	-3.6697	74%	26%	-9.5740	-8.0325	-3.1076	-1.0811	-0.8592	-2.0560	1	1
			Excess(%)	-0.1295	96%	4%	-0.4183	-0.3540	-0.1173	0.0446	0.0761	0.0597	0	1
		T→P	Durations	14.8604	84%	17%	10.1990	10.7817	14.4286	20.4000	22.1056	17.5455	1	1
			Amplitudes(%)	15.0662	85%	15%	9.7433	10.3510	14.5741	21.4961	23.2498	18.4390	1	1
			Cumulative(%)	187.0852	83%	17%	67.4674	76.7452	159.8227	389.0327	466.6939	260.9763	1	1
			Excess(%)	-0.1063	100%	0%	-0.7760	-0.6431	-0.1054	0.4244	0.5353	0.8959	0	0
	GMM Volatility Break (3) Switching Residual Resample (Per 2/Per 1) First Equation	P→T	Durations	1.9150	71%	30%	1.2500	1.3333	1.8667	2.6923	2.8750	2.0909	1	1
			Amplitudes(%)	-1.9803	60%	40%	-3.6158	-3.2859	-1.8918	-1.0175	-0.8932	-1.7135	1	1
			Cumulative(%)	-3.7052	74%	26%	-9.9715	-8.3012	-3.1158	-1.0489	-0.8271	-2.0560	1	1
			Excess(%)	-0.1288	97%	3%	-0.4180	-0.3484	-0.1162	0.0415	0.0785	0.0597	0	1
		T→P	Durations	14.9355	83%	18%	10.1667	10.7101	14.4667	20.8819	23.0000	17.5455	1	1
			Amplitudes(%)	15.1435	84%	16%	9.6732	10.2605	14.5615	21.9512	24.2855	18.4390	1	1
			Cumulative(%)	188.7988	83%	17%	67.3497	75.5305	160.6279	398.1964	484.4197	260.9763	1	1
			Excess(%)	-0.1074	100%	0%	-0.8253	-0.6767	-0.1021	0.4344	0.5523	0.8959	0	0

Appendix Table 2A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	GMM Volatility Break (3) Switching (Vol 1/Vol2)	P→T	Durations	2.9861	49%	53%	2.0000	2.1000	2.9000	4.1250	4.4000	2.8889	1	1
			Amplitudes(%)	-1.9878	28%	72%	-3.1080	-2.8977	-1.9291	-1.2644	-1.1785	-2.2380	1	1
			Cumulative(%)	-3.9080	62%	38%	-9.2225	-7.7589	-3.4255	-1.5365	-1.3124	-2.9379	1	1
			Excess(%)	-0.0024	63%	37%	-0.1724	-0.1418	-0.0027	0.1365	0.1643	0.0244	1	1
		T→P	Durations	16.4525	57%	43%	8.5505	9.3167	15.5000	26.7321	30.0833	16.4444	1	1
			Amplitudes(%)	17.1425	69%	31%	8.8560	9.8015	16.1112	27.8674	30.5471	18.9260	1	1
			Cumulative(%)	271.3283	58%	42%	52.3629	65.4817	205.0907	689.0923	852.2526	238.2669	1	1
			Excess(%)	0.0966	98%	2%	-0.7375	-0.5770	0.0857	0.8144	1.0324	1.0412	0	0
	GMM Volatility Break (3) Reversing (Vol 2/Vol1)	P→T	Durations	2.8847	58%	43%	1.7778	1.8889	2.8000	4.2000	4.5357	2.8889	1	1
			Amplitudes(%)	-1.8284	21%	79%	-3.1695	-2.8877	-1.7465	-1.0233	-0.8896	-2.2380	1	1
			Cumulative(%)	-3.5240	50%	50%	-9.4125	-7.8387	-2.9363	-1.1130	-0.9362	-2.9379	1	1
			Excess(%)	-0.0003	62%	38%	-0.2011	-0.1575	-0.0006	0.1566	0.1972	0.0244	1	1
		T→P	Durations	24.0511	26%	74%	8.4286	9.5000	22.4444	43.8000	52.0000	16.4444	1	1
			Amplitudes(%)	22.7720	40%	60%	8.7201	9.8541	21.0171	40.7692	47.8750	18.9260	1	1
			Cumulative(%)	633.6710	29%	71%	48.2445	63.7288	441.8839	1817.7328	2331.4906	238.2669	1	1
			Excess(%)	-0.1567	98%	2%	-1.4600	-1.1303	-0.1156	0.6765	0.8871	1.0412	0	0
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) Both Equations	P→T	Durations	2.7566	63%	38%	1.7778	1.8889	2.6923	3.8571	4.1181	2.8889	1	1
			Amplitudes(%)	-3.4581	91%	9%	-5.7686	-5.3311	-3.3454	-2.0373	-1.8412	-2.2380	1	1
			Cumulative(%)	-7.2953	92%	8%	-18.2918	-14.9835	-6.3323	-2.6105	-2.2242	-2.9379	1	1
			Excess(%)	-0.2379	93%	7%	-0.6816	-0.5905	-0.2217	0.0557	0.1183	0.0244	1	1
		T→P	Durations	18.1658	44%	56%	9.9045	10.6667	17.2222	28.1548	31.8452	16.4444	1	1
			Amplitudes(%)	20.6220	47%	53%	11.1159	12.1257	19.3862	32.7295	37.0144	18.9260	1	1
			Cumulative(%)	344.2475	42%	58%	77.9779	96.9489	269.6127	834.5972	1033.4105	238.2669	1	1
			Excess(%)	0.0988	95%	5%	-1.0097	-0.8027	0.0597	1.0664	1.4092	1.0412	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) First Equation	P→T	Durations	2.7620	62%	39%	1.7500	1.8750	2.7000	3.8333	4.1111	2.8889	1	1
			Amplitudes(%)	-3.4680	91%	9%	-5.7882	-5.2870	-3.3773	-2.0284	-1.8186	-2.2380	1	1
			Cumulative(%)	-7.2599	92%	8%	-17.8637	-15.0994	-6.3531	-2.5838	-2.1343	-2.9379	1	1
			Excess(%)	-0.2327	92%	8%	-0.6919	-0.5903	-0.2202	0.0706	0.1314	0.0244	1	1
		T→P	Durations	18.2947	43%	57%	9.8889	10.8819	17.2222	29.1429	32.7083	16.4444	1	1
			Amplitudes(%)	20.7610	46%	54%	11.2784	12.2588	19.4387	33.5104	38.2419	18.9260	1	1
			Cumulative(%)	350.7471	42%	59%	79.2574	98.0795	273.7747	839.6711	1045.3798	238.2669	1	1
			Excess(%)	0.1043	94%	6%	-1.0039	-0.7720	0.0703	1.1181	1.3709	1.0412	1	1
	GMM Volatility Break (3) Reversing Residual Resample (Per 2/Per 1) Both Equations	P→T	Durations	2.5873	73%	28%	1.5714	1.6667	2.5000	3.7500	4.1429	2.8889	1	1
			Amplitudes(%)	-3.1321	80%	20%	-5.6597	-5.0924	-2.9687	-1.6920	-1.4928	-2.2380	1	1
			Cumulative(%)	-6.3574	83%	17%	-17.7539	-14.3435	-5.2817	-1.8932	-1.5519	-2.9379	1	1
			Excess(%)	-0.2216	91%	9%	-0.7036	-0.6054	-0.1976	0.0834	0.1594	0.0244	1	1
		T→P	Durations	24.3090	15%	85%	10.8571	12.7857	22.7778	41.0000	47.0000	16.4444	1	1
