Acceptance Sampling

Tutorial Outline

In the Supplement to Chapter 6, "Statistical Process Control," we briefly introduced the topic of acceptance sampling. **Acceptance sampling** is a form of testing that involves taking random samples of "lots," or batches, of finished products and measuring them against predetermined standards. In this tutorial, we extend our introduction to acceptance sampling by discussing sampling plans, how to build an operating characteristic (OC) curve, and average outgoing quality.

SAMPLING PLANS

A "lot," or batch, of items can be inspected in several ways, including the use of single, double, or sequential sampling.

Single Sampling

Two numbers specify a **single sampling** plan: They are the number of items to be sampled (n) and a prespecified acceptable number of defects (c). If there are fewer or equal defects in the lot than the acceptance number, c, then the whole batch will be accepted. If there are more than c defects, the whole lot will be rejected or subjected to 100% screening.

Double Sampling

Often a lot of items is so good or so bad that we can reach a conclusion about its quality by taking a smaller sample than would have been used in a single sampling plan. If the number of defects in this smaller sample (of size n_1) is less than or equal to some lower limit (c_1), the lot can be accepted. If the number of defects exceeds an upper limit (c_2), the whole lot can be rejected. But if the number of defects in the n_1 sample is between c_1 and c_2, a second sample (of size n_2) is drawn. The cumulative results determine whether to accept or reject the lot. The concept is called **double sampling**.

Sequential Sampling

Multiple sampling is an extension of double sampling, with smaller samples used sequentially until a clear decision can be made. When units are randomly selected from a lot and tested one by one, with the cumulative number of inspected pieces and defects recorded, the process is called **sequential sampling**.

If the cumulative number of defects exceeds an upper limit specified for that sample, the whole lot will be rejected. Or if the cumulative number of rejects is less than or equal to the lower limit, the lot will be accepted. But if the number of defects falls within these two boundaries, we continue to sample units from the lot. It is possible in some sequential plans for the whole lot to be tested, unit by unit, before a conclusion is reached.

Selection of the best sampling approach—single, double, or sequential—depends on the types of products being inspected and their expected quality level. A very low-quality batch of goods, for example, can be identified quickly and more cheaply with sequential sampling. This means that the inspection, which may be costly and/or destructive, can end sooner. On the other hand, in many cases a single sampling plan is easier and simpler for workers to conduct even though the number sampled may be greater than under other plans.

OPERATING CHARACTERISTIC (OC) CURVES

The **operating characteristic (OC) curve** describes how well an acceptance plan discriminates between good and bad lots. A curve pertains to a specific plan, that is, a combination of n (sample size) and c (acceptance level). It is intended to show the probability that the plan will accept lots of various quality levels.

Naturally, we would prefer a highly discriminating sampling plan and OC curve. If the entire shipment of parts has an unacceptably high level of defects, we hope the sample will reflect that fact with a very high probability (preferably 100%) of rejecting the shipment.

Figure T2.1(a) shows a perfect discrimination plan for a company that wants to reject all lots with more than $2\frac{1}{2}\%$ defectives and accept all lots with less than $2\frac{1}{2}\%$ defectives. Unfortunately, the only way to assure 100% acceptance of good lots and 0% acceptance of bad lots is to conduct a full inspection, which is often very costly.

Figure T2.1(b) reveals that no OC curve will be as steplike as the one in Figure T2.1(a); nor will it be discriminating enough to yield 100% error-free inspection. Figure T2.1(b) does indicate, though, that for the same sample size ($n = 100$ in this case), a smaller value of c (of acceptable
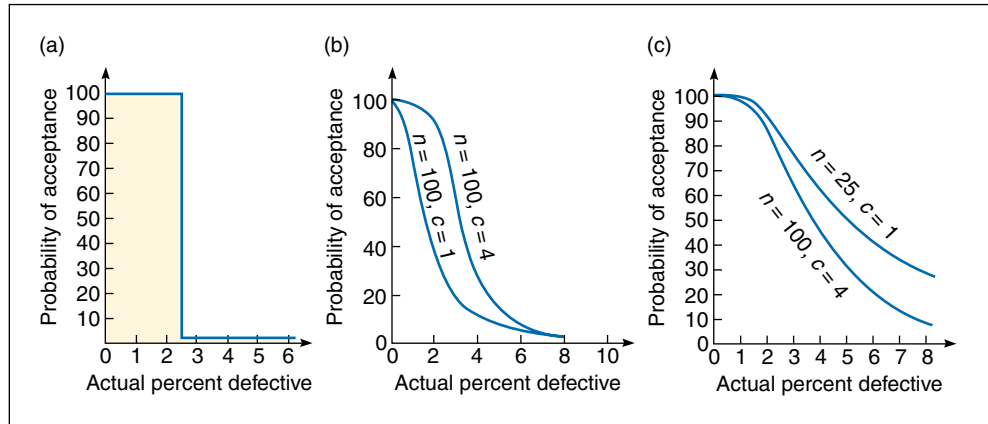
Figure T2.1 ■ (a) Perfect Discrimination for Inspection Plan. (b) OC Curves for Two Different Acceptable Levels of Defects ($c = 1$, $c = 4$) for the Same Sample Size ($n = 100$). (c) OC Curves for Two Different Sample Sizes ($n = 25$, $n = 100$) but Same Acceptance Percentages (4%). Larger sample size shows better discrimination.

