Online Tutorial 3

The Simplex Method of Linear Programming

Tutorial Outline

Most real-world linear programming problems have more than two variables and thus are too complex for graphical solution. A procedure called the **simplex method** may be used to find the optimal solution to multivariable problems. The simplex method is actually an algorithm (or a set of instructions) with which we examine corner points in a methodical fashion until we arrive at the best solution—highest profit or lowest cost. Computer programs and spreadsheets are available to handle the simplex calculations for you. But you need to know what is involved behind the scenes in order to best understand their valuable outputs.

CONVERTING THE CONSTRAINTS TO EQUATIONS

The first step of the simplex method requires that we convert each inequality constraint in an LP formulation into an equation. Less-than-or-equal-to constraints (≤) can be converted to equations by adding *slack variables*, which represent the amount of an unused resource.

We formulate the Shader Electronics Company's product mix problem as follows, using linear programming:

$$\text{Maximize profit} = \$7X_1 + \$5X_2$$

subject to LP constraints:

$$2X_1 + 1X_2 \leq 100$$
$$4X_1 + 3X_2 \leq 240$$

where X_1 equals the number of Walkmans produced and X_2 equals the number of Watch-TVs produced.

To convert these inequality constraints to equalities, we add slack variables S_1 and S_2 to the left side of the inequality. The first constraint becomes

$$2X_1 + 1X_2 + S_1 = 100$$

and the second becomes

$$4X_1 + 3X_2 + S_2 = 240$$

To include all variables in each equation (a requirement of the next simplex step), we add slack variables not appearing in each equation with a coefficient of zero. The equations then appear as

$$2X_1 + 1X_2 + 1S_1 + 0S_2 = 100$$
$$4X_1 + 3X_2 + 0S_1 + 1S_2 = 240$$

Because slack variables represent unused resources (such as time on a machine or labor-hours available), they yield no profit, but we must add them to the objective function with zero profit coefficients. Thus, the objective function becomes

$$\text{Maximize profit} = \$7X_1 + \$5X_2 + \$0S_1 + \$0S_2$$

SETTING UP THE FIRST SIMPLEX TABLEAU

To simplify handling the equations and objective function in an LP problem, we place all of the coefficients into a tabular form. We can express the preceding two constraint equations as

SOLUTION MIX	X_1	X_2	S_1	S_2	QUANTITY (RHS)
S_1	2	1	1	0	100
S_2	4	3	0	1	240

The numbers (2, 1, 1, 0) and (4, 3, 0, 1) represent the coefficients of the first equation and second equation, respectively.

As in the graphical approach, we begin the solution at the origin, where $X_1 = 0$, $X_2 = 0$, and profit = 0. The values of the two other variables, S_1 and S_2, then, must be nonzero. Because $2X_1 + 1X_2 + 1S_1 = 100$, we see that $S_1 = 100$. Likewise, $S_2 = 240$. These two slack variables comprise the initial solution mix—as a matter of fact, their values are found in the quantity column across from each variable. Because X_1 and X_2 are not in the solution mix, their initial values are automatically equal to zero.

This initial solution is called a *basic feasible solution* and can be described in vector, or column, form as

$$\begin{bmatrix} X_1 \\ X_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 240 \end{bmatrix}$$

Variables in the solution mix, which is often called the *basis* in LP terminology, are referred to as *basic variables*. In this example, the basic variables are S_1 and S_2. Variables not in the solution mix—or basis—(X_1 and X_2, in this case) are called *nonbasic variables*.

Table T3.1 shows the complete initial simplex tableau for Shader Electronics. The terms and rows that you have not seen before are as follows:

C_j: Profit contribution per unit of each variable. C_j applies to both the top row and first column. In the row, it indicates the unit profit for all variables in the LP objective function. In the column, C_j indicates the unit profit for each variable *currently* in the solution mix.

Z_j: In the quantity column, Z_j provides the total contribution (gross profit in this case) of the given solution. In the other columns (under the variables) it represents the gross profit *given up* by adding one unit of this variable into the current solution. The Z_j value for each column is found by multiplying the C_j of the row by the number in that row and jth column and summing.

