APPENDIX TO CHAPTER 24

More on Hedging with Financial Derivatives

This appendix provides more detail and examples of hedging for four cases: with interest-rate futures, with futures options, with interest-rate swaps, and with stock index futures.

Hedging with Financial Futures

Managers of financial institutions such as banks, insurance companies, pension funds, finance companies, and mutual funds make use of two basic kinds of hedging strategies involving forward markets and futures markets to reduce interest-rate risk: the micro hedge and the macro hedge. When a financial institution hedges the interest-rate risk for a specific asset it is holding, it is conducting a *micro hedge.* When the financial institution is hedging interest-rate risk on its overall portfolio, it is conducting a *macro hedge.* To illustrate these hedging strategies, let's look at how a financial institution manager—say, the manager of the First National Bank—can use futures markets to engage in both a micro and a macro hedge.

Micro Hedge

Suppose that in March 2012, the First National Bank is holding $10 million face value of 10%-coupon-rate Treasury bonds selling at par that mature in the year 2022, referred to as the "10s of 2022." We have already seen that fluctuations in interest rates on these long-term bonds can cause major price fluctuations that result in large capital gains or losses in the coming year. One way that this risk could be hedged over the coming year is with the forward contract that was described earlier in the chapter in which the bank agrees to sell, at today's price and interest rate, $10 million of this bond to another party one year in the future, that is, in March 2013. However, as we have seen, finding a counterparty for this transaction might be difficult, so Mona, the manager of the First National Bank, decides to use the financial futures market instead.

Mona's problem is that there is no financial futures contract that corresponds exactly to the 10s of 2022 Treasury bond whose price she would like to lock in for next year. So she looks for a widely traded futures contract whose underlying asset price moves closely with the price of the asset her bank is holding. She decides that the Treasury bond contract traded at the Chicago Board of Trade is the best one for her hedge. This hedge is called a *cross hedge* because the underlying asset in the futures contract is not the same as the asset being hedged.

Mona knows that she needs to take a short position and sell Treasury bond futures contracts to hedge the interest-rate risk on the Treasury bonds. She figures this out by recognizing that if there is a fall in bond prices that would cause her bank to suffer losses on the bonds it is holding, the bank needs an equal offsetting gain on futures contracts. If the bank has taken a short position, then when the bond price falls, the bank can buy the bonds in the market at a lower price than the price at which it agreed to deliver the securities, thereby making the profit needed to offset the losses on the bonds it is holding.

Although the bank manager knows that she needs to take a short position, she still has to decide how many Treasury bond contracts she must sell to make sure that the change in the value of these futures contracts over the coming year is likely to offset the change in the value of the Treasury bonds she is hedging. Her first step in this process is to calculate the so-called *hedge ratio,* which tells her how many points the price of the hedged asset moves on average for a 1-point change in the futures contract used for the hedge. For example, if, when the price of the futures contract increases by 1 point, the price of the Treasury 10s of 2022 on average were to increase by 1.1 points, the hedge ratio would be 1.1.

The hedge ratio is important because it tells the bank manager the par dollar amount of futures contract needed per par dollar of the asset being hedged in order to provide the best hedge. Thus with a hedge ratio of 1.1, Mona should sell \$1.10 face value of the futures contract for every dollar of face value of the Treasury 10s of 2022; that is, she should sell \$11 million face value (= 1.1 × \$10 million) of Treasury bond futures contracts. Doing this makes intuitive sense because a good hedge is one in which on average any fall in the value of the hedged assets is offset by the gains on the futures contract. The hedge ratio indicates that if the \$10 million of Treasury 10s of 2022 fall in price by 1.1 points, for a loss of \$110,000 (1.1% of \$10 million), she wants to have a gain on average of \$110,000 on the Treasury bond futures contract she has sold. Since the hedge ratio indicates that when the Treasury 10s of 2022 have a price decline of 1.1 points, the Treasury bond futures contract on average has a decline of 1 point, she will have a profit of \$110,000 on the futures contract (1% of \$11 million).