			Amplitudes(%)	24.2600	29%	71%	11.6086	13.2174	22.5307	41.0526	47.5770	18.9260	1	1
			Cumulative(%)	562.6086	21%	79%	90.7797	119.0374	424.3482	1419.3887	1834.6935	238.2669	1	1
			Excess(%)	-0.3204	98%	2%	-1.9604	-1.5696	-0.2521	0.6986	0.9685	1.0412	0	0
	GMM Volatility Break (3) Switching Residual Resample (Per 2/Per 1) First Equation	P→T	Durations	2.5782	74%	27%	1.5556	1.6962	2.5000	3.7778	4.0500	2.8889	1	1
			Amplitudes(%)	-3.1391	80%	20%	-5.6796	-5.1510	-2.9656	-1.6777	-1.4851	-2.2380	1	1
			Cumulative(%)	-6.3190	83%	17%	-17.1043	-14.2710	-5.2111	-1.8855	-1.5265	-2.9379	1	1
			Excess(%)	-0.2233	91%	9%	-0.7019	-0.6083	-0.1989	0.0764	0.1510	0.0244	1	1
		T→P	Durations	24.4371	17%	83%	10.7143	12.7778	22.7500	41.6667	47.7500	16.4444	1	1
			Amplitudes(%)	24.4133	30%	70%	11.4562	12.9341	22.5811	42.3037	48.9746	18.9260	1	1
			Cumulative(%)	573.0572	21%	79%	85.6096	118.6310	423.1142	1460.6726	1942.3004	238.2669	1	1
			Excess(%)	-0.3418	98%	2%	-2.1251	-1.5817	-0.2621	0.7170	0.9614	1.0412	0	0

Appendix Table 2A: U.S. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	GMM Volatility Break (3) Switching (Vol 1/Vol2)	P→T	Durations	1.5641	23%	78%	1.2414	1.2835	1.5455	1.9091	2.0000	1.4167	1	1
			Amplitudes(%)	-0.9845	30%	70%	-1.4748	-1.3754	-0.9608	-0.6583	-0.6140	-1.0789	1	1
			Cumulative(%)	-1.2061	54%	46%	-2.5930	-2.2569	-1.0955	-0.5503	-0.4838	-1.0494	1	1
			Excess(%)	-0.0004	29%	71%	-0.0553	-0.0445	-0.0008	0.0460	0.0563	-0.0145	1	1
		T→P	Durations	6.9042	73%	27%	4.7029	5.0286	6.7692	9.1905	9.8500	7.5833	1	1
			Amplitudes(%)	7.5680	79%	21%	5.0690	5.3407	7.3773	10.3766	11.2197	8.7447	1	1
			Cumulative(%)	63.3282	89%	11%	23.0252	26.5097	55.4915	127.0221	153.7809	101.6148	1	1
			Excess(%)	0.0225	96%	4%	-0.2028	-0.1642	0.0201	0.2130	0.2652	0.2383	0	1
	GMM Volatility Break (3) Reversing (Vol 2/Vol1)	P→T	Durations	1.4948	39%	62%	1.1640	1.2000	1.4737	1.8571	1.9535	1.4167	1	1
			Amplitudes(%)	-0.8480	15%	85%	-1.3717	-1.2729	-0.8185	-0.5211	-0.4762	-1.0789	1	1
			Cumulative(%)	-1.0139	37%	63%	-2.3903	-2.0785	-0.8879	-0.3939	-0.3323	-1.0494	1	1
			Excess(%)	-0.0002	29%	71%	-0.0575	-0.0469	-0.0002	0.0465	0.0577	-0.0145	1	1
		T→P	Durations	9.4245	17%	83%	6.2367	6.6091	9.1364	13.2009	14.3571	7.5833	1	1
			Amplitudes(%)	9.3400	44%	56%	5.8949	6.2938	9.0295	13.5307	14.7731	8.7447	1	1
			Cumulative(%)	113.6368	54%	46%	37.8405	43.6746	95.9949	237.8567	287.2581	101.6148	1	1
			Excess(%)	-0.0285	98%	2%	-0.3155	-0.2646	-0.0236	0.1913	0.2337	0.2383	0	0
	GMM Volatility Break (3) Switching Residual	P→T	Durations	1.5637	24%	77%	1.2381	1.2778	1.5385	1.9130	2.0000	1.4167	1	1
			Amplitudes(%)	-1.8108	96%	4%	-2.9673	-2.7315	-1.7558	-1.1030	-0.9852	-1.0789	0	1
			Cumulative(%)	-2.5703	95%	5%	-5.7335	-4.9931	-2.3041	-1.0606	-0.9037	-1.0494	0	1
			Excess(%)	-0.1211	92%	8%	-0.3218	-0.2750	-0.1108	0.0011	0.0182	-0.0145	1	1
	Resample (Per 1/Per 2) Both Equations	T→P	Durations	8.2930	38%	62%	5.4791	5.8333	8.0833	11.5000	12.3661	7.5833	1	1
			Amplitudes(%)	9.7516	37%	63%	6.0916	6.5036	9.4298	14.0233	15.3819	8.7447	1	1
			Cumulative(%)	88.6604	72%	28%	31.0448	36.0731	77.0443	177.5427	215.8628	101.6148	1	1
			Excess(%)	-0.0086	89%	11%	-0.4074	-0.3334	-0.0181	0.3426	0.4319	0.2383	1	1
	GMM Volatility Break (3) Switching Residual	P→T	Durations	1.5577	25%	76%	1.2308	1.2743	1.5417	1.9000	2.0000	1.4167	1	1
			Amplitudes(%)	-1.7995	95%	5%	-2.9326	-2.7137	-1.7444	-1.0825	-0.9849	-1.0789	0	1
			Cumulative(%)	-2.5268	95%	5%	-5.5168	-4.8348	-2.2711	-1.0293	-0.8770	-1.0494	1	1
			Excess(%)	-0.1181	91%	9%	-0.3076	-0.2710	-0.1086	0.0036	0.0191	-0.0145	1	1
	Resample (Per 1/Per 2) First Equation	T→P	Durations	8.3001	37%	63%	5.4768	5.8666	8.0833	11.6111	12.6360	7.5833	1	1
			Amplitudes(%)	9.7844	37%	63%	6.1328	6.5628	9.4440	14.2129	15.6030	8.7447	1	1
			Cumulative(%)	87.7763	72%	28%	31.4158	36.3475	76.8451	178.3881	209.1809	101.6148	1	1
			Excess(%)	-0.0072	89%	11%	-0.4069	-0.3347	-0.0158	0.3412	0.4208	0.2383	1	1
	GMM Volatility Break (3) Reversing Residual	P→T	Durations	1.4876	40%	61%	1.1538	1.2000	1.4706	1.8500	1.9393	1.4167	1	1
			Amplitudes(%)	-1.5731	85%	15%	-2.7204	-2.4760	-1.5073	-0.8835	-0.7900	-1.0789	1	1
			Cumulative(%)	-2.1381	85%	15%	-5.1996	-4.4305	-1.8691	-0.7538	-0.6223	-1.0494	1	1
			Excess(%)	-0.1058	89%	11%	-0.3068	-0.2603	-0.0945	0.0086	0.0281	-0.0145	1	1
	Resample (Per 2/Per 1) Both Equations	T→P	Durations	10.6891	7%	93%	6.9213	7.3333	10.3889	15.3077	16.5644	7.5833	1	1
			Amplitudes(%)	11.1002	20%	80%	6.7648	7.2457	10.6989	16.4156	17.8641	8.7447	1	1
			Cumulative(%)	129.4005	43%	57%	44.2380	50.4143	110.6976	263.9962	329.6832	101.6148	1	1