defects) yields a steeper curve than does a larger value of c. So one way to increase the probability of accepting only good lots and rejecting only bad lots with random sampling is to set very tight acceptance levels.

A second way to develop a steeper, and thereby sounder, OC curve is to increase the sample size. Figure T2.1(c) illustrates that even when the acceptance number is the same proportion of the sample size, a larger value of n will increase the likelihood of accurately measuring the lot's quality. In this figure, both curves use a maximum defect rate of 4% (equal to $4/100 = 1/25$). Yet if you take a straightedge or ruler and carefully examine Figure T2.1(c), you will be able to see that the OC curve for $n = 25$, $c = 1$ rejects more good lots and accepts more bad lots than the second plan. Here are a few measurements to illustrate that point.

WHEN THE ACTUAL PERCENT OF DEFECTS IN THE LOT IS:	THEN THE PROBABILITY (APPROXIMATE) OF ACCEPTING THE WHOLE LOT IS:	
	FOR $n = 100$, $c = 4$	FOR $n = 25$, $c = 1$
1%	99%	97%
3%	81%	83%
5%	44%	64%
7%	17%	48%

In other words, the probability of accepting a more than satisfactory lot (one with only 1% defects) is 99% for $n = 100$, but only 97% for $n = 25$. Likewise, the chance of accepting a "bad" lot (one with 5% defects) is only 44% for $n = 100$, whereas it is 64% using the smaller sample size.[1] Of course, were it not for the cost of extra inspection, every firm would opt for larger sample sizes.

PRODUCER'S AND CONSUMER'S RISK

With acceptance sampling, two parties are usually involved: the producer of the product and the consumer of the product. When specifying a sampling plan, each party wants to avoid costly mistakes in accepting or rejecting a lot. The producer wants to avoid the mistake of having a good lot rejected

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[1] It bears repeating that sampling always runs the danger of leading to an erroneous conclusion. Let us say in this example that the total population under scrutiny is a load of 1,000 computer chips, of which in reality only 30 (or 3%) are defective. This means that we would want to accept the shipment of chips, because 4% is the allowable defect rate. But if a random sample of $n = 50$ chips were drawn, we could conceivably end up with zero defects and accept that shipment (that is, it is OK) or we could find all 30 defects in the sample. If the latter happened, we could wrongly conclude that the whole population was 60% defective and reject them all.

(*producer's risk*) because he or she usually must replace the rejected lot. Conversely, the customer or consumer wants to avoid the mistake of accepting a bad lot because defects found in a lot that has already been accepted are usually the responsibility of the customer (*consumer's risk*). The OC curve shows the features of a particular sampling plan, including the risks of making a wrong decision.

To help you understand the theory underlying the use of sampling plans, we will illustrate how an OC curve is constructed statistically.

In attribute sampling, where products are determined to be either good or bad, a binomial distribution is usually employed to build the OC curve. The binomial equation is

$$P(x) = \frac{n!}{x!(n-x)!}\, p^x (1-p)^{n-x} \tag{T2-1}$$

where n = number of items sampled (called trials)
    p = probability that an x (defect) will occur on any one trial
    $P(x)$ = probability of exactly x results in n trials

When the sample size (n) is large and the percent defective (p) is small, however, the Poisson distribution can be used as an approximation of the binomial formula. This is convenient because binomial calculations can become quite complex, and because cumulative Poisson tables are readily available. Our Poisson table appears in Appendix II of the text.

In a Poisson approximation of the binomial distribution, the mean of the binomial, which is np, is used as the mean of the Poisson, which is λ; that is,

$$\lambda = np \tag{T2-2}$$

Example T1

A shipment of 2,000 portable battery units for microcomputers is about to be inspected by a Malaysian importer. The Korean manufacturer and the importer have set up a sampling plan in which the α risk is limited to 5% at an acceptable quality level (AQL) of 2% defective, and the β risk is set to 10% at Lot Tolerance Percent Defective (LTPD) = 7% defective. We want to construct the OC curve for the plan of $n = 120$ sample size and an acceptance level of $c \leq 3$ defectives. Both firms want to know if this plan will satisfy their quality and risk requirements.