The hedge ratio is calculated in two steps, as is indicated by the formula given by Equation 1:

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af} \tag{1}$$

where HR = hedge ratio

ΔP_a = change in the price of the hedged asset as a percentage of par in response to a 1% change in the interest rate

ΔP_f = change in the price of the futures contract as a percentage of par in response to a 1% change in the interest rate

β_{af} = average change in the interest rate of the hedged asset for a given change in the interest rate of the futures contract

The first step is to calculate the first term in the formula, $\Delta P_a/\Delta P_f$, which tells us how much the value of the hedged asset changes relative to the futures contract when there is a change in interest rates. The hedge ratio formula gives the intuitive result that when there is a greater change in the hedged asset's value relative to the futures contract for a given change in the interest rate—that is, when $\Delta P_a/\Delta P_f$ is higher, meaning that *HR* is higher—more Treasury bond futures contracts are needed to complete the hedge.

The change in the values of the hedged asset and the futures contract when interest rates change by, say, 1 percentage point can be calculated using the duration concept described in Chapter 3 or by other methods. Let's say that Mona finds that when the interest rate rises from 10% to 11% in March 2013, the 10s of 2022 Treasury bonds her bank is holding would decline by 6.58% of par (that is, by 6.58 points), while the Treasury bond futures contract would decline by 5.98% of par (5.98 points). The relative change in the value of the hedged asset relative to the futures contract would then equal 6.58/5.98 = 1.10.

The second step is to calculate the second term in the formula, β_{af}, which tells us how the interest rates for the hedged asset and the financial futures contract move together. When $\beta_{af} = 0$, for example, the interest rate on the hedged asset does not tend to move at all with changes in the interest rate on the futures contract. When $\beta_{af} = 1$, the interest rate on the hedged asset rises 1 percentage point on average when the interest rate on the futures contract rises by 1 percentage point; that is, on average they move in tandem. If $\beta_{af} = 2$, the interest rate on the hedged asset on average rises by 2 percentage points when there is a 1-percentage-point rise in the interest rate on the futures contract.

As the formula indicates, when β_{af} is lower, *HR* is lower, yielding the intuitive result that if the interest rate on the hedged assets does not on average change much with a change in the interest rate on the futures contract, a smaller amount of futures contracts should be used in the hedge. For example, if the interest rates on the hedged asset and the futures contract did not move together at all ($\beta_{af} = 0$), the futures contract would not be at all helpful in constructing a hedge and so should not be used. That is exactly what the hedge ratio formula indicates because when the interest rates on the hedged asset and the futures contract do not move together, plugging $\beta_{af} = 0$ into the formula yields $HR = 0$.

The β_{af} term is calculated by means of a statistical analysis of past data that determines how much interest rates on the hedged asset change on average for a given change in the interest rate of the futures contract. The bank manager calculates that when the interest rate on the futures contract changes by 1 percentage point, the interest rate on the Treasury 10s of 2022 changes on average by 1 percentage point, meaning that $\beta_{af} = 1$. Plugging the estimates of $\beta_{af} = 1$ and $\Delta P_a/\Delta P_f = 1.10$ into the formula in Equation 1, she calculates the hedge ratio to be

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af} = 1.10 \times 1 = 1.10$$

Now Mona is almost done with her calculation of how many Treasury bond contracts she needs to sell to hedge the 10s of 2022 bonds her bank is holding. Recall that the hedge ratio is the par dollar amount of futures contract per par dollar of the asset being hedged, so to calculate the number of contracts, Mona just has to multiply the hedge ratio by the face value of the amount of bonds she is hedging and divide through by the face value of the futures contract. Expressed as a formula, the number of contracts she needs to sell is

$$\text{Contracts} = HR \times \frac{PV_a}{PV_f} \qquad (2)$$

where HR = hedge ratio (1.10 in our example)
PV_a = par (face) value of the asset hedged ($10 million of Treasury 10s of 2022)
PV_f = par (face) value of the futures contract ($100,000 per Treasury bond futures contract)

In our example,

$$\text{Contracts} = 1.10 \times \frac{\$10{,}000{,}000}{\$100{,}000} = 1.10 \times 100 = 110$$

After calculating that she needs to sell 110 futures contracts, Mona now calls her broker and puts in an order to sell 110 of the March 2013 Treasury bond futures contracts at the Chicago Board of Trade.