The calculations for the values of Z_j in Table T3.1 are as follows:

$$\begin{aligned}
Z_j \text{ (for column } X_1) &= 0(2) + 0(4) = 0 \\
Z_j \text{ (for column } X_2) &= 0(1) + 0(3) = 0 \\
Z_j \text{ (for column } S_1) &= 0(1) + 0(0) = 0 \\
Z_j \text{ (for column } S_2) &= 0(0) + 0(1) = 0 \\
Z_j \text{ (for total profit)} &= 0(100) + 0(240) = 0
\end{aligned}$$

$C_j - Z_j$: This number represents the net profit (that is, the profit gained minus the profit given up), which will result from introducing one unit of each product (variable) into the solution. It is not calculated for the quantity column. To compute these numbers, we simply subtract the Z_j total from the C_j value at the very top of each variable's column.

The calculations for the net profit per unit ($C_j - Z_j$) row in this example are as follows:

	COLUMN			
	X_1	X_2	S_1	S_2
C_j for column	\$7	\$5	\$0	\$0
Z_j for column	0	0	0	0
$C_j - Z_j$ for column	\$7	\$5	\$0	\$0

It was obvious to us when we computed a profit of \$0 that this initial solution was not optimal. Examining numbers in the $C_j - Z_j$ row of Table T3.1, we see that total profit can be increased by \$7 for each unit of X_1 (Walkmans) and by \$5 for each unit of X_2 (Watch-TVs) added to the solution mix. A negative number in the $C_j - Z_j$ row would tell us that profits would *decrease* if the corresponding variable were added to the solution mix. An optimal solution is reached in the simplex method when the $C_j - Z_j$ row contains no positive numbers. Such is not the case in our initial tableau.

TABLE T3.1

Completed Initial Simplex Tableau

$C_j \rightarrow$ ↓	SOLUTION MIX	$7 X_1	$5 X_2	$0 S_1	$0 S_2	QUANTITY (RHS)
$0	S_1	2	1	1	0	100
$0	S_2	4	3	0	1	240
	Z_j	$0	$0	$0	$0	$0
	$C_j - Z_j$	$7	$5	$0	$0	(total profit)

SIMPLEX SOLUTION PROCEDURES

Once we have completed an initial tableau, we proceed through a series of five steps to compute all of the numbers we need for the next tableau. The calculations are not difficult, but they are sufficiently complex that the smallest arithmetic error can produce a very wrong answer.

We first list the five steps and then apply them in determining the second and third tableau for the data in the Shader Electronics example.

1. Determine which variable to enter into the solution mix next. Identify the column—hence the variable—with the largest positive number in the $C_j - Z_j$ row of the previous tableau. This step means that we will now be producing some of the product contributing the greatest additional profit per unit.
2. Determine which variable to replace. Because we have just chosen a new variable to enter into the solution mix, we must decide which variable currently in the solution to remove to make room for it. To do so, we divide each amount in the quantity column by the corresponding number in the column selected in step 1. The row with the *smallest nonnegative number* calculated in this fashion will be replaced in the next tableau (this smallest number, by the way, gives the maximum number of units of the variable that we may place in the solution). This row is often referred to as the **pivot row**, and the column identified in step 1 is called the **pivot column**. The number at the intersection of the pivot row and pivot column is the **pivot number**.
3. Compute new values for the pivot row. To find them, we simply divide every number in the row by the *pivot number*.
4. Compute new values for each remaining row. (In our sample problems there have been only two rows in the LP tableau, but most larger problems have many more rows.) All remaining row(s) are calculated as follows:

$$\begin{pmatrix}\text{New row}\\ \text{numbers}\end{pmatrix} = \begin{pmatrix}\text{numbers}\\ \text{in old row}\end{pmatrix} - \left[\begin{pmatrix}\text{number in old row}\\ \text{above or below}\\ \text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{corresponding number in}\\ \text{the new row, i.e., the row}\\ \text{replaced in step 3}\end{pmatrix}\right]$$

5. Compute the Z_j and $C_j - Z_j$ rows, as demonstrated in the initial tableau. If all numbers in the $C_j - Z_j$ row are zero or negative, we have found an optimal solution. If this is not the case, we must return to step 1.

All of these computations are best illustrated by using an example. The initial simplex tableau computed in Table T3.1 is repeated below. We can follow the five steps just described to reach an optimal solution to the LP problem.

	$C_j \rightarrow$ ↓	SOLUTION MIX	$7 X_1	$5 X_2	$0 S_1	$0 S_2	QUANTITY
1ST TABLEAU		pivot number					
	$0	S_1	2	1	1	0	100 ← pivot
	$0	S_2	4	3	0	1	240 row
		Z_j	$0	$0	$0	$0	$0
		$C_j - Z_j$	$7	$5	$0	$0	$0
			↑ pivot column (maximum $C_j - Z_j$ values)				

Step 1: Variable X_1 enters the solution next because it has the highest contribution to profit value, $C_j - Z_j$. Its column becomes the pivot column.