To solve the problem, we turn to the cumulative Poisson table in Appendix II of the text, whose columns are set up in terms of the acceptance level, c. We are interested only in the $c = 3$ column for this example. The rows in the table are $\lambda\ (= np)$, which represents the number of defects we would expect to find in each sample.

By varying the percent defectives (p) from .01 (1%) to .08 (8%) and holding the sample size at $n = 120$, we can compute the probability of acceptance of the lot at each chosen level. The values for P (acceptance) calculated in what follows are then plotted to produce the OC curve shown in Figure T2.2.

FIGURE T2.2 ■

OC Curve Constructed for Example T1
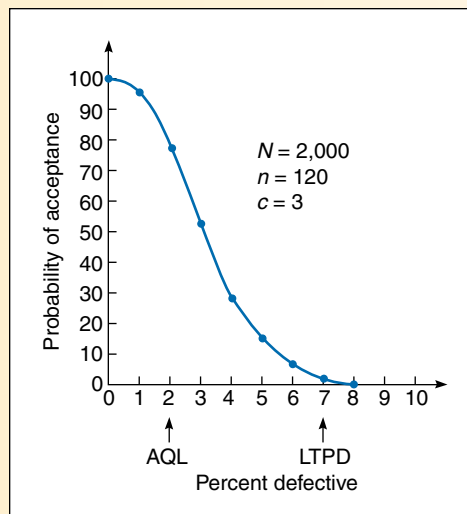
SELECTED VALUES OF % DEFECTIVE	MEAN OF POISSON, $\lambda = np$	P (ACCEPTANCE) FROM APPENDIX II
.01	1.20	.966
.02	2.40	$\boxed{.779}$ ← $1 - \alpha$ at AQL
.03	3.60	.515
.04	4.80	.294
.05	6.00	.151
.06	7.20	.072
.07	8.40	$\boxed{.032^a}$ ← β level at LTPD
.08	9.60	.014[a]

[a]Interpolated from value.

Now back to the issue of whether this OC curve satisfies the quality and risk needs of the consumer and producer of the batteries. For the AQL of $p = .02 = 2\%$ defects, the P (acceptance) of the lot = .779. This yields an α risk of $1 - .779 = .221$, or 22.1%, which exceeds the 5% level desired by the producer. The β risk of .032, or 3.2%, is well under the 10% sought by the consumer. It appears that new calculations are necessary with a larger sample size if the α level is to be lowered.

In Example T1, we set n and c values for a sampling plan and then computed the α and β risks to see if they were within desired levels. Often, organizations instead develop an OC curve for preset values and an AQL and then substitute values of n and c until the plan also satisfies the β and LTPD demands.

AVERAGE OUTGOING QUALITY

In most sampling plans, when a lot is rejected, the entire lot is inspected and all of the defective items are replaced. Use of this replacement technique improves the average outgoing quality in terms of percent defective. In fact, given (1) any sampling plan that replaces all defective items encountered and (2) the true incoming percent defective for the lot, it is possible to determine the **average outgoing quality (AOQ)** in percent defective. The equation for AOQ is

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N} \tag{T2-3}$$

where
P_d = true percent defective of the lot
P_a = probability of accepting the lot
N = number of items in the lot
n = number of items in the sample

Example T2

The percent defective from an incoming lot in Example T1 is 3%. An OC curve showed the probability of acceptance to be .515. Given a lot size of 2,000 and a sample of 120, what is the average outgoing quality in percent defective?

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N}$$

$$= \frac{(.03)(.515)(2,000 - 120)}{2,000} = .015$$

Thus, an acceptance sampling plan changes the quality of the lots in percent defective from 3% to 1.5% on the average. Acceptance sampling significantly increases the quality of the inspected lots.

In most cases, we do not know the value of P_a; we must determine it from the particular sampling plan. The fact that we seldom know the true incoming percent defective presents another problem. In most cases, several different incoming percent defective values are assumed. Then we can determine the average outgoing quality for each value.

Example T3

To illustrate the AOQ relationship, let us use the data we developed for the OC curve in Example T1. The lot size in that case was $N = 2,000$ and the sample size was $n = 120$. We assume that any defective batteries found during inspection are replaced by good ones. Then using the formula for AOQ given before and the probabilities of acceptance from Example T1, we can develop the following numbers:

P_d	$\times$	P_a	$\times$	$(N - n)/N$	$=$	AOQ
.01		.966		.94		.009
.02		.779		.94		.015
.03		.515		.94		.015
.04		.294		.94		.011
.05		.151		.94		.007
.06		.072		.94		.004
.07		.032		.94		.002
.08		.014		.94		.001

These numbers are graphed in Figure T2.3 as the average outgoing quality as a function of incoming quality.