To see that the bank manager has indeed hedged the interest-rate risk on the $10 million of Treasury 10s of 2022 her bank is holding, let's see what happens if interest rates on both the futures contract and the 10s of 2022 Treasury bonds rise from 10% in March 2012 to 11% in March 2013. As we have seen, the rise in the interest rate from 10% to 11% would result in a decline in the price of the Treasury 10s of 2022 by 6.58% of par. Thus on the $10 million face value of bonds that the bank is holding, it would have a loss of $658,000 over the course of the year. Conversely, as we have seen, the rise in the interest rate for the futures contract from 10% to 11% would result in a decline in the futures contract's price by 5.98% of par, which is $5980 per $100,000 contract. Since the bank manager has sold 110 of these contracts short, the decline in price results in a profit of $657,800 (110 × $5980), which almost exactly offsets the loss on the bonds the bank is holding. (The hedge is not perfect—that is, the loss is slightly higher than the gain—because the manager can sell only a whole number of contracts.)

EXAMPLE 1 Micro Hedge

Suppose that in October 2012, a pension fund you are managing is holding $100 million face value of 8s of 2035 and you estimate that when interest rates rise from 12% to 13% in October 2013, the 8s of 2035 Treasury bonds you are holding would decline by 6.72% of par, or 6.72 points. In addition, the Treasury bonds futures contract would decline by 5.84%, or 5.84 points. You also calculated that when interest rates on the futures contracts change by 1 percentage point, the interest rate on the Treasury bonds changes on average by 2 percentage points. What is the hedge ratio for these Treasury bonds, and how many futures contracts should you sell?

Solution

The hedge ratio is 2.30, and you should sell 2300 Treasury futures contracts.

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af}$$

where

ΔP_a = change in the price of the hedged asset as a percentage of par in response to a 1% change in interest rate = 6.72

ΔP_f = change in the price of the futures contract as a percentage of par in response to a 1% change in interest rate = 5.84

β_{af} = average change in the interest rate of the hedged asset for a given change in the interest rate of the futures contract = 2

Thus

$$HR = \frac{6.72}{5.84} \times 2 = 2.30$$

Now solve for the number of futures contracts.

$$\text{Contracts} = HR \times \frac{PV_a}{PV_f}$$

where

HR = hedge ratio = 2.30

PV_a = par value of hedged asset = \$100 million

PV_f = par value of futures contract = \$100,000

Thus

$$HR = 2.30 \times \frac{\$100 \text{ million}}{\$100{,}000} = 2300$$

Macro Hedge

Instead of just hedging the Treasury bonds and other individual assets and liabilities of the First National Bank, the bank manager might decide that it would be better to try to hedge the entire balance sheet of the bank in one fell swoop. Recall from Chapter 24 that the First National Bank, which has \$100 million of assets, calculated that it had a duration gap of 1.72 years. If in March 2012 the bank manager sells \$100 million of March 2013 futures contracts whose underlying bonds also have an average duration of 1.72 years, then a rise in interest rates over the coming year, which would cause the value of the bank's net worth to fall, would be offset by the profits earned on the short position from selling the futures contracts. In other words, the macro hedge would be such that

$$V_f \times DUR_f = -V_a \times DUR_{gap} \tag{3}$$

where
V_f = value of the futures contracts
V_a = value of total bank assets
DUR_f = average duration of the underlying bonds in the futures contracts
DUR_{gap} = duration gap measurement for the bank

Solving for V_f, the value of the futures contracts, by dividing both sides of Equation 3 by DUR_f:

$$V_f = \frac{-V_a \times DUR_{gap}}{DUR_f}$$

If the duration of the deliverable bonds in the five-year \$100,000 Treasury note futures contract equaled 1.72 years, it is pretty straightforward for Mona, the bank manager, to calculate that she would need to sell just \$100 million of these futures contracts (= $-1.72 \times$ \$100 million/1.72). If these contracts were selling at the face value of \$100,000, Mona would call her broker and put in an order to sell 1000 March 2013 five-year Treasury note contracts.