Step 2: Divide each number in the quantity column by the corresponding number in the X_1 column: 100/2 = 50 for the first row and 240/4 = 60 for the second row. The smaller of these numbers—50—identifies the pivot row, the pivot number, and the variable to be replaced. The pivot row is identified above by an arrow, and the pivot number is circled. Variable X_1 replaces variable S_1 in the solution mix column, as shown in the second tableau.

Step 3: Replace the pivot row by dividing every number in it by the pivot number (2/2 = 1, 1/2 = 1/2, 1/2 = 1/2, 0/2 = 0, 100/2 = 50). This new version of the entire pivot row appears below:

C_j	SOLUTION MIX	X_1	X_2	S_1	S_2	QUANTITY
\$7	X_1	1	1/2	1/2	0	50

Step 4: Calculate the new values for the S_2 row.

$$\begin{pmatrix}\text{Number in}\\ \text{new } S_2 \text{ row}\end{pmatrix} = \begin{pmatrix}\text{number in}\\ \text{old } S_2 \text{ row}\end{pmatrix} - \left[\begin{pmatrix}\text{number below}\\ \text{pivot number}\\ \text{in old row}\end{pmatrix} \times \begin{pmatrix}\text{corresponding}\\ \text{number in the}\\ \text{new } X_1 \text{ row}\end{pmatrix}\right]$$

$$\begin{array}{rcrcccr}
0 & = & 4 & - & [(4) & \times & (1)] \\
1 & = & 3 & - & [(4) & \times & (1/2)] \\
-2 & = & 0 & - & [(4) & \times & (1/2)] \\
1 & = & 1 & - & [(4) & \times & (0)] \\
40 & = & 240 & - & [(4) & \times & (50)]
\end{array}$$

C_j	SOLUTION MIX	X_1	X_2	S_1	S_2	QUANTITY
\$7	X_1	1	1/2	1/2	0	50
0	S_2	0	1	−2	1	40

Step 5: Calculate the Z_j and $C_j - Z_j$ rows.

$$\begin{array}{lll}
Z_j \text{ (for } X_1 \text{ column)} & = \$7(1) + 0(0) = \$7 & C_j - Z_j = \$7 - \$7 = 0 \\
Z_j \text{ (for } X_2 \text{ column)} & = \$7(1/2) + 0(1) = \$7/2 & C_j - Z_j = \$5 - \$7/2 = \$3/2 \\
Z_j \text{ (for } S_1 \text{ column)} & = \$7(1/2) + 0(-2) = \$7/2 & C_j - Z_j = 0 - \$7/2 = -\$7/2 \\
Z_j \text{ (for } S_2 \text{ column)} & = \$7(0) + 0(1) = 0 & C_j - Z_j = 0 - 0 = 0 \\
Z_j \text{ (for total profit)} & = \$7(50) + 0(40) = \$350 &
\end{array}$$

2ND TABLEAU

$C_j \rightarrow$ $\downarrow$	SOLUTION MIX	\$7 X_1	\$5 X_2	\$0 S_1	\$0 S_2	QUANTITY
		pivot number				
\$7	X_1	1	1/2	1/2	0	50
\$0	S_2	0	1	−2	1	40 ← pivot row
	Z_j	\$7	\$7/2	\$7/2	\$0	\$350 (total profit)
	$C_j - Z_j$	\$0	\$3/2 ↑ pivot column	\$−7/2	\$0	

Because not all numbers in the $C_j - Z_j$ row of this latest tableau are zero or negative, the solution (that is, $X_1 = 50$, $S_2 = 40$, $X_2 = 0$, $S_1 = 0$; profit = \$350) is not optimal; we then proceed to a third tableau and repeat the five steps.

Step 1: Variable X_2 enters the solution next because its $C_j - Z_j = 3/2$ is the largest (and only) positive number in the row. Thus, for every unit of X_2 that we start to produce, the objective function will increase in value by \$3/2, or \$1.50.

Step 2: The pivot row becomes the S_2 row because the ratio 40/1 = 40 is smaller than the ratio $50/(\frac{1}{2}) = 100$.

Step 3: Replace the pivot row by dividing every number in it by the (circled) pivot number. Because every number is divided by 1, there is no change.