			Excess(%)	-0.1133	95%	5%	-0.5851	-0.4798	-0.1087	0.2429	0.3384	0.2383	1	1
	GMM Volatility Break (3) Switching Residual	P→T	Durations	1.4863	40%	61%	1.1450	1.1875	1.4706	1.8421	1.9333	1.4167	1	1
			Amplitudes(%)	-1.5831	86%	14%	-2.7697	-2.5177	-1.5116	-0.8842	-0.7980	-1.0789	1	1
			Cumulative(%)	-2.1548	86%	14%	-5.3188	-4.4878	-1.8697	-0.7354	-0.6219	-1.0494	1	1
			Excess(%)	-0.1064	89%	11%	-0.3137	-0.2706	-0.0934	0.0088	0.0264	-0.0145	1	1
	Resample (Per 2/Per 1) First Equation	T→P	Durations	10.6508	7%	93%	6.9272	7.3333	10.2632	15.4286	16.7098	7.5833	1	1
			Amplitudes(%)	11.0634	21%	79%	6.7115	7.2131	10.6196	16.4709	18.0965	8.7447	1	1
			Cumulative(%)	130.7648	44%	56%	43.4335	50.5913	109.5048	288.1602	341.7431	101.6148	1	1
			Excess(%)	-0.1120	95%	5%	-0.5786	-0.4803	-0.1038	0.2339	0.3120	0.2383	0	1

Appendix Table 2B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	GMM Volatility	P→T	Durations	1.6850	88%	13%	1.1765	1.2330	1.6471	2.2667	2.4286	2.0667	1	1
	Break (3)		Amplitudes(%)	-1.0018	1%	99%	-1.4662	-1.3803	-0.9850	-0.6758	-0.6214	-1.6087	0	0
	Switching (Vol		Cumulative(%)	-1.1329	1%	99%	-2.4106	-2.1018	-1.0365	-0.4966	-0.4272	-3.0887	0	0
	1/Vol2)		Excess(%)	0.0019	28%	72%	-0.0724	-0.0578	0.0015	0.0613	0.0724	-0.0163	1	1
		T→P	Durations	8.7408	39%	62%	6.5000	6.7647	8.4667	11.5505	12.2857	8.0714	1	1
			Amplitudes(%)	8.0706	40%	60%	5.9169	6.1595	7.9282	10.5064	11.0744	7.6091	1	1
			Cumulative(%)	50.2369	48%	52%	22.5644	24.4064	41.9427	106.8980	122.7930	41.1670	1	1
			Excess(%)	0.0638	5%	95%	-0.2623	-0.2074	0.0628	0.3414	0.3973	-0.2132	0	1
	GMM Volatility	P→T	Durations	1.5977	86%	14%	1.0000	1.0000	1.5000	2.4286	2.7143	2.0667	1	1
	Break (3)		Amplitudes(%)	-0.8967	3%	97%	-1.6328	-1.4743	-0.8532	-0.4534	-0.3873	-1.6087	0	1
	Reversing (Vol		Cumulative(%)	-0.9853	2%	98%	-2.8503	-2.3262	-0.7902	-0.2717	-0.2226	-3.0887	0	0
	2/Vol1)		Excess(%)	0.0038	29%	71%	-0.1020	-0.0788	0.0000	0.0888	0.1101	-0.0163	1	1
		T→P	Durations	16.0053	19%	81%	6.0000	6.5000	15.7000	28.8333	31.5500	8.0714	1	1
			Amplitudes(%)	12.8179	19%	81%	5.3358	5.8090	12.4163	21.8539	23.9627	7.6091	1	1
			Cumulative(%)	293.6796	23%	77%	18.1558	21.3125	186.5476	953.8459	1104.4644	41.1670	1	1
			Excess(%)	0.0067	20%	80%	-0.5289	-0.4317	0.0070	0.4446	0.5417	-0.2132	1	1
	GMM Volatility	P→T	Durations	1.5689	94%	7%	1.1111	1.1500	1.5294	2.1213	2.2667	2.0667	1	1
	Break (3)		Amplitudes(%)	-2.0021	75%	25%	-3.2691	-3.0367	-1.9486	-1.1939	-1.0767	-1.6087	1	1
	Switching		Cumulative(%)	-2.0669	15%	85%	-5.1027	-4.2133	-1.8051	-0.8152	-0.7072	-3.0887	1	1
	Residual		Excess(%)	0.0353	27%	73%	-0.1474	-0.1113	0.0255	0.2076	0.2629	-0.0163	1	1
	Resample (Per	T→P	Durations	9.9848	9%	92%	7.5000	7.8377	9.8333	12.6429	13.4167	8.0714	1	1
	1/Per 2) Both		Amplitudes(%)	9.3808	11%	89%	6.7783	7.1268	9.2336	12.1277	12.8432	7.6091	1	1
	Equations		Cumulative(%)	65.6262	14%	86%	30.6170	34.2580	60.1956	116.6231	134.7945	41.1670	1	1
			Excess(%)	-0.0133	21%	79%	-0.6255	-0.4863	0.0038	0.4074	0.4946	-0.2132	1	1
	GMM Volatility	P→T	Durations	1.5790	93%	7%	1.1111	1.1538	1.5333	2.1538	2.2941	2.0667	1	1
	Break (3)		Amplitudes(%)	-1.9971	74%	26%	-3.2913	-3.0253	-1.9283	-1.1933	-1.0855	-1.6087	1	1
	Switching		Cumulative(%)	-2.0852	16%	84%	-5.0939	-4.2983	-1.7980	-0.8047	-0.7099	-3.0887	1	1
	Residual		Excess(%)	0.0316	28%	72%	-0.1541	-0.1110	0.0227	0.1967	0.2601	-0.0163	1	1
	Resample (Per	T→P	Durations	9.9911	8%	92%	7.5000	7.8500	9.8333	12.6291	13.2792	8.0714	1	1
	1/Per 2) First		Amplitudes(%)	9.3950	11%	89%	6.8323	7.1882	9.2642	12.1218	12.8774	7.6091	1	1
	Equation		Cumulative(%)	66.2771	14%	86%	31.2179	34.5781	60.1763	118.6951	137.2588	41.1670	1	1
			Excess(%)	-0.0047	20%	80%	-0.5987	-0.4813	0.0099	0.4273	0.5298	-0.2132	1	1
	GMM Volatility	P→T	Durations	1.3593	97%	3%	1.0000	1.0000	1.3000	1.9231	2.1000	2.0667	0	1
	Break (3)		Amplitudes(%)	-1.6657	48%	52%	-3.0673	-2.7890	-1.5785	-0.8428	-0.7303	-1.6087	1	1
	Reversing		Cumulative(%)	-1.4810	7%	93%	-4.4064	-3.5235	-1.1828	-0.4873	-0.4124	-3.0887	1	1
	Residual		Excess(%)	0.0225	25%	75%	-0.1448	-0.1085	0.0065	0.1809	0.2522	-0.0163	1	1
	Resample (Per	T→P	Durations	14.8551	0%	100%	10.1765	10.6962	14.3846	20.6667	22.4143	8.0714	0	0
	2/Per 1) Both		Amplitudes(%)	12.3174	1%	99%	8.3242	8.7254	11.9283	17.0281	18.6570	7.6091	0	0
	Equations		Cumulative(%)	155.4531	0%	100%	55.4286	63.4890	130.9045	324.8324	394.8310	41.1670	0	0
			Excess(%)	-0.0240	22%	78%	-0.7167	-0.5601	0.0076	0.4072	0.5069	-0.2132	1	1
	GMM Volatility	P→T	Durations	1.3623	98%	2%	1.0000	1.0000	1.3077	1.9091	2.0000	2.0667	0	0
	Break (3)		Amplitudes(%)	-1.6693	49%	51%	-3.0124	-2.7264	-1.6008	-0.8390	-0.7272	-1.6087	1	1