To see that Mona has correctly hedged the interest-rate risk of the bank's portfolio, let's again look at what happens if interest rates rise from 10% to 11% from March 2012 to March 2013. Using the Equation 5 formula from Chapter 24, the change in the bank's net worth as a percentage of its assets is

$$\frac{\Delta NW}{A} = -DUR_{gap} \times \frac{\Delta i}{1 + i} = -1.72 \times \frac{0.01}{1.10} = -0.016 = -1.6\%$$

That is, when the interest rate rises by 1% from 10% to 11%, the First National Bank's net worth declines by \$1.6 million (1.6% of \$100 million of assets) to \$98.4 million. But we can calculate the percentage change in the price of the futures contract using the Equation 3 formula from Chapter 24 as follows:

$$\%\,\Delta P = -DUR_f \times \frac{\Delta i}{1 + i} = -\left(1.72 \times \frac{0.01}{1 + 0.10}\right) = -0.016 = -1.6\%$$

So when the interest rate rises by 1% from 10% to 11%, the price of the futures contract falls by 1.6%. Because the bank manager sold \$100 million of these contracts, the 1.6% decline in price results in a profit for the First National Bank of \$1.6 million (1.6% of \$100 million). The \$1.6 million gain on the futures contract exactly offsets the \$1.6 million decline in the bank's net worth from the rise in interest rates.

Unfortunately, the bank manager's job is not this easy because it is unlikely that she would find a futures contract whose underlying securities had a duration exactly equal to the bank's duration gap. To overcome this problem, the bank manager can mix futures contracts for bonds of different maturities into a \$100 million portfolio of futures contracts, making sure that the portfolio's duration exactly equals the bank's duration gap of 1.72 years.

EXAMPLE 2 Macro Hedge

If the duration of the Treasury futures contract is 3.44 years, rather than 1.72 years, what amount of these contracts does the bank manager need to sell to hedge the balance sheet of the bank?

Solution

The bank manager would need to sell \$50 million of futures contracts.

$$V_f = \frac{V_a \times DUR_{gap}}{DUR_f}$$

where

V_a = value of total bank assets = \$100 million

DUR_{gap} = duration gap measurement = 1.72

DUR_f = average duration in the future contracts = 3.44

Thus

$$V_f = \frac{\$100 \text{ million} \times 1.72}{3.44} = -\$50 \text{ million}$$

Hedging with Futures Options

Futures options are also particularly useful for offsetting risk created when the bank extends option-like commitments to certain bank customers. Banks sometimes make fixed-rate loan commitments to their customers, allowing customers to decide at their own discretion whether to borrow up to a certain amount from the bank at the specified fixed interest rate.[1] In effect, each such customer has been given the option to sell a bond to (borrow from) the bank at a given interest rate. Thus a loan commitment with a set interest rate is very similar to the bank's selling the customer a put option on bonds (an option to sell bonds). Because selling put options on bonds can expose the bank (the seller of the option) to substantial risk, the bank would like to hedge this risk by *buying* a put option that will cancel out the put option it has *sold*.

To see how this could be done, let's see how the bank manager might want to offset the risk created by a loan commitment to the First National Bank's good customer, Frivolous Luxuries, Inc. Suppose that in January, First National extends a \$2 million loan commitment to Frivolous Luxuries for a four-year loan at an interest rate of 7% and the commitment lasts for two months. Mona the Bank Manager knows that the four-year CD rate is currently 6%, which represents the cost of funds for the loan, so she figures that if interest rates remain the same as today's, the bank will have a comfortable profit margin of 1 percentage point on the loan. The problem is that Frivolous Luxuries is very likely to exercise the option provided by the loan commitment and take the loan if interest rates rise but is unlikely to take the loan if interest rates fall. If within the next two months interest rates rose by 2 percentage points, Frivolous Luxuries would almost surely take out the loan, and the First National Bank would be suffering a big loss because its cost of funds would be 1 percentage point higher than the interest rate on the loan.

Mona knows that to hedge this risk, she has to buy put options on a financial instrument whose interest rate moves closely with the rate on four-year CDs and whose expiration date is close to that on the loan commitment. She decides that March put options written on five-year Treasury note futures are her best bet. To decide on the number of put option contracts to buy, she goes through the same analysis she conducted when she carried out a micro hedge using futures. First she calculates the hedge ratio:

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af}$$

where
HR = hedge ratio
ΔP_a = change in the price of the hedged asset as a percentage of par in response to a 1% change in the interest rate
ΔP_a = change in the price of the futures contract as a percentage of par in response to a 1% change in the interest rate
β_{af} = average change in the interest rate of the hedged asset for a given change in the interest rate of the futures contract

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[1]Banks can also hedge loan commitments by having the interest rate on the loan tied to a market interest rate like the CD or T-bill rate. Variable-rate loan commitments of this type do not expose the bank to the interest-rate risk described in the text because the interest rate on the loan will move with the bank's cost of acquiring funds. Understandably, variable-rate loan commitments are more common than fixed-rate loan commitments.