Step 4: Compute the new values for the X_1 row.

$$\begin{pmatrix}\text{Number in}\\ \text{new } X_1 \text{ row}\end{pmatrix} = \begin{pmatrix}\text{number in}\\ \text{old } X_1 \text{ row}\end{pmatrix} - \left[\begin{pmatrix}\text{number above}\\ \text{pivot number}\end{pmatrix} \times \begin{pmatrix}\text{corresponding}\\ \text{number in the}\\ \text{new } X_2 \text{ row}\end{pmatrix}\right]$$

$$\begin{array}{ccccccc}
1 & = & 1 & - & [(1/2) & \times & (0)] \\
0 & = & 1/2 & - & [(1/2) & \times & (1)] \\
3/2 & = & 1/2 & - & [(1/2) & \times & (-2)] \\
-1/2 & = & 0 & - & [(1/2) & \times & (1)] \\
30 & = & 50 & - & [(1/2) & \times & (40)]
\end{array}$$

Step 5: Calculate the Z_j and $C_j - Z_j$ rows.

$$\begin{array}{lll}
Z_j \text{ (for } X_1 \text{ column)} & = \$7(1) + \$5(0) = \$7 & C_j - Z_j = \$7 - 7 = \$0 \\
Z_j \text{ (for } X_2 \text{ column)} & = \$7(0) + \$5(1) = \$5 & C_j - Z_j = \$5 - 5 = \$0 \\
Z_j \text{ (for } S_1 \text{ column)} & = \$7(3/2) + \$5(-2) = \$1/2 & C_j - Z_j = \$0 - 1/2 = \$-1/2 \\
Z_j \text{ (for } S_2 \text{ column)} & = \$7(-1/2) + \$5(1) = \$3/2 & C_j - Z_j = \$0 - 3/2 = \$-3/2 \\
Z_j \text{ (for total profit)} & = \$7(30) + \$5(40) = \$410 &
\end{array}$$

The results for the third and final tableau are seen in Table T3.2.

Because every number in the third tableau's $C_j - Z_j$ row is zero or negative, we have reached an optimal solution. That solution is: $X_1 = 30$ (Walkmans), and $X_2 = 40$ (Watch-TVs), $S_1 = 0$ (slack in first resource), $S_2 = 0$ (slack in second resource), and profit = \$410.

TABLE T3.2

Completed Initial Simplex Tableau

$C_j \rightarrow$ / $\downarrow$	SOLUTION MIX	\$7 X_1	\$5 X_2	\$0 S_1	\$0 S_2	QUANTITY
\$7	X_1	1	0	3/2	−1/2	30
\$5	X_2	0	1	−2	1	40
	Z_j	\$7	\$5	\$1/2	\$3/2	\$410
	$C_j - Z_j$	\$0	\$0	−\$1/2	−\$3/2	

SUMMARY OF SIMPLEX STEPS FOR MAXIMIZATION PROBLEMS

The steps involved in using the simplex method to help solve an LP problem in which the objective function is to be maximized can be summarized as follows:

1. Choose the variable with the greatest positive $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting the one with the smallest (non-negative) ratio of quantity to pivot column.
3. Calculate the new values for the pivot row.
4. Calculate the new values for the other row(s).
5. Calculate the C_j and $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers greater than zero, return to step 1.

ARTIFICIAL AND SURPLUS VARIABLES

Constraints in linear programming problems are seldom all of the "less-than-or-equal-to" (≤) variety seen in the examples thus far. Just as common are "greater-than-or-equal-to" (≥) constraints and equalities. To use the simplex method, each of these also must be converted to a special form. If they are not, the simplex technique is unable to set an initial feasible solution in the first tableau. Example T1 shows how to convert such constraints.

Example T1

The following constraints were formulated for an LP problem for the Joyce Cohen Publishing Company. We shall convert each constraint for use in the simplex algorithm.

Constraint 1. $25X_1 + 30X_2 = 900$. To convert an *equality*, we simply add an "artificial" variable (A_1) to the equation:

$$25X_1 + 30X_2 + A_1 = 900$$

An *artificial variable* is a variable that has no physical meaning in terms of a real-world LP problem. It simply allows us to create a basic feasible solution to start the simplex algorithm. An artificial variable is not allowed to appear in the final solution to the problem.

Constraint 2. $5X_1 + 13X_2 + 8X_3 \geq 2{,}100$. To handle ≥ constraints, a "surplus" variable (S_1) is first subtracted and then an artificial variable (A_2) is added to form a new equation:

$$5X_1 + 13X_2 + 8X_3 - S_1 + A_2 = 2{,}100$$

A **surplus variable** *does* have a physical meaning—it is the amount over and above a required minimum level set on the right-hand side of a greater-than-or-equal-to constraint.