	Switching		Cumulative(%)	-1.4998	7%	93%	-4.1242	-3.4353	-1.2058	-0.4939	-0.4071	-3.0887	1	1
	Residual		Excess(%)	0.0201	27%	73%	-0.1550	-0.1109	0.0050	0.1821	0.2530	-0.0163	1	1
	Resample (Per	T→P	Durations	14.8008	0%	100%	10.0000	10.5625	14.3333	20.6458	22.2500	8.0714	0	0
	2/Per 1) First		Amplitudes(%)	12.2812	1%	99%	8.2330	8.7320	11.9028	17.2082	18.5612	7.6091	0	0
	Equation		Cumulative(%)	153.6137	1%	99%	54.3404	62.1370	129.5051	323.2534	405.6100	41.1670	0	0
			Excess(%)	-0.0444	25%	75%	-0.7629	-0.6185	-0.0086	0.3977	0.4920	-0.2132	1	1

Appendix Table 2B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	GMM Volatility Break (3) Switching (Vol 1/Vol2)	P→T	Durations	2.4454	99%	2%	1.6000	1.7143	2.4000	3.3333	3.5556	3.7143	0	0
			Amplitudes(%)	-1.5018	0%	100%	-2.1696	-2.0448	-1.4831	-1.0239	-0.9380	-2.7174	0	0
			Cumulative(%)	-2.0300	0%	100%	-4.2431	-3.7112	-1.8472	-0.9491	-0.8292	-6.3894	0	0
			Excess(%)	0.0003	27%	73%	-0.1428	-0.1164	0.0007	0.1167	0.1427	-0.0406	1	1
		T→P	Durations	15.7341	83%	17%	8.8397	9.5000	14.5000	25.7500	30.3333	19.6667	1	1
		Missing	Amplitudes(%)	13.7504	82%	18%	7.5402	8.1963	12.7223	22.5838	26.4066	16.9030	1	1
		2	Cumulative(%)	176.0252	84%	16%	43.4235	51.7118	132.2446	438.5115	577.7320	258.2457	1	1
			Excess(%)	0.0248	20%	80%	-0.6937	-0.5310	0.0248	0.5980	0.7434	-0.2523	1	1
	GMM Volatility Break (3) Reversing (Vol 2/Vol1)	P→T	Durations	2.4210	93%	7%	1.0000	1.3333	2.2500	4.0000	4.5000	3.7143	1	1
		Missing	Amplitudes(%)	-1.5024	2%	98%	-2.6606	-2.4157	-1.4576	-0.7535	-0.6327	-2.7174	0	0
		66	Cumulative(%)	-2.0148	2%	98%	-5.8491	-4.8293	-1.6233	-0.5795	-0.4685	-6.3894	0	0
			Excess(%)	-0.0003	33%	67%	-0.2439	-0.1905	0.0000	0.1904	0.2434	-0.0406	1	1
		T→P	Durations	14.1080	83%	17%	5.0000	5.6000	12.0000	30.0000	38.6667	19.6667	1	1
		Missing	Amplitudes(%)	12.3523	82%	18%	3.9779	4.8114	10.4329	26.7082	33.6218	16.9030	1	1
		401	Cumulative(%)	134.9478	88%	12%	10.4779	14.6807	74.9881	439.0391	680.4279	258.2457	1	1
			Excess(%)	0.0331	27%	73%	-1.1721	-0.8949	0.0331	0.9738	1.3144	-0.2523	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) Both Equations	P→T	Durations	2.0259	100%	1%	1.2500	1.3333	2.0000	2.9231	3.2000	3.7143	0	0
			Amplitudes(%)	-3.0842	63%	37%	-4.9425	-4.5954	-2.9714	-1.9432	-1.7844	-2.7174	1	1
			Cumulative(%)	-3.4679	7%	93%	-8.3827	-7.0664	-2.9697	-1.4201	-1.2560	-6.3894	1	1
			Excess(%)	0.0515	26%	74%	-0.2572	-0.1989	0.0394	0.3439	0.4292	-0.0406	1	1
		T→P	Durations	16.1449	81%	19%	9.4226	10.2000	15.2222	25.0000	27.5857	19.6667	1	1
			Amplitudes(%)	14.5764	76%	24%	8.4036	9.0912	13.7257	22.6368	24.9461	16.9030	1	1
			Cumulative(%)	194.7254	79%	21%	53.2163	61.7760	155.0220	443.9064	575.7920	258.2457	1	1
			Excess(%)	0.0815	19%	81%	-0.8590	-0.6564	0.0848	0.7948	0.9700	-0.2523	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) First Equation	P→T	Durations	2.0276	99%	1%	1.2500	1.3333	2.0000	2.9231	3.2000	3.7143	0	0
			Amplitudes(%)	-3.0946	64%	36%	-4.9490	-4.5786	-3.0054	-1.9128	-1.7589	-2.7174	1	1
			Cumulative(%)	-3.5006	8%	92%	-8.9925	-7.4175	-2.9661	-1.4219	-1.2350	-6.3894	1	1
			Excess(%)	0.0514	25%	75%	-0.2748	-0.2012	0.0380	0.3342	0.4292	-0.0406	1	1
		T→P	Durations	16.3108	81%	19%	9.5000	10.1909	15.4545	25.3333	28.3333	19.6667	1	1
			Amplitudes(%)	14.7159	76%	24%	8.4114	9.1146	13.9345	22.8115	25.4579	16.9030	1	1
			Cumulative(%)	198.5400	79%	21%	52.3324	62.8092	161.3391	466.9850	577.3940	258.2457	1	1
			Excess(%)	0.0828	19%	81%	-0.8779	-0.6562	0.0951	0.7787	0.9618	-0.2523	1	1
	GMM Volatility Break (3) Reversing Residual Resample (Per 2/Per 1) Both Equations	P→T	Durations	1.7515	99%	1%	1.0000	1.0000	1.6364	2.8000	3.1667	3.7143	0	0
		Missing	Amplitudes(%)	-2.9019	51%	49%	-5.2942	-4.7790	-2.7303	-1.5823	-1.4113	-2.7174	1	1
		2	Cumulative(%)	-2.7953	5%	95%	-8.1982	-6.5719	-2.1987	-0.9822	-0.8537	-6.3894	1	1
			Excess(%)	0.0435	25%	75%	-0.3223	-0.2252	0.0136	0.3746	0.5139	-0.0406	1	1
		T→P	Durations	28.2449	24%	76%	10.3667	12.6667	25.5714	51.4167	60.8333	19.6667	1	1
		Missing	Amplitudes(%)	22.5652	30%	70%	8.9462	10.6695	20.4501	41.0307	47.8737	16.9030	1	1
		11	Cumulative(%)	632.6793	26%	74%	57.9996	92.4059	443.0934	1777.9238	2286.7334	258.2457	1	1
			Excess(%)	-0.0695	31%	69%	-1.5476	-1.1487	0.0010	0.8291	1.0476	-0.2523	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 2/Per 1) First Equation	P→T	Durations	1.7416	99%	1%	1.0000	1.0000	1.6250	2.8000	3.2000	3.7143	0	0
		Missing	Amplitudes(%)	-2.8836	51%	49%	-5.2625	-4.6780	-2.7319	-1.5542	-1.3774	-2.7174	1	1
		2	Cumulative(%)	-2.8059	6%	94%	-8.9310	-6.8337	-2.1790	-0.9421	-0.8150	-6.3894	1	1
			Excess(%)	0.0369	25%	75%	-0.3094	-0.2222	0.0132	0.3474	0.4775	-0.0406	1	1