She finds that the change in the price of four-year CDs in response to a 1% rise in the interest rate is 3.80% of par, and the change in the price of the five-year Treasury bond is 4.40% of par, giving a value for $\Delta P_a/\Delta P_f$ of 0.86 (= 3.80/4.40). Mona then estimates that the interest rate on four-year CDs on average changes by 1.05 percentage points when the interest rate on the Treasury notes futures contract changes by 1 percentage point. Her calculation of the hedge ratio is thus

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af} = 0.86 \times 1.05 = 0.90$$

To calculate the number of contracts, she again uses the formula

$$\text{Contracts} = HR \times \frac{PV_a}{PV_f}$$

where HR = hedge ratio (0.90 in our example)
PV_a = par value of the asset hedged ($2 million of the four-year CDs)
PV_f = par value of the futures contract ($100,000 per five-year Treasury note futures contract)

In our example,

$$\text{Contracts} = 0.90 \times \frac{\$2 \text{ million}}{\$100{,}000} = 0.90 \times 20 = 18$$

Choosing the strike price to be close to the current price, Mona completes the hedge by purchasing 18 put option contracts on five-year Treasury notes with a premium of, say, $2000 per contract. Thus for a total cost of $36,000 (= 18 × $2000), she has hedged the First National Bank's loan commitment. If interest rates rise and the loan commitment is exercised, the profits on the put option the bank manager has bought will offset the loss on the loan.

If interest rates fall, the bank manager will not exercise the options, and so the bank will not be exposed to any additional losses. Here we can see the advantage of hedging the loan commitment with put option contracts rather than selling futures contracts. If Mona had sold futures contracts to hedge the loan commitment, the decline in interest rates would have produced losses on the futures contracts. However, because interest rates have fallen, it is likely that Frivolous Luxuries will decide not to take the loan offered under the commitment because it will not want to pay the high 7% interest rate. In this case, the First National Bank would have a loss on the futures but would not have an offsetting gain in profits from making the loan. Clearly, if the bank manager put the bank into this unhappy situation, she might get fired. Using futures options rather than futures contracts prevents this situation from occurring and is another reason why financial institution managers find hedging with futures options so attractive.

EXAMPLE 3 Hedging with Futures Options

Suppose that the pension fund you are managing is holding \$60 million of five-year bonds, and if their interest rates fall by 1%, the bonds rise in price by 4.3 points. When the interest rates fall by 1%, the five-year Treasury bond contract rises by 5.7 points. The interest rate on five-year Treasury bonds on average changes by 1.30 when the interest rate on the Treasury bond futures contract changes by 1 point. The value of the future contract is \$125,000 per five-year Treasury bond. What should you do in the option market to hedge the interest-rate risk on the \$60 million of five-year bonds?

Solution

To hedge the interest-rate risk, you should purchase 470 put option contracts on five-year Treasury bonds.

$$HR = \frac{\Delta P_a}{\Delta P_f} \times \beta_{af}$$

where

ΔP_a = change in the price of the hedged asset as a percentage of par in response to a 1% change in interest rate = 4.3

ΔP_f = change in the price of the futures contract as a percentage of par in response to a 1% change in interest rate = 5.7

β_{af} = average change in the interest rate of the hedged asset for a given change in the interest rate of the futures contract = 1.30

Thus

$$HR = \frac{4.3}{5.7} \times 1.30 = 0.7544 \times 1.30 = 0.98$$

Now solve for the number of contracts.

$$\text{Contracts} = HR \times \frac{PV_a}{PV_f}$$

where

HR = hedge ratio = 0.98

PV_a = par value of the asset hedged = \$60 million

PV_f = par value of the futures contract = \$125,000

Thus

$$\text{Contracts} = 0.98 \times \frac{\$60 \text{ million}}{\$125{,}000} = 0.98 \times 480 = 470.4$$

Hedging with Interest-Rate Swaps

We have already seen how the Midwest Savings Bank and the Friendly Finance Company can hedge interest-rate risk using interest-rate swaps, but to make our understanding of hedging with interest-rate swaps even more concrete, let's return again to the bank manager's hedging problem. Recall from Chapter 24 that the First National Bank has \$32 million of rate-sensitive assets and \$49.5 million of rate-sensitive liabilities. The bank thus has a gap of −\$17.5 million, and, as we saw in Chapter 24, if interest rates rise by 5 percentage points, the change in bank income is 5% × −\$17.5 million = −\$0.9 million.