FIGURE T2.3 ■

A Typical AOQ Curve Using Data from Example T3
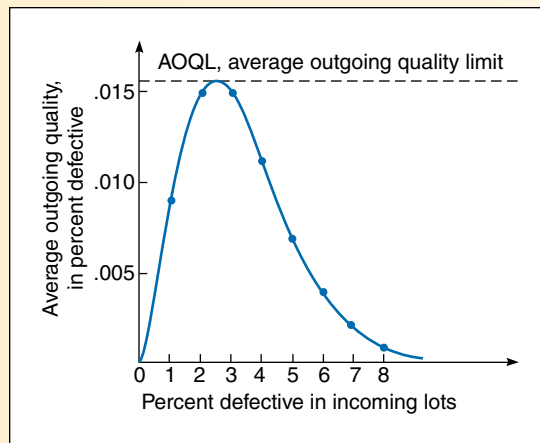
Did you notice how AOQ changed for different percent defectives? When the percent defective of the incoming lots is either very high or very low, the percent defective of the outgoing lots is low. AOQ at 1% was .009, and AOQ at 8% was .001. For moderate levels of the incoming percent defective, AOQ is higher: AOQ at 2% to 3% was .015. Thus, AOQ is low for small values of the incoming percent defective. As the incoming percent defective increases, the AOQ increases up to a point. Then, for increasing incoming percent defective, AOQ decreases.

The maximum value on the AOQ curve corresponds to the highest average percent defective or the lowest average quality for the sampling plan. It is called the *average outgoing quality limit (AOQL)*. In Figure T2.3, the AOQL is just over 1.5%, meaning the batteries are about 98.4% good when the incoming quality is between 2% and 3%.

Acceptance sampling is useful for screening incoming lots. When the defective parts are replaced with good parts, acceptance sampling helps to increase the quality of the lots by reducing the outgoing percent defective.

SUMMARY

Acceptance sampling is a major statistical tool of quality control. Sampling plans and operating characteristic (OC) curves facilitate acceptance sampling and provide the manager with tools to evaluate the quality of a production run or shipment.

KEY TERMS

Acceptance sampling *(p. T2-2)*
Single sampling *(p. T2-2)*
Double sampling *(p. T2-2)*

Sequential sampling *(p. T2-2)*
Operating characteristic (OC) curve *(p. T2-2)*
Average outgoing quality (AOQ) *(p. T2-5)*

SOLVED PROBLEM

Solved Problem T2.1

In an acceptance sampling plan developed for lots containing 1,000 units, the sample size n is 85 and c is 3. The percent defective of the incoming lots is 2%, and the probability of acceptance, which was obtained from an OC curve, is 0.64.

What is the average outgoing quality?

SOLUTION

$$\text{AOQ} = \frac{(P_d)(P_a)(N-n)}{N} = \frac{(.02)(.64)(1,000-85)}{1,000}$$
$$= .012 \text{ or AOQ} = 1.2\%$$

DISCUSSION QUESTIONS

1. Explain the difference between single, double, and sequential sampling.
2. Define AQL and LTPD.
3. What is "average outgoing quality"?
4. What is the AOQL?

PROBLEMS*

T2.1 Eighty items are randomly drawn from a lot of 6,000 talking toy animals, and the total lot is accepted if there are $c \leq 2$ defects. Develop an OC curve for this sample plan.

T2.2 A load of 200 desk lamps has just arrived at the warehouse of Lighting, Inc. Random samples of $n = 5$ lamps are checked. If more than one lamp is defective, the whole lot is rejected. Set up the OC curve for this plan.

T2.3 Develop the AOQ curve for Problem T2.2.

T2.4 Each week, Melissa Bryant Ltd. receives a batch of 1,000 popular Swiss watches for its chain of East Coast boutiques. Bryant and the Swiss manufacturer have agreed on the following sampling plan: $\alpha = 5\%$, $\beta = 10\%$, AQL = 1%, LTPD = 5%. Develop the OC curve for a sampling plan of $n = 100$ and $c \leq 2$. Does this plan meet the producer's and consumer's requirements?

T2.5 Kristi Conlin's firm in Waco, Texas, has designed an OC curve that shows a $\frac{2}{3}$ chance of accepting lots with a true percentage defective of 2%. Lots of 1,000 units are produced at a time, with 100 of each lot sampled randomly. What is the average outgoing quality level?

*Note: **P** means the problem may be solved with POM for Windows; ✕ means the problem may be solved with Excel OM; and **P✕** means the problem may be solved with POM for Windows and/or Excel OM.