		T→P	Durations	28.3200	23%	77%	10.0000	12.9545	25.8571	51.0000	62.0000	19.6667	1	1
		Missing	Amplitudes(%)	22.5942	30%	70%	9.0017	10.8937	20.7167	41.1308	49.2615	16.9030	1	1
		24	Cumulative(%)	635.6150	26%	74%	55.3881	92.4338	442.4367	1829.5517	2425.0364	258.2457	1	1
			Excess(%)	-0.0675	32%	68%	-1.5499	-1.1185	-0.0149	0.8158	1.0675	-0.2523	1	1

Appendix Table 2B: U.K. Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	GMM Volatility Break (3) Switching (Vol 1/Vol2)	P→T	Durations	1.2707	99%	1%	1.0938	1.1111	1.2581	1.4643	1.5000	1.5385	0	0
			Amplitudes(%)	-0.8060	1%	99%	-1.0672	-1.0122	-0.7998	-0.6193	-0.5841	-1.1539	0	0
			Cumulative(%)	-0.6104	0%	100%	-1.0056	-0.9165	-0.5854	-0.3904	-0.3608	-1.4308	0	0
			Excess(%)	0.0001	21%	79%	-0.0339	-0.0281	0.0004	0.0271	0.0333	-0.0123	1	1
		T→P	Durations	4.0078	69%	32%	2.7500	2.8750	3.8519	5.6667	6.0426	4.2800	1	1
			Amplitudes(%)	4.3773	61%	39%	3.1045	3.2461	4.2664	5.8978	6.2151	4.4987	1	1
			Cumulative(%)	18.9888	70%	30%	6.4734	7.1325	14.2924	47.0130	54.4675	20.0276	1	1
			Excess(%)	0.0057	65%	35%	-0.1354	-0.1133	0.0049	0.1281	0.1519	0.0326	1	1
	GMM Volatility Break (3) Reversing (Vol 2/Vol1)	P→T	Durations	1.2511	95%	6%	1.0000	1.0000	1.2308	1.5385	1.6250	1.5385	0	1
			Amplitudes(%)	-0.7576	3%	97%	-1.1691	-1.0853	-0.7433	-0.4830	-0.4392	-1.1539	0	1
			Cumulative(%)	-0.5723	1%	99%	-1.2107	-1.0411	-0.5178	-0.2794	-0.2483	-1.4308	0	0
			Excess(%)	0.0004	26%	74%	-0.0507	-0.0391	0.0000	0.0425	0.0512	-0.0123	1	1
		T→P	Durations	8.2569	26%	74%	2.5000	2.7692	8.0000	15.6364	17.2000	4.2800	1	1
			Amplitudes(%)	7.1995	25%	75%	2.8020	3.0928	6.9504	12.5150	13.7848	4.4987	1	1
			Cumulative(%)	149.4410	29%	71%	5.1348	6.2981	95.7761	482.5951	566.2390	20.0276	1	1
			Excess(%)	-0.0250	67%	33%	-0.2973	-0.2392	-0.0207	0.1759	0.2167	0.0326	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) Both Equations	P→T	Durations	1.1871	100%	0%	1.0357	1.0588	1.1786	1.3455	1.3871	1.5385	0	0
			Amplitudes(%)	-1.4757	83%	17%	-2.2344	-2.0803	-1.4473	-0.9794	-0.9088	-1.1539	1	1
			Cumulative(%)	-0.9733	8%	93%	-1.6712	-1.5147	-0.9270	-0.5767	-0.5270	-1.4308	1	1
			Excess(%)	0.0135	23%	77%	-0.0642	-0.0486	0.0086	0.0873	0.1129	-0.0123	1	1
		T→P	Durations	5.4924	9%	91%	3.8714	4.0728	5.3772	7.2727	7.7727	4.2800	1	1
			Amplitudes(%)	5.7398	9%	91%	4.0286	4.2687	5.6081	7.6282	8.1197	4.4987	1	1
			Cumulative(%)	31.3709	21%	79%	12.9801	14.5774	28.2499	59.1178	67.4312	20.0276	1	1
			Excess(%)	-0.0373	68%	32%	-0.3625	-0.2927	-0.0272	0.1841	0.2325	0.0326	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 1/Per 2) First Equation	P→T	Durations	1.1891	100%	0%	1.0357	1.0588	1.1818	1.3500	1.3871	1.5385	0	0
			Amplitudes(%)	-1.4903	84%	16%	-2.2440	-2.0984	-1.4617	-0.9860	-0.9168	-1.1539	1	1
			Cumulative(%)	-0.9853	8%	92%	-1.7204	-1.5581	-0.9360	-0.5762	-0.5295	-1.4308	1	1
			Excess(%)	0.0153	22%	78%	-0.0610	-0.0463	0.0103	0.0935	0.1169	-0.0123	1	1
		T→P	Durations	5.4910	9%	91%	3.8524	4.0857	5.3667	7.3333	7.8341	4.2800	1	1
			Amplitudes(%)	5.7492	9%	91%	4.0672	4.2807	5.6223	7.7007	8.1779	4.4987	1	1
			Cumulative(%)	31.5642	20%	80%	12.9654	14.5942	28.1737	60.3615	69.4049	20.0276	1	1
			Excess(%)	-0.0380	68%	32%	-0.3617	-0.2949	-0.0263	0.1778	0.2174	0.0326	1	1
	GMM Volatility Break (3) Reversing Residual Resample (Per 2/Per 1) Both Equations	P→T	Durations	1.1373	100%	0%	1.0000	1.0000	1.1250	1.3000	1.3515	1.5385	0	0
			Amplitudes(%)	-1.3537	66%	34%	-2.2606	-2.0819	-1.3070	-0.7706	-0.7010	-1.1539	1	1
			Cumulative(%)	-0.8470	6%	94%	-1.6620	-1.4696	-0.7846	-0.4307	-0.3818	-1.4308	1	1
			Excess(%)	0.0115	22%	78%	-0.0739	-0.0535	0.0026	0.0942	0.1278	-0.0123	1	1
		T→P	Durations	9.6050	0%	100%	6.2981	6.7083	9.2632	13.6923	15.0000	4.2800	0	0
			Amplitudes(%)	8.4016	0%	100%	5.5178	5.8751	8.0740	11.9406	13.0124	4.4987	0	0
			Cumulative(%)	94.5478	0%	100%	32.3437	36.8974	79.6154	203.1724	241.3295	20.0276	0	0
			Excess(%)	-0.0572	67%	33%	-0.5121	-0.4037	-0.0335	0.2082	0.2587	0.0326	1	1
	GMM Volatility Break (3) Switching Residual Resample (Per 2/Per 1) First Equation	P→T	Durations	1.1386	100%	0%	1.0000	1.0000	1.1250	1.3158	1.3750	1.5385	0	0
			Amplitudes(%)	-1.3551	66%	34%	-2.3013	-2.0988	-1.3126	-0.7673	-0.6846	-1.1539	1	1
			Cumulative(%)	-0.8463	6%	94%	-1.6621	-1.4556	-0.7920	-0.4251	-0.3757	-1.4308	1	1
			Excess(%)	0.0106	22%	78%	-0.0691	-0.0510	0.0019	0.0935	0.1278	-0.0123	1	1
		T→P	Durations	9.7338	0%	100%	6.3077	6.6800	9.4118	14.0000	15.4167	4.2800	0	0
			Amplitudes(%)	8.4890	0%	100%	5.5276	5.8869	8.1782	12.0441	13.1680	4.4987	0	0
			Cumulative(%)	97.1649	0%	100%	33.1218	37.9865	81.1496	206.7967	256.8943	20.0276	0	0