How large an interest-rate swap does Mona the Bank Manager have to arrange to hedge this interest-rate risk and prevent the decline in profits when interest rates rise? The answer is straightforward: Mona has to arrange an interest-rate swap in which she exchanges income on \$17.5 million of rate-insensitive assets for income on \$17.5 million of rate-sensitive assets. Then the rate-sensitive income will in effect be on \$49.5 million of rate-sensitive assets (= \$32 million + \$17.5 million), which exactly matches the \$49.5 million of rate-sensitive liabilities, so $GAP = 0$. Now when interest rates rise by 5 percentage points, the income on the rate-sensitive assets will rise by 5% × \$49.5 million = \$2.5 million while the cost on the rate-sensitive liabilities will rise by the same 5% × \$49.5 million = \$2.5 million. The net result is that profits and the net interest margin do not change, and the hedge is successful.

Instead of using interest-rate swaps to eliminate interest-rate risk for the bank's income, the bank manager could have decided to hedge interest-rate risk for the bank's net worth. Suppose that the Friendly Finance Company offers Mona's bank the same interest-rate swap it offered Midwest Savings: The First National Bank would receive interest payments of 1% plus the one-year Treasury bill rate over the next ten years in exchange for a 7% fixed-rate payment. How much notional principal of this swap should the bank manager agree to if she wants fully to hedge the interest-rate risk on the bank's net worth?

Mona's first step is to calculate the effective duration of the interest-rate swap she is being offered. Because the one-year Treasury bill is a pure discount bond, she knows that its duration is simply one year regardless of the interest rate. Thus the duration of the 1% plus the one-year Treasury bill rate interest payment is one year. She calculates the duration of the 7% fixed-rate payment made over the next ten years to be 8.1 years. Since the interest payments are made on the same notional principal, the duration of the swap is simply the duration of the payments her bank receives (the asset) minus the duration of the payments her bank makes (the liability), which in this case equals 1 − 8.1 = −7.1.

Just as in the macro hedge she conducted using futures, Mona knows that she wants a rise in interest rates to cause the value of the interest-rate swap to rise by exactly the same amount as the bank's net worth would fall, thereby offsetting this decline. That will occur when the notional principal of the swap multiplied by the swap's duration is the same as the bank's assets multiplied by the duration gap:

$$V_s \times DUR_s = -V_a \times DUR_{gap} \tag{4}$$

where
V_s = notional principal of the swap
V_a = value of total bank assets
DUR_s = duration of the swap
DUR_{gap} = duration gap measure for the bank

Dividing both sides by DUR_s gives the formula for V_s, the notional principal of the swap:

$$V_s = \frac{-V_a \times DUR_{gap}}{DUR_s} \tag{5}$$

Since the bank manager's earlier duration gap analysis revealed that the bank's duration gap is 1.72 years on \$100 million of assets and she has calculated the duration of the swap to be −7.1 years, Mona plugs these numbers into the formula in Equation 4 to get

$$V_s = \frac{-V_a \times DUR_{gap}}{DUR_s} = \frac{-\$100 \text{ million} \times 1.72}{-7.1} = \$24.2 \text{ million}$$

She contacts the Friendly Finance Company, and they agree to the swap for \$24.2 million of notional principal.

To check that she has done her calculations correctly, Mona now goes through the thought experiment that we went through earlier in the chapter for assessing what happens if interest rates rise by 1 percentage point, from 7% to 8%. Recall that the change in net worth as a percentage of assets would be

$$\frac{\Delta NW}{A} = -DUR_{gap} \times \frac{\Delta i}{1+i} = -1.72 \times \frac{0.01}{1+0.07} = -0.016 = -1.6\%$$

The 1.6% decline in net worth on the \$100 million of assets thus translates into a \$1.6 million decline in net worth. Using the Equation 3 formula from Chapter 24, Mona determines that the percentage change in the value of the swaps is

$$\%\Delta P = -DUR_s \times \frac{\Delta i}{1+i} = -(-7.1) \times \frac{0.01}{1+0.07} = 0.066 = 6.6\%$$

When this gain of 6.6% is multiplied by the \$24.2 million notional principal of the swaps, Mona sees that she will have a gain of \$1.6 million in the value of the swaps. Knowing that the decline in the bank's assets minus its liabilities is exactly matched by the increase in the value of the swaps, Mona now takes comfort in her knowledge that the value of the bank is fully protected from changes in interest rates.