Whenever an artificial or surplus variable is added to one of the constraints, it must also be included in the other equations and in the problem's objective function, just as was done for slack variables. Each artificial variable is assigned an extremely high cost to ensure that it does not appear in the final solution. Rather than set an actual dollar figure of $10,000 or $1 million, however, we simply use the symbol $M to represent a very large number. Surplus variables, like slack variables, carry a zero cost. Example T2 shows how to figure in such variables.

Example T2

The Memphis Chemical Corp. must produce 1,000 lb of a special mixture of phosphate and potassium for a customer. Phosphate costs $5/lb and potassium costs $6/lb. No more than 300 lb of phosphate can be used, and at least 150 lb of potassium must be used.

We wish to formulate this as a linear programming problem and to convert the constraints and objective function into the form needed for the simplex algorithm. Let

X_1 = number of pounds of phosphate in the mixture
X_2 = number of pounds of potassium in the mixture

Objective function: minimize cost = $\$5X_1 + \$6X_2$.
Objective function in simplex form:

$$\text{Minimize costs} = \$5X_1 + \$6X_2 + \$0S_1 + \$0S_2 + \$MA_1 + \$MA_2$$

REGULAR FORM		SIMPLEX FORM
1st constraint:	$1X_1 + 1X_2 = 1{,}000$	$1X_1 + 1X_2 + 1A_1 = 1{,}000$
2nd constraint:	$1X_1 \leq 300$	$1X_1 + 1S_1 = 300$
3rd constraint:	$1X_2 \geq 150$	$1X_2 - 1S_2 + 1A_2 = 150$

SOLVING MINIMIZATION PROBLEMS

Now that you have worked a few examples of LP problems with the three different types of constraints, you are ready to solve a minimization problem using the simplex algorithm. Minimization problems are quite similar to the maximization problem tackled earlier. The one significant difference involves the $C_j - Z_j$ row. Because our objective is now to minimize costs, the new variable to enter the solution in each tableau (the pivot column) will be the one with the *largest negative* number in the $C_j - Z_j$ row. Thus, we will be choosing the variable that decreases costs the most. In minimization problems, an optimal solution is reached when all numbers in the $C_j - Z_j$ row are *zero* or *positive*—just the opposite from the maximization case. All other simplex steps, as shown, remain the same.

1. Choose the variable with the largest negative $C_j - Z_j$ to enter the solution.
2. Determine the row to be replaced by selecting the one with the smallest (non-negative) quantity-to-pivot-column ratio.
3. Calculate new values for the pivot row.
4. Calculate new values for the other rows.
5. Calculate the $C_j - Z_j$ values for this tableau. If there are any $C_j - Z_j$ numbers less than zero, return to step 1.

SUMMARY

This tutorial treats a special kind of model, linear programming. LP has proven to be especially useful when trying to make the most effective use of an organization's resources. All LP problems can also be solved with the simplex method, either by computer or by hand. This method is more complex mathematically than graphical LP, but it also produces such valuable economic information as shadow prices. LP is used in a wide variety of business applications.

KEY TERMS

Simplex method *(p. T3-2)*
Pivot row *(p. T3-4)*
Pivot column *(p. T3-4)*
Pivot number *(p. T3-4)*
Surplus variable *(p. T3-7)*

SOLVED PROBLEM

Solved Problem T3.1

Convert the following constraints and objective function into the proper form for use in the simplex method.

Objective function: Minimize cost $= 4X_1 + 1X_2$

Subject to the constraints:

$$3X_1 + X_2 = 3$$
$$4X_1 + 3X_2 \geq 6$$
$$X_1 + 2X_2 \leq 3$$

SOLUTION

Minimize cost $= 4X_1 + 1X_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

Subject to:

$$3X_1 + 1X_2 + 1A_1 = 3$$
$$4X_1 + 3X_2 - 1S_1 + 1A_2 = 6$$
$$1X_1 + 2X_2 + 1S_2 = 3$$

DISCUSSION QUESTIONS

1. Explain the purpose and procedures of the simplex method.
2. How do the graphic and simplex methods of solving linear programming problems differ? In what ways are they the same? Under what circumstances would you prefer to use the graphic approach?
3. What are the simplex rules for selecting the pivot column? The pivot row? The pivot number?
4. A particular linear programming problem has the following objective function:

$$\text{Maximize profit} = \$8X_1 + \$6X_2 + \$12X_3 - \$2X_4$$

Which variable should enter at the second simplex tableau? If the objective function was

$$\text{Minimize cost} = \$2.5X_1 + \$2.9X_2 + \$4.0X_3 + \$7.9X_4$$

which variable would be the best candidate to enter the second tableau?