			Excess(%)	-0.0499	65%	35%	-0.4900	-0.3951	-0.0277	0.2168	0.2711	0.0326	1	1

Appendix Table 2C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,5)	GMM Volatility	P→T	Durations	2.6068	61%	39%	1.0000	1.2000	2.4000	4.6667	5.2500	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.8935	24%	76%	-2.4758	-2.0669	-0.7450	-0.2048	-0.1504	-1.1728	1	1
	Switching (Vol	17	Cumulative(%)	-2.3229	33%	67%	-10.1846	-7.6411	-1.3550	-0.1354	-0.0852	-2.2481	1	1
	1/Vol2)		Excess(%)	-0.0001	84%	16%	-0.1230	-0.0934	0.0000	0.0958	0.1338	0.0447	1	1
		T→P	Durations	23.4247	65%	36%	8.4000	9.8571	20.0000	49.0000	60.0000	23.6667	1	1
		Missing	Amplitudes(%)	24.0598	67%	33%	5.7736	7.7307	19.8113	55.0965	68.7489	25.2807	1	1
		115	Cumulative(%)	552.2002	56%	44%	33.4889	53.8592	298.8355	1816.0470	2583.8373	348.9563	1	1
			Excess(%)	0.1471	74%	26%	-1.5355	-1.1049	0.0945	1.5686	2.1913	0.4702	1	1
	GMM Volatility	P→T	Durations	2.4915	67%	33%	1.0000	1.0000	2.0000	5.3333	6.3333	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.8086	21%	79%	-3.0303	-2.3029	-0.5642	-0.0903	-0.0576	-1.1728	1	1
	Reversing (Vol	162	Cumulative(%)	-2.1340	25%	75%	-12.4013	-8.3496	-0.7959	-0.0481	-0.0292	-2.2481	1	1
	2/Vol1)		Excess(%)	0.0002	85%	15%	-0.1547	-0.1106	0.0000	0.1154	0.1622	0.0447	1	1
		T→P	Durations	28.8554	54%	47%	5.0000	6.0000	22.0000	74.0000	101.0000	23.6667	1	1
		Missing	Amplitudes(%)	27.5761	60%	40%	2.1753	3.4402	20.6581	73.4423	98.1439	25.2807	1	1
		743	Cumulative(%)	832.3866	54%	46%	5.4678	12.0656	298.9061	3310.0529	5137.9728	348.9563	1	1
			Excess(%)	-0.3170	82%	18%	-3.8695	-2.6360	-0.0864	1.3038	1.8007	0.4702	1	1
	GMM Volatility	P→T	Durations	1.8201	84%	16%	1.0000	1.0000	1.5000	4.0000	5.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3833	5%	95%	-1.4788	-1.1350	-0.2544	-0.0251	-0.0128	-1.1728	0	1
	Switching	1032	Cumulative(%)	-0.6764	6%	94%	-3.9911	-2.6099	-0.2159	-0.0129	-0.0064	-2.2481	1	1
	Residual		Excess(%)	-0.0179	96%	4%	-0.1609	-0.1086	0.0000	0.0330	0.0600	0.0447	0	1
	Resample (Per	T→P	Durations	34.9563	37%	63%	4.0000	6.0000	29.0000	83.5000	94.0000	23.6667	1	1
	1/Per 2) Both	Missing	Amplitudes(%)	29.8238	52%	48%	1.2913	2.3731	23.9298	76.2631	89.8114	25.2807	1	1
	Equations	2681	Cumulative(%)	849.6781	45%	55%	2.6713	7.4345	429.0288	3142.9583	4018.3124	348.9563	1	1
			Excess(%)	-0.1856	78%	22%	-2.8726	-2.2444	-0.1315	1.7551	2.3737	0.4702	1	1
	GMM Volatility	P→T	Durations	1.8696	82%	18%	1.0000	1.0000	1.5000	4.0000	5.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.4045	5%	95%	-1.5175	-1.1875	-0.2727	-0.0287	-0.0173	-1.1728	1	1
	Switching	1041	Cumulative(%)	-0.7466	8%	92%	-4.4649	-3.0084	-0.2432	-0.0145	-0.0086	-2.2481	1	1
	Residual		Excess(%)	-0.0193	96%	4%	-0.1533	-0.1170	0.0000	0.0328	0.0608	0.0447	0	1
	Resample (Per	T→P	Durations	34.9525	37%	63%	4.5000	6.0000	29.5000	85.0000	97.0000	23.6667	1	1
	1/Per 2) First	Missing	Amplitudes(%)	29.7428	52%	48%	1.4654	2.2969	24.2621	77.9723	92.1003	25.2807	1	1
	Equation	2688	Cumulative(%)	864.1640	44%	56%	3.1636	6.6223	434.1202	3341.0008	4498.9944	348.9563	1	1
			Excess(%)	-0.1703	79%	21%	-2.7310	-2.1338	-0.1555	1.8123	2.5152	0.4702	1	1
	GMM Volatility	P→T	Durations	1.7843	83%	17%	1.0000	1.0000	1.3333	4.0000	5.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3748	6%	94%	-1.5709	-1.2922	-0.2229	-0.0224	-0.0107	-1.1728	1	1
	Reversing	2145	Cumulative(%)	-0.6478	7%	93%	-4.4501	-2.9368	-0.1484	-0.0112	-0.0053	-2.2481	1	1
	Residual		Excess(%)	-0.0172	96%	4%	-0.1594	-0.1119	0.0000	0.0384	0.0653	0.0447	0	1
	Resample (Per	T→P	Durations	27.9382	58%	42%	4.0000	5.0000	20.0000	84.0000	110.0000	23.6667	1	1
	2/Per 1) Both	Missing	Amplitudes(%)	22.4737	70%	30%	1.0688	1.4967	14.6542	75.0518	102.7113	25.2807	1	1
	Equations	3905	Cumulative(%)	642.6204	66%	34%	1.7979	3.2934	160.0302	3197.0973	5466.4324	348.9563	1	1
			Excess(%)	-0.2572	85%	15%	-2.6981	-1.8509	-0.1488	1.1528	1.6882	0.4702	1	1
	GMM Volatility	P→T	Durations	1.8124	83%	17%	1.0000	1.0000	1.5000	4.0000	5.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3877	6%	94%	-1.7166	-1.2667	-0.2263	-0.0222	-0.0089	-1.1728	1	1
	Switching	2052	Cumulative(%)	-0.7126	7%	93%	-4.9361	-3.0084	-0.1551	-0.0111	-0.0045	-2.2481	1	1
	Residual		Excess(%)	-0.0200	96%	4%	-0.1952	-0.1212	0.0000	0.0364	0.0610	0.0447	0	1
	Resample (Per	T→P	Durations	28.8876	57%	43%	4.0000	4.5000	20.0000	95.0000	122.5000	23.6667	1	1
	2/Per 1) First	Missing	Amplitudes(%)	23.3589	68%	32%	0.9198	1.4155	14.8355	83.1971	106.8409	25.2807	1	1
	Equation	3866	Cumulative(%)	728.7784	66%	34%	1.7606	2.9478	151.2441	4024.4131	6380.2014	348.9563	1	1
			Excess(%)	-0.2551	84%	16%	-2.5732	-1.9899	-0.1464	1.2408	1.7095	0.4702	1	1