EXAMPLE 4 Hedging with Interest-Rate Swaps

Suppose that you are offered a 12% fixed-rate payment over the next 15 years. The duration of the swap is 26.4. You are managing a firm with \$3.7 million of assets and a duration gap of 1.7 years. How much notional principal would you want the swap to have?

Solution

The swap should have \$982,812.50 of notional principal.

$$V_s = \frac{-V_a \times DUR_{gap}}{DUR_s}$$

where

V_a = value of total bank assets = \$3.7 million

DUR_{gap} = duration gap = 1.7

DUR_s = duration of the swap = 26.4

Thus

$$V_s = \frac{-\$3.7 \text{ million} \times 1.7}{-6.4} = \$982{,}812.50$$

Hedging with Stock Index Futures

Financial institution managers use stock index futures contracts to cope with stock market risk in two principal ways: to reduce systematic risk and to lock in stock prices. Let's first look at how the portfolio manager for the Rock Solid Insurance Company would go about hedging against systematic risk in the company's portfolio.

Reducing Systematic Risk

Even if a portfolio of assets is fully diversified, there still remains a component of risk, called *systematic risk*, that cannot be diversified away. Systematic risk is measured by a concept called *beta*, a measure of the sensitivity of the portfolio's return to changes in the value of the entire market of assets. When on average a 1% rise in the value of the market leads to a 2% rise in the value of a portfolio, the beta of this portfolio is calculated to be 2.0. Conversely, if the value of a portfolio on average rises by only 0.5% when the market rises by 1%, the asset's beta is 0.5. If we take the market to be represented by the S&P 500 Index, a broad measure of how well the market is doing, then the beta of a portfolio can be measured using statistical methods to determine how much on average the portfolio's value changes for a 1-percentage-point change in the S&P index.

Suppose that in March 2012, Mort, the portfolio manager, does this statistical calculation and finds that Rock Solid's portfolio of \$100 million of stocks on average moves percentagewise one-for-one with the S&P index and so has a beta of 1; that is, if the value of the S&P index changes by 10%, the portfolio value changes by 10%. Suppose also that the March 2013 S&P 500 Index contracts are currently selling at a price of 1000. How many of these contracts should Mort sell so that the beta of the combined portfolio, including the stock portfolio and the futures contracts, is equal to zero and hence Rock Solid is not exposed to any systematic risk over the coming year?

Because the beta of his stock portfolio is 1 (its value changes in exact proportion to the change in the S&P 500 Index), this calculation is quite easy. To immunize his portfolio against systematic risk, Mort must sell \$100 million of S&P index futures, thereby agreeing to a delivery amount due of \$250 times the S&P index in March 2013. At a price of 1000 (\$250,000 per contract), Mort sells \$100 million/\$250,000 = 400 contracts. If the S&P index falls 10% to 900, on average the \$100 million portfolio will suffer a \$10 million loss. At the same time, however, Mort makes a profit of 100 × \$250 = \$25,000 per contract because he agreed to receive \$250,000 for each contract when the price was originally at 1000, but at a price of 900 on the expiration date he has a delivery amount due of only \$225,000 (900 × \$250). Multiplied by the 400 contracts, the \$250,000 profit per contract yields a total profit of \$10 million. The \$10 million profit on the futures contracts exactly offsets the loss on Rock Solid's stock portfolio, so the portfolio manager has been successful in hedging the stock market risk due to overall market swings.

If Mort's calculations reveal that Rock Solid's portfolio has a beta of 2—meaning that it has twice as much systematic risk as the market—then selling 500 contracts of the S&P stock index futures will not eliminate the systematic risk of the portfolio. With a beta of 2, if the S&P index goes down by 10% to 900, then on average the \$100 million portfolio would suffer a 20% (\$20 million) loss. As before, the 400 futures contracts would yield a profit of \$10 million, but this profit would offset only half of the \$20 million loss on the portfolio, for a net loss of \$10 million.