5. To solve a problem by the simplex method, when are slack variables added?
6. List the steps in a simplex maximization problem.
7. What is a surplus variable? What is an artificial variable?

PROBLEMS*

T3.1 Each coffee table produced by John Alessi Designers nets the firm a profit of \$9. Each bookcase yields a \$12 profit. Alessi's firm is small and its resources limited. During any given production period (of one week), 10 gallons of varnish and 12 lengths of high-quality redwood are available. Each coffee table requires approximately 1 gallon of varnish and 1 length of redwood. Each bookcase takes 1 gallon of varnish and 2 lengths of wood.

Formulate Alessi's production mix decision as a linear programming problem and solve, using the simplex method. How many tables and bookcases should be produced each week? What will the maximum profit be?

T3.2 a) Set up an initial simplex tableau, given the following two constraints and objective function:

$$\begin{aligned} 1X_1 + 4X_2 &\le 24 \\ 1X_1 + 2X_2 &\le 16 \\ \text{Maximize profit} = \$3X_1 &+ \$9X_2 \end{aligned}$$

You will have to add slack variables.

b) Briefly list the iterative steps necessary to solve the problem in part (a).

c) Determine the next tableau from the one you developed in part (a). Determine whether it is an optimum solution.

d) If necessary, develop another tableau and determine whether it is an optimum solution. Interpret this tableau.

e) Start with the same initial tableau from part (a) but use X_1 as the first pivot column. Continue to iterate it (a total of twice) until you reach an optimum solution.

T3.3 Solve the following linear programming problem graphically. Then set up a simplex tableau and solve the problem, using the simplex method. Indicate the corner points generated at each iteration by the simplex on your graph.

$$\begin{aligned} &\text{Maximize profit} = \$3X_1 + \$5X_2 \\ &\text{Subject to:} \quad X_2 \le 6 \\ &\qquad 3X_1 + 2X_2 \le 18 \\ &\qquad X_1, X_2 \ge 0 \end{aligned}$$

T3.4 Solve the following linear programming problem, first graphically and then by simplex algorithm.

$$\begin{aligned} &\text{Minimize cost} = 4X_1 + 5X_2 \\ &\text{Subject to:} \quad X_1 + 2X_2 \ge 80 \\ &\qquad 3X_1 + X_2 \ge 75 \\ &\qquad X_1, X_2 \ge 0 \end{aligned}$$

What are the values of the basic variables at each iteration? Which are the nonbasic variables at each iteration?

T3.5 Barrow Distributors packages and distributes industrial supplies. A standard shipment can be packaged in a Class A container, a Class K container, or a Class T container. A single Class A container yields a profit of \$8; a Class K container, a profit of \$6; and a Class T container, a profit of \$14. Each shipment prepared requires a certain amount of packing material and a certain amount of time.

Resources Needed per Standard Shipment

CLASS OF CONTAINER	PACKING MATERIAL (POUNDS)	PACKING TIME (HOURS)
A	2	2
K	1	6
T	3	4
Total amount of resource available each week	120 pounds	240 hours

*Note: P means the problem may be solved with POM for Windows; means the problem may be solved with Excel OM; and means the problem may be solved with POM for Windows and/or Excel OM.

Joe Barrow, head of the firm, must decide the optimal number of each class of container to pack each week. He is bound by the previously mentioned resource restrictions but also decides that he must keep his six full-time packers employed all 240 hours (6 workers × 40 hours) each week.

Formulate and solve this problem, using the simplex method.

T3.6 Set up a complete initial tableau for the data (repeated below) that were first presented in Solved Problem T3.1.

$$\begin{array}{ll}
\text{Minimize cost} = & 4X_1 + 1X_2 + 0S_1 + 0S_2 + MA_1 + MA_2 \\
\text{Subject to:} & 3X_1 + 1X_2 + 1A_1 = 3 \\
 & 4X_1 + 3X_2 - 1S_1 + 1A_2 = 6 \\
 & 1X_1 + 2X_2 + 1S_2 = 3
\end{array}$$

a) Which variable will enter the solution next?

b) Which variable will leave the solution?

T3.7 Solve Problem T3.6 for the optimal solution, using the simplex method.