Appendix Table 2C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(2,2,5)	GMM Volatility	P→T	Durations	3.6692	70%	38%	1.5000	2.0000	3.3333	6.5000	7.6667	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-1.3613	17%	83%	-3.7613	-3.0975	-1.1344	-0.3808	-0.2882	-1.9868	1	1
	Switching (Vol	181	Cumulative(%)	-3.9340	31%	69%	-17.7712	-12.5100	-2.2645	-0.3210	-0.2429	-3.9013	1	1
	1/Vol2)		Excess(%)	0.0007	84%	16%	-0.2116	-0.1572	0.0000	0.1599	0.2181	0.0782	1	1
		T→P	Durations	29.7893	74%	27%	6.0000	8.0000	24.5000	72.5000	87.0000	36.0000	1	1
		Missing	Amplitudes(%)	29.4450	77%	23%	2.5988	4.8052	23.3437	76.7813	96.4336	38.6480	1	1
		798	Cumulative(%)	797.7234	73%	27%	7.7064	21.8293	398.1018	2989.7103	4359.8401	842.6579	1	1
			Excess(%)	0.0655	89%	11%	-2.6664	-1.9638	0.0217	2.2189	3.1299	1.3878	1	1
	GMM Volatility	P→T	Durations	3.5314	74%	36%	1.0000	1.5000	3.0000	7.0000	8.0000	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-1.2647	17%	83%	-4.3066	-3.3394	-0.9445	-0.2357	-0.1729	-1.9868	1	1
	Reversing (Vol	802	Cumulative(%)	-3.5953	25%	75%	-19.7577	-13.7687	-1.5620	-0.1907	-0.1290	-3.9013	1	1
	2/Vol1)		Excess(%)	0.0010	84%	16%	-0.2371	-0.1815	0.0000	0.1849	0.2585	0.0782	1	1
		T→P	Durations	27.6419	77%	24%	3.0000	4.5833	19.0000	80.7500	107.0000	36.0000	1	1
		Missing	Amplitudes(%)	25.3950	79%	21%	1.1769	1.8002	17.1849	77.4381	102.1230	38.6480	1	1
		2204	Cumulative(%)	748.6394	78%	22%	1.9635	4.4188	194.1609	3837.1264	5309.4910	842.6579	1	1
			Excess(%)	-0.2285	93%	7%	-3.8113	-2.5224	-0.0548	1.7227	2.3035	1.3878	1	1
	GMM Volatility	P→T	Durations	2.7865	89%	20%	1.0000	1.0000	2.2500	5.5000	6.0000	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-0.6920	3%	97%	-2.1672	-1.7360	-0.5420	-0.1357	-0.0976	-1.9868	0	1
	Switching	2481	Cumulative(%)	-1.4312	8%	92%	-6.5953	-4.8794	-0.7213	-0.1034	-0.0752	-3.9013	1	1
	Residual		Excess(%)	-0.0417	97%	3%	-0.2522	-0.1843	-0.0275	0.0628	0.0945	0.0782	0	1
	Resample (Per	T→P	Durations	34.8381	61%	40%	3.0000	4.0000	29.0000	87.0000	95.5000	36.0000	1	1
	1/Per 2) Both	Missing	Amplitudes(%)	28.6863	71%	29%	0.7255	1.1999	22.7782	76.9759	88.0187	38.6480	1	1
	Equations	4163	Cumulative(%)	835.9628	68%	32%	1.1093	2.0958	368.3180	3305.1129	4200.7623	842.6579	1	1
			Excess(%)	-0.2314	93%	7%	-3.2764	-2.6121	-0.1331	1.9678	2.8304	1.3878	1	1
	GMM Volatility	P→T	Durations	2.8085	89%	21%	1.0000	1.0000	2.3333	5.5000	6.0000	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-0.6999	4%	96%	-2.2487	-1.8214	-0.5497	-0.1319	-0.0846	-1.9868	0	1
	Switching	2374	Cumulative(%)	-1.4332	8%	92%	-7.1637	-5.0118	-0.7305	-0.1006	-0.0738	-3.9013	1	1
	Residual		Excess(%)	-0.0383	96%	4%	-0.2548	-0.1830	-0.0251	0.0694	0.1012	0.0782	0	1
	Resample (Per	T→P	Durations	35.7445	59%	42%	3.0000	4.0000	30.0000	87.0000	98.0000	36.0000	1	1
	1/Per 2) First	Missing	Amplitudes(%)	29.4481	71%	29%	0.6255	1.0705	23.6943	77.5988	90.8521	38.6480	1	1
	Equation	4168	Cumulative(%)	878.0715	67%	33%	1.0607	1.8490	404.4217	3312.9590	4386.9958	842.6579	1	1
			Excess(%)	-0.1254	90%	10%	-2.8688	-2.1555	-0.0731	2.2585	2.8809	1.3878	1	1
	GMM Volatility	P→T	Durations	2.7233	90%	20%	1.0000	1.0000	2.0000	5.0000	6.0000	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-0.6704	3%	97%	-2.3086	-1.7040	-0.5078	-0.1307	-0.0948	-1.9868	0	1
	Reversing	3405	Cumulative(%)	-1.3425	7%	93%	-6.7290	-4.5953	-0.6775	-0.0948	-0.0712	-3.9013	1	1
	Residual		Excess(%)	-0.0386	96%	4%	-0.2534	-0.1878	-0.0233	0.0669	0.1057	0.0782	0	1
	Resample (Per	T→P	Durations	22.4967	81%	20%	2.0000	3.0000	17.7500	57.0000	80.7500	36.0000	1	1
	2/Per 1) Both	Missing	Amplitudes(%)	17.6638	86%	14%	0.4076	0.6032	11.6566	53.2622	71.2336	38.6480	1	1
	Equations	4696	Cumulative(%)	397.1259	87%	13%	0.4217	0.8106	107.3759	1447.2264	2968.9936	842.6579	1	1
			Excess(%)	-0.1679	93%	7%	-2.7602	-1.7229	-0.0726	1.4877	2.1243	1.3878	1	1
	GMM Volatility	P→T	Durations	2.7755	90%	20%	1.0000	1.0000	2.0000	5.2500	6.0000	4.0000	1	1
	Break (3)	Missing	Amplitudes(%)	-0.7098	4%	96%	-2.4788	-1.9069	-0.5432	-0.1279	-0.0909	-1.9868	0	1
	Switching	3368	Cumulative(%)	-1.4368	9%	91%	-7.4646	-5.5114	-0.7029	-0.0993	-0.0693	-3.9013	1	1
	Residual		Excess(%)	-0.0383	95%	5%	-0.2688	-0.1933	-0.0221	0.0778	0.1148	0.0782	0	1
	Resample (Per	T→P	Durations	22.8634	83%	17%	3.0000	3.0000	18.0000	57.5000	84.0000	36.0000	1	1
	2/Per 1) First	Missing	Amplitudes(%)	17.3952	91%	9%	0.3096	0.4546	11.8469	50.4188	77.4393	38.6480	1	1
	Equation	4667	Cumulative(%)	398.5073	90%	10%	0.3559	0.6725	106.4637	1337.8082	3207.1791	842.6579	1	1
			Excess(%)	-0.2307	95%	5%	-2.7456	-2.1692	-0.0941	1.3363	1.6130	1.3878	0	1

Appendix Table 2C: Australia Simulation Peak to Trough and Trough to Peak Characteristics by Algorithm, (Minimum Phase Length, Window Range ±, Minimum Total Cycle)