Because Mort is a smart guy, he realizes that all he has to do to prevent this loss is sell twice as many futures contracts. In other words, if he sells 800 futures contracts, when the S&P index goes down by 10% to 900, Rock Solid will have a profit of 100 × \$250 = \$25,000 per contract, which multiplied by the 800 contracts yields a total profit of \$20 million. Once again, the hedge position in S&P 500 Index futures provides a profit that exactly offsets the average loss on the portfolio due to overall market swings.

The portfolio manager has discovered that to hedge the systematic risk of a stock portfolio, the number of futures contracts sold must be adjusted proportionally to the beta of the portfolio. The following formula reveals the number of contracts that must to be sold to hedge systematic risk:

$$\text{Contracts} = \beta \times \frac{\text{value of portfolio}}{\text{value of contract}} \tag{6}$$

We have now seen how hedging with stock index futures can immunize a portfolio from a decline in the overall market, but one consequence is that when the overall market rises, the company will not reap the profits. A rise in the overall market by 10% will on average produce a 20% increase in Rock Solid's portfolio if its beta is 2, for a profit of \$20 million, but the \$20 million loss on the 800 futures contracts the portfolio manager has sold (= 800 × \$25,000 loss per contract) will yield a net profit of zero.

Why would the portfolio manager be willing to forgo profits when the stock market rises? The first reason is that he might be worried that a bear market was imminent and so wants to protect Rock Solid's portfolio from the coming decline. This feature of the stock index futures hedge is one reason why this type of hedging has been dubbed *portfolio insurance*. The second reason is that the portfolio manager may feel that he is particularly good at picking individual stocks that will do well but wants to minimize the risk due to overall swings in the market. With a stock index futures hedge, if he has been successful at picking good stocks that perform better than the market, his results will still look good even if there has been a sharp decline in the overall market.

EXAMPLE 5 Reducing Systematic Risk

Suppose that the stock portfolio you are managing has a value of $50 million with a beta of 1.6. The S&P 500 Index futures contracts are currently selling for 375. How many contracts would you have to sell to hedge the systematic risk of your portfolio?

Solution

To hedge the systematic risk of the portfolio, you would have to sell 534 contracts.

$$\text{Contracts} = \beta \times \frac{\text{value of portfolio}}{\text{value of contract}}$$

where

$$\beta = \text{beta} \qquad = 1.6$$

$$\text{Value of portfolio} = \$50 \text{ million}$$

$$\text{Value of contract} = 1100 \times \$250 = \$275{,}000$$

Thus

$$\text{Contracts} = 1.6 \times \frac{\$50 \text{ million}}{\$275{,}000} = 290.91 \approx 291$$

Locking in Stock Prices

Suppose that the portfolio manager knows that his company will receive an inflow of funds in the future that have to be invested and believes that a stock market boom is imminent. In this case, he would like to be able to lock in the stock prices at which he will invest these funds in the future at current levels. Although he cannot do this for individual stocks, he can use stock index futures to do this for the overall market.

To understand this use of stock index futures, let us suppose that in January, Mort is informed by his boss that Rock Solid's insurance agents have been doing such a great job selling insurance recently that an additional $20 million of cash payments from insurance premiums will arrive at the firm in March. If the price of the March S&P index contract is 1000 and the portfolio manager expects that the S&P index will rise by 5% to 1050 by March, he can lock in the price of 400 on the $20 million by taking a long position and purchasing $20 million of S&P 500 Index futures. Since each contract is selling for $250,000 (1000 × $250), the portfolio manager will put in an order to purchase 80 contracts (= $20 million/$250,000). With this purchase of futures contracts, the portfolio manager has in effect assured his company that he can buy the same number of shares with the $20 million coming in March that he could when the overall market was at the level represented by an S&P index price of 1000. If stock prices go up 5% as expected with the S&P 500 Index rising to 1050, the portfolio manager has a profit of $12,500 per contract because he has agreed to pay $250,000 per contract but has a cash payment due of $262,500 (1050 × $250). Multiplying the $12,500 profit per contract times the 80 contracts he bought yields a total profit of $1 million. When the $20 million in premiums arrives in March, Mort now has $21 million to invest—the $20 million in premiums plus the $1 million in profit. Even though the same amount of shares that Mort would have bought for $20 million will now cost 5% more, or $21 million, he is able to buy them because he has $21 million as a result of his futures contract purchase.