The MODI and VAM Methods of Solving Transportation Problems

Tutorial Outline

This tutorial deals with two techniques for solving transportation problems: the MODI method and Vogel's Approximation Method (VAM).

MODI METHOD

The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths. Because of this, it can often provide considerable time savings over other methods for solving transportation problems.

MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace only one closed path. This path helps determine the maximum number of units that can be shipped via the best unused route.

How to Use the MODI Method

In applying the MODI method, we begin with an initial solution obtained by using the northwest corner rule or any other rule. But now we must compute a value for each row (call the values R_1, R_2, R_3 if there are three rows) and for each column (K_1, K_2, K_3) in the transportation table. In general, we let

$$R_i = \text{value assigned to row } i$$
$$K_j = \text{value assigned to column } j$$
$$C_{ij} = \text{cost in square } ij \text{ (cost of shipping from source } i \text{ to destination } j)$$

The MODI method then requires five steps:

1. To compute the values for each row and column, set

$$R_i + K_j = C_{ij}$$

but *only for those squares that are currently used or occupied*. For example, if the square at the intersection of row 2 and column 1 is occupied, we set $R_2 + K_1 = C_{21}$.
2. After all equations have been written, set $R_1 = 0$.
3. Solve the system of equations for all R and K values.
4. Compute the improvement index for each unused square by the formula improvement index $(I_{ij}) = C_{ij} - R_i - K_j$.
5. Select the largest negative index and proceed to solve the problem as you did using the stepping-stone method.

Solving the Arizona Plumbing Problem with MODI

Let us try out these rules on the Arizona Plumbing problem. The initial northwest corner solution is shown in Table T4.1. MODI will be used to compute an improvement index for each unused square. Note that the only change in the transportation table is the border labeling the $R_i s$ (rows) and $K_j s$ (columns).

We first set up an equation for each occupied square:

1. $R_1 + K_1 = 5$
2. $R_2 + K_1 = 8$
3. $R_2 + K_2 = 4$
4. $R_3 + K_2 = 7$
5. $R_3 + K_3 = 5$

Letting $R_1 = 0$, we can easily solve, step by step, for K_1, R_2, K_2, R_3, and K_3.

1. $R_1 + K_1 = 5$
   $0 + K_1 = 5$ $K_1 = 5$
2. $R_2 + K_1 = 8$
   $R_2 + 5 = 8$ $R_2 = 3$
3. $R_2 + K_2 = 4$
   $3 + K_2 = 4$ $K_2 = 1$

TABLE T4.1

Initial Solution to Arizona Plumbing Problem in the MODI Format

R_i / K_j		K_1 ALBUQUERQUE	K_2 BOSTON	K_3 CLEVELAND	FACTORY CAPACITY
R_1	DES MOINES	5 / 100	4	3	100
R_2	EVANSVILLE	8 / 200	4 / 100	3	300
R_3	FORT LAUDERDALE	9	7 / 100	5 / 200	300
	WAREHOUSE REQUIREMENTS	300	200	200	700

4. $R_3 + K_2 = 7$
   $R_2 + 1 = 7$ $R_3 = 6$
5. $R_3 + K_3 = 5$
   $6 + K_3 = 5$ $K_3 = -1$

You can observe that these R and K values will not always be positive; it is common for zero and negative values to occur as well. After solving for the Rs and Ks in a few practice problems, you may become so proficient that the calculations can be done in your head instead of by writing the equations out.

The next step is to compute the improvement index for each unused cell. That formula is

$$\text{improvement index} = I_{ij} = C_{ij} - R_i - K_j$$

We have:

$$\text{Des Moines–Boston index} = I_{DB} \text{ (or } I_{12}) = C_{12} - R_1 - K_2 = 4 - 0 - 1$$
$$= +\$3$$
$$\text{Des Moines–Cleveland index} = I_{DC} \text{ (or } I_{13}) = C_{13} - R_1 - K_3 = 3 - 0 - (-1)$$
$$= +\$4$$
$$\text{Evansville–Cleveland index} = I_{EC} \text{ (or } I_{23}) = C_{23} - R_2 - K_3 = 3 - 3 - (-1)$$
$$= +\$1$$
$$\text{Fort Lauderdale–Albuquerque index} = I_{FA} \text{ (or } I_{31}) = C_{31} - R_3 - K_1 = 9 - 6 - 5$$
$$= -\$2$$

Because one of the indices is negative, the current solution is not optimal. Now it is necessary to trace only the one closed path, for Fort Lauderdale–Albuquerque, in order to proceed with the solution procedures.

The steps we follow to develop an improved solution after the improvement indices have been computed are outlined briefly:

1. Beginning at the square with the best improvement index (Fort Lauderdale–Albuquerque), trace a closed path back to the original square via squares that are currently being used.
2. Beginning with a plus (+) sign at the unused square, place alternate minus (−) signs and plus signs on each corner square of the closed path just traced.
3. Select the smallest quantity found in those squares containing minus signs. *Add* that number to all squares on the closed path with plus signs; *subtract* the number from all squares assigned minus signs.
4. Compute new improvement indices for this new solution using the MODI method.

TABLE T4.2

Second Solution to the
Arizona Plumbing
Problem

TO / FROM	A	B	C	FACTORY
D	$5 100	$4	$3	100
E	$8 100	$4 200	$3	300
F	$9 100	$7	$5 200	300
WAREHOUSE	300	200	200	700

TABLE T4.3

Third and Optimal
Solution to Arizona
Plumbing Problem

TO / FROM	A	B	C	FACTORY
D	$5 100	$4	$3	100
E	$8	$4 200	$3 100	300
F	$9 200	$7	$5 100	300
WAREHOUSE	300	200	200	700

Following this procedure, the second and third solutions to the Arizona Plumbing Corporation problem can be found. See Tables T4.2 and T4.3. With each new MODI solution, we must recalculate the R and K values. These values then are used to compute new improvement indices in order to determine whether further shipping cost reduction is possible.

VOGEL'S APPROXIMATION METHOD: ANOTHER WAY TO FIND AN INITIAL SOLUTION

In addition to the northwest corner and intuitive lowest-cost methods of setting an initial solution to transportation problems, we introduce one other important technique—*Vogel's approximation method* (VAM). VAM is not quite as simple as the northwest corner approach, but it facilitates a very good initial solution—as a matter of fact, one that is often the *optimal* solution.

Vogel's approximation method tackles the problem of finding a good initial solution by taking into account the costs associated with each route alternative. This is something that the northwest corner rule did not do. To apply the VAM, we first compute for each row and column the penalty faced if we should ship over the *second best* route instead of the *least-cost* route.

TO / FROM	Warehouse at Albuquerque	Warehouse at Boston	Warehouse at Cleveland	Factory Capacity
Des Moines factory	$5	$4	$3	100
Evansville factory	$8	$4	$3	300
Fort Lauderdale factory	$9	$7	$5	300
Warehouse requirements	300	200	200	700

Des Moines capacity constraint

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Cleveland warehouse demand

Total demand and total supply

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

TABLE T4.4

Transportation Table for Arizona Plumbing Corporation

The six steps involved in determining an initial VAM solution are illustrated on the Arizona Plumbing Corporation data. We begin with Table T4.4.

VAM Step 1: For each row and column of the transportation table, find the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution cost on the *best* route in the row or column and the *second best* route in the row or column. (This is the *opportunity cost* of not using the best route.)

Step 1 has been done in Table T4.5. The numbers at the heads of the columns and to the right of the rows represent these differences. For example, in row *E* the three transportation costs are $8, $4, and $3. The two lowest costs are $4 and $3; their difference is $1.

VAM Step 2: Identify the row or column with the greatest opportunity cost, or difference. In the case of Table T4.5, the row or column selected is column *A*, with a difference of 3.

TABLE T4.5

Transportation Table with VAM Row and Column Differences Shown

TO / FROM	ALBUQUERQUE A	BOSTON B	CLEVELAND C	TOTAL AVAILABLE	
	3	0	0		
DES MOINES D	5	4	3	100	1
EVANSVILLE E	8	4	3	300	1
FORT LAUDERDALE F	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

TABLE T4.6

VAM Assignment with *D*'s Requirements Satisfied

TO / FROM	$\cancel{3}$ 1 A	$\cancel{0}$ 3 B	$\cancel{0}$ 2 C	TOTAL AVAILABLE	
D	5 / 100	4 / X	3 / X	100	$\cancel{X}$
E	8	4	3	300	1
F	9	7	5	300	2
TOTAL REQUIRED	300	200	200	700	

VAM Step 3: Assign as many units as possible to the lowest cost square in the row or column selected.

Step 3 has been done in Table T4.6. Under Column *A*, the lowest-cost route is *D–A* (with a cost of $5), and 100 units have been assigned to that square. No more were placed in the square because doing so would exceed *D*'s availability.

VAM Step 4: Eliminate any row or column that has just been completely satisfied by the assignment just made. This can be done by placing *X*s in each appropriate square.

Step 4 has been done in Table T4.6 *D* row. No future assignments will be made to the *D–B* or *D–C* routes.

VAM Step 5: Recompute the cost differences for the transportation table, omitting rows or columns crossed out in the preceding step.

This is also shown in Table T4.6. *A*'s, *B*'s, and *C*'s differences each change. *D*'s row is eliminated, and *E*'s and *F*'s differences remain the same as in Table T4.5.

VAM Step 6: Return to step 2 and repeat the steps until an initial feasible solution has been obtained.

TABLE T4.7

Second VAM Assignment with *B*'s Requirements Satisfied

TO / FROM	$\cancel{3}$ 1 A	$\cancel{0}$ 3 B	$\cancel{0}$ 2 C	TOTAL AVAILABLE	
D	5 / 100	4 / X	3 / X	100	$\cancel{X}$
E	8	4 / 200	3	300	$\cancel{1}$ 5
F	9	7 / X	5	300	$\cancel{2}$ 4
TOTAL REQUIRED	300	200	200	700	

TABLE T4.8

Third VAM Assignment
with C's Requirements
Satisfied

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	5 100	4 X	3 X	100
E	8 X	4 200	3 100	300
F	9 	7 X	5 	300
TOTAL REQUIRED	300	200	200	700

In our case, column *B* now has the greatest difference, which is 3. We assign 200 units to the low-est-cost square in column *B* that has not been crossed out. This is seen to be *E–B*. Since *B*'s require-ments have now been met, we place an X in the *F–B* square to eliminate it. Differences are once again recomputed. This process is summarized in Table T4.7.

The greatest difference is now in row *E*. Hence, we shall assign as many units as possible to the lowest-cost square in row *E*, that is, *E–C* with a cost of $3. The maximum assignment of 100 units depletes the remaining availability at *E*. The square *E–A* may therefore be crossed out. This is illus-trated in Table T4.8.

The final two allocations, at *F–A* and *F–C*, may be made by inspecting supply restrictions (in the rows) and demand requirements (in the columns). We see that an assignment of 200 units to *F–A* and 100 units to *F–C* completes the table (see Table T4.9).

The cost of this VAM assignment is = (100 units × $5) + (200 units × $4) + (100 units × $3) + (200 units × $9) + (100 units × $5) = $3,900.

It is worth noting that the use of Vogel's approximation method on the Arizona Plumbing Corporation data produces the optimal solution to this problem. Even though VAM takes many more calculations to find an initial solution than does the northwest corner rule, it almost always produces a much better initial solution. Hence VAM tends to minimize the total number of compu-tations needed to reach an optimal solution.

TABLE T4.9

Final Assignments to
Balance Column and
Row Requirements

FROM \ TO	A	B	C	TOTAL AVAILABLE
D	5 100	4 X	3 X	100
E	8 X	4 200	3 100	300
F	9 200	7 X	5 100	300
TOTAL REQUIRED	300	200	200	700

DISCUSSION QUESTIONS

1. Why does Vogel's approximation method provide a good initial feasible solution? Could the northwest corner rule ever provide an initial solution with as low a cost?
2. How do the MODI and stepping-stone methods differ?

PROBLEMS*

P :

T4.1 The Hardrock Concrete Company has plants in three locations and is currently working on three major construction projects, each located at a different site. The shipping cost per truckload of concrete, daily plant capacities, and daily project requirements are provided in the accompanying table.

To ⟍ From	Project A	Project B	Project C	Plant Capacities
Plant 1	$10	$ 4	$11	70
Plant 2	12	5	8	50
Plant 3	9	7	6	30
Project Requirements	40	50	60	150

a) Formulate an initial feasible solution to Hardrock's transportation problem using VAM.
b) Then solve using the MODI method.
c) Was the initial solution optimal?

P :

T4.2 Hardrock Concrete's owner has decided to increase the capacity at his smallest plant (see Problem T4.1). Instead of producing 30 loads of concrete per day at plant 3, that plant's capacity is doubled to 60 loads. Find the new optimal solution using VAM and MODI. How has changing the third plant's capacity altered the optimal shipping assignment?

P .

T4.3 The Saussy Lumber Company ships pine flooring to three building supply houses from its mills in Pineville, Oak Ridge, and Mapletown. Determine the best transportation schedule for the data given in the accompanying table. Use the northwest corner rule and the MODI method.

TO ⟍ FROM	SUPPLY HOUSE 1	SUPPLY HOUSE 2	SUPPLY HOUSE 3	MILL CAPACITY (TONS)
PINEVILLE	$3	$3	$2	25
OAK RIDGE	4	2	3	40
MAPLETOWN	3	2	3	30
SUPPLY HOUSE DEMAND (TONS)	30	30	35	95

Note: **P** means the problem may be solved with POM for Windows; ✖ means the problem may be solved with Excel OM; and **P✖** means the problem may be solved with POM for Windows and/or Excel OM.

P✕:   **T4.4**   The Krampf Lines Railway Company specializes in coal handling. On Friday, April 13, Krampf had empty cars at the following towns in the quantities indicated:

TOWN	SUPPLY OF CARS
Morgantown	35
Youngstown	60
Pittsburgh	25

By Monday, April 16, the following towns will need coal cars:

TOWN	DEMAND FOR CARS
Coal Valley	30
Coaltown	45
Coal Junction	25
Coalsburg	20

Using a railway city-to-city distance chart, the dispatcher constructs a mileage table for the preceding towns. The result is

	To			
FROM	COAL VALLEY	COALTOWN	COAL JUNCTION	COALSBURG
Morgantown	50	30	60	70
Youngstown	20	80	10	90
Pittsburgh	100	40	80	30

Minimizing total miles over which cars are moved to new locations, compute the best shipment of coal cars. Use the northwest corner rule and the MODI method.

P✕:   **T4.5**   The Jessie Cohen Clothing Group owns factories in three towns (W, Y, and Z) that distribute to three Cohen retail dress shops (in A, B, and C). Factory availabilities, projected store demands, and unit shipping costs are summarized in the table that follows:

FROM \ TO	A	B	C	FACTORY AVAILABILITY
W	4	3	3	35
Y	6	7	6	50
Z	8	2	5	50
STORE DEMAND	30	65	40	135

Use Vogel's approximation method to find an initial feasible solution to this transportation problem. Is your VAM solution optimal?

P✕:   **T4.6**   The state of Missouri has three major power-generating companies (A, B, and C). During the months of peak demand, the Missouri Power Authority authorizes these companies to pool their excess supply and to distribute it to smaller independent power companies that do not have generators large enough to handle the demand. Excess supply is distributed on the basis of cost per kilowatt hour transmitted. The following table shows the

demand and supply in millions of kilowatt hours and the costs per kilowatt hour of transmitting electric power to four small companies in cities *W*, *X*, *Y*, and *Z*.

To ⟍ From	W	X	Y	Z	Excess Supply
A	12¢	4¢	9¢	5¢	55
B	8¢	1¢	6¢	6¢	45
C	1¢	12¢	4¢	7¢	30
Unfilled Power Demand	40	20	50	20	

Use Vogel's approximation method to find an initial transmission assignment of the excess power supply. Then apply the MODI technique to find the least-cost distribution system.