				Average	%>Avg	%<Avg	2.50%	5%	50%	95%	97.50%	Actual	Actual <90%>	Actual <95%>
(1,1,2)	GMM Volatility	P→T	Durations	2.0559	85%	16%	1.0000	1.1250	2.0000	3.4286	3.8333	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.7056	13%	87%	-1.8617	-1.5693	-0.6085	-0.1898	-0.1386	-1.1728	1	1
	Switching (Vol	19	Cumulative(%)	-1.5344	19%	81%	-6.9442	-5.1375	-0.9145	-0.1167	-0.0809	-2.2481	1	1
	1/Vol2)		Excess(%)	-0.0004	90%	10%	-0.0832	-0.0632	0.0000	0.0636	0.0874	0.0447	1	1
		T→P	Durations	17.6027	82%	19%	5.4365	6.6000	14.6000	38.5000	47.5000	23.6667	1	1
		Missing	Amplitudes(%)	18.2654	82%	18%	3.8482	5.2244	14.6004	43.3174	54.5497	25.2807	1	1
		71	Cumulative(%)	408.7879	67%	33%	20.1744	34.4037	221.9405	1352.1166	1926.8515	348.9563	1	1
			Excess(%)	0.0889	82%	18%	-1.0879	-0.8219	0.0406	1.1503	1.5931	0.4702	1	1
	GMM Volatility	P→T	Durations	1.9737	83%	17%	1.0000	1.0000	1.7500	3.6667	4.3333	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.6385	13%	87%	-2.1531	-1.7353	-0.4779	-0.0879	-0.0574	-1.1728	1	1
	Reversing (Vol	140	Cumulative(%)	-1.3413	16%	84%	-7.7532	-5.1951	-0.5632	-0.0479	-0.0292	-2.2481	1	1
	2/Vol1)		Excess(%)	0.0004	89%	11%	-0.1078	-0.0766	0.0000	0.0795	0.1118	0.0447	1	1
		T→P	Durations	22.6378	67%	33%	2.0000	3.0000	16.0000	64.0000	87.0000	23.6667	1	1
		Missing	Amplitudes(%)	21.9218	71%	29%	0.5468	1.4191	15.2889	64.4027	86.5218	25.2807	1	1
		547	Cumulative(%)	681.6568	59%	41%	0.5208	2.7043	221.5220	2835.1707	4390.6184	348.9563	1	1
			Excess(%)	-0.3227	89%	11%	-3.4487	-2.2015	-0.0813	0.8832	1.2858	0.4702	1	1
	GMM Volatility	P→T	Durations	1.6361	90%	10%	1.0000	1.0000	1.5000	3.0000	4.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3450	4%	96%	-1.3231	-1.0065	-0.2371	-0.0301	-0.0142	-1.1728	0	1
	Switching	953	Cumulative(%)	-0.5360	4%	96%	-3.0849	-2.1264	-0.1886	-0.0151	-0.0071	-2.2481	0	1
	Residual		Excess(%)	-0.0153	98%	2%	-0.1271	-0.0907	0.0000	0.0248	0.0442	0.0447	0	0
	Resample (Per	T→P	Durations	28.8815	51%	49%	1.0000	1.0000	23.3333	80.0000	93.0000	23.6667	1	1
	1/Per 2) Both	Missing	Amplitudes(%)	24.6759	62%	38%	0.1036	0.2159	19.0636	72.5603	87.2822	25.2807	1	1
	Equations	2415	Cumulative(%)	728.4062	52%	48%	0.0524	0.1261	315.1451	2863.8581	4118.0546	348.9563	1	1
			Excess(%)	-0.2214	85%	15%	-2.7016	-1.9657	-0.0819	1.2782	1.8641	0.4702	1	1
	GMM Volatility	P→T	Durations	1.6518	90%	10%	1.0000	1.0000	1.3333	3.0000	4.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3501	4%	96%	-1.3589	-1.0148	-0.2377	-0.0300	-0.0134	-1.1728	0	1
	Switching	939	Cumulative(%)	-0.5768	5%	95%	-3.2653	-2.1842	-0.1788	-0.0151	-0.0067	-2.2481	0	1
	Residual		Excess(%)	-0.0156	97%	3%	-0.1229	-0.0929	0.0000	0.0253	0.0470	0.0447	0	1
	Resample (Per	T→P	Durations	28.3164	51%	49%	1.0000	1.5000	23.3333	77.0000	90.0000	23.6667	1	1
	1/Per 2) First	Missing	Amplitudes(%)	24.2136	63%	37%	0.1311	0.2636	18.8597	71.7197	83.0169	25.2807	1	1
	Equation	2365	Cumulative(%)	693.3891	53%	47%	0.0705	0.1800	309.0104	2827.6619	3748.3722	348.9563	1	1
			Excess(%)	-0.1399	83%	17%	-2.3332	-1.7588	-0.0720	1.4099	2.0092	0.4702	1	1
	GMM Volatility	P→T	Durations	1.6073	89%	11%	1.0000	1.0000	1.0000	3.0000	4.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3331	4%	96%	-1.4132	-1.0541	-0.2075	-0.0230	-0.0123	-1.1728	0	1
	Reversing	1954	Cumulative(%)	-0.5203	5%	95%	-3.4906	-2.1544	-0.1297	-0.0115	-0.0061	-2.2481	0	1
	Residual		Excess(%)	-0.0153	98%	2%	-0.1373	-0.0994	0.0000	0.0222	0.0431	0.0447	0	0
	Resample (Per	T→P	Durations	20.2236	73%	27%	1.0000	1.0000	12.0000	67.0000	104.0000	23.6667	1	1
	2/Per 1) Both	Missing	Amplitudes(%)	16.1515	79%	21%	0.0519	0.1038	8.0855	57.4804	94.8890	25.2807	1	1
	Equations	3592	Cumulative(%)	485.8481	77%	23%	0.0259	0.0519	64.7157	2457.2121	4838.3064	348.9563	1	1
			Excess(%)	-0.1922	90%	10%	-2.6947	-1.6022	-0.0493	0.8694	1.2698	0.4702	1	1
	GMM Volatility	P→T	Durations	1.6194	89%	11%	1.0000	1.0000	1.2500	3.0000	4.0000	2.7143	1	1
	Break (3)	Missing	Amplitudes(%)	-0.3402	4%	96%	-1.4707	-1.0778	-0.2176	-0.0194	-0.0105	-1.1728	0	1
	Switching	1985	Cumulative(%)	-0.5252	5%	95%	-3.5147	-2.2331	-0.1400	-0.0097	-0.0052	-2.2481	0	1
	Residual		Excess(%)	-0.0169	97%	3%	-0.1521	-0.1057	0.0000	0.0277	0.0493	0.0447	0	1
	Resample (Per	T→P	Durations	20.5674	71%	29%	1.0000	1.0000	13.0000	69.2500	101.0000	23.6667	1	1
	2/Per 1) First	Missing	Amplitudes(%)	16.5295	79%	21%	0.0669	0.1261	8.5164	59.4823	85.2939	25.2807	1	1
	Equation	3555	Cumulative(%)	485.6323	76%	24%	0.0335	0.0658	70.8565	2337.4493	4347.3272	348.9563	1	1
			Excess(%)	-0.2482	91%	9%	-2.5479	-1.7988	-0.0577	0.8061	1.2005	0.4702	